NEW SCHOOL ARITHMETIC

BOOK TWO

JONES *and* ROGERS

$X - 5 = 10$

$X = 10 + 5$

$X = 15 \text{ anos}$

BOOK

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ARITHMETIC

BOOK TWO

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COLUMBIAN BOOK COMPANY

NEW SCHOOL ARITHMETIC

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PREFACE

While the authors have assumed that students entering the sixth grade have acquired reasonable facility in ordinary computations, they have included in this volume a more extended treatment of each of the four fundamental operations with ample provision for application and drill. Since success in all arithmetical work is directly dependent upon efficiency in these fundamentals, the authors feel that the space and time given to these fundamentals are justified.

Practically all the other topics included in this volume are those that have found secure places in text-books of similar grade in general use. In addition, however, special attention is given to a few features which seem to demand place in a modern text-book on arithmetic.

First, a somewhat extended study of graphs is deemed necessary because of the use being made of them in general publications. Not only technical journals but magazines of all classes, as well as newspapers, are employing graphs for purposes of comparison in one of the several forms presented in this volume. The mathematics of the graphs here treated is sufficiently elementary to be mastered easily by the pupils in the upper grammar grades.

Second, the equation has long been recognized as the most powerful instrument for problem solving, and yet in the field of elementary arithmetic its value has not only been ignored, but unnatural and forced processes have been employed in order to avoid the natural method based upon the equation. The authors have allowed the formula and equation to perform their logical functions as they have evolved in the regular work in mensuration and in problems requiring the indirect process.

Third, a few new applications of percentage seem to demand attention in a modern text-book on arithmetic. The interest developed in stocks and bonds through thrift and war stamps, liberty bonds, etc., vitalizes an old and uninteresting topic almost to the extent of making it new. While most of the mathematics of the Federal Farm Loan Associations is embodied in prepared tables, the manipulation of these tables and the informational side of the subject are important and are given proper space. Phases of farming, commerce, and industry not always included in a text-book are treated not only for the arithmetic involved but also for the general informational value to the student.

Fourth, the authors have attempted a combination of the types of text-books represented on the one hand by the extreme spiral organization, and on the other by the rigid topical organization. To avoid the too frequent changes of topics resulting often in mere confusion, and to eliminate the long grind of holding the

young mind to a complete mastery of each topic before introducing a psychologically correlated topic have been the aims of the authors. To illustrate: the topic of percentage is introduced in an elementary form in the latter part of the fifth grade in its natural connection with decimal fractions; it is treated in more detail again in the sixth grade; and its applications are emphasized in the seventh and eighth grades.

Fifth, since many children leave school by the end of the fifth year, the essentials of arithmetic have been given early with a view to providing the child with such arithmetical skill as will meet the usual demands outside the schoolroom and at the same time to preparing the child who remains in school for continuous development of his mathematical powers.

Sixth, in the lower grades the purpose has been to provide the child with the tools of arithmetical processes; that is, to habituate the fundamental operations with integers, fractions, and denominate numbers; while in the upper grades the purpose has been to develop by means of industrial and economic applications the thinking and judging ability of the child. Ample provision for the first is found in motivated drills and simple applications; while problems with complete data gathered from the child's personal or reading experience, as well as problems with incomplete data to be completed by the child, provide natural and interesting means of developing judgment and reason.

R. L. J.
C. E. R.

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PART I

1. Review of Fundamental Operations

NOTATION AND NUMERATION

A unit is a single thing of any kind or a group of things considered as a whole, by which any other quantity of the same kind is measured; as one dollar, one horse, one apple, one pound, one dozen, the number 1.

A number is a unit or a collection of units; as one man, ten dollars, fifty bushels, three dozen, the number 6.

Notation is a system of writing numbers by the use of figures or letters.

The Arabic notation expresses numbers by the use of the following figures: 1, 2, 3, 4, 5, 6, 7, 8, 9, 0.

Each of these figures represents a number, except 0 which is used only to help other figures express numbers; thus, 1 and 0 written together (10) represent the number ten.

The number represented by a figure standing alone is its *simple value*. Which figure has the largest *simple* value? Which the smallest? Which has no value? When more than one figure is used to represent a number, the value of each figure, in addition to its *simple value*, is determined by the *place* it occupies. Which figure represents the largest *place value* and which represents the smallest place value in each of the following numbers: 675, 576, 765, 196, 961, 619, 1867, 8176?

For convenience in reading numbers, the figures are generally separated by the comma into periods of three figures each; thus, 4,165,384. The first period counting from the right is called units' period, the second thousands' period, the third millions', the fourth billions', the fifth trillions', etc.

Trillions	Hundred-billions Ten-billions Billions	Hundred-millions Ten-millions Millions	Hundred-thousands Ten-thousands Thousands	Hundreds Tens Units
9	270	084	756	802
5th	4th	3d	2d	1st
Trillions	Billions	Millions	Thousands	Units

The number represented above is read: *nine trillion, two hundred seventy billion, eighty-four million, seven hundred fifty-six thousand, eight hundred two.*

In reading whole numbers do not use *and* between periods.

2. Reading Numbers

Point off into periods and read each of the following numbers:

	a	*b*	*c*	*d*
1.	5684	703786	870024506	9083265898
2.	89654	9786018	1001010001	9006086401
3.	2905468	891000105	9086432108	87000054300
4.	8653878	1905603	190004387	52000538
5.	47000000	84007006	73869281	86300869
6.	8005002	983467	8000009	380008416
7.	93456780	542821	93002008	76000056
8.	5389001	2345678	10897342	58043007
9.	8003020	1001001	8043206	700200002
10.	10010010	3030030	50005005	25025025

3. Writing Numbers

Write in words the following numbers:

1.	5000000	**2.**	57870006	**3.**	98000040
4.	10005003030	**5.**	201018645	**6.**	738145237801

What is the largest number that can be expressed by two figures?

What is the smallest number that can be expressed by two figures?

What is the smallest number that can be written with three figures?

How much larger is the smallest number that can be written with three figures than the largest number that can be written with two figures?

Write the smallest numbers that can be expressed by the figures 2, 0, 1. Write the largest number that can be expressed by the figures 2, 7, 9. Write three numbers expressed by the figures 4, 5, 6.

Write:

1. 301 billion, 10 million, 148 thousand, 325.

2. Twenty billion, one hundred twenty-five million, four hundred seventeen thousand, six hundred four.

3. One hundred fifty billion, ten million, nine hundred seventy-eight thousand, two hundred one.

4. Nine trillion, nine billion, nine million, nine thousand, nine.

5. Four hundred eighty-four million, nine hundred twenty-five thousand, four hundred seventy-nine.

6. Eighty-five billion, seventy-five million, sixty-five thousand, sixty-five.

7. One hundred eighty-five billion, one hundred eighty-five.

8. Ten billion, ninety million, ninety-five.

9. Forty-seven million, seventeen thousand, seventy-seven.

10. One trillion, ten million, forty thousand, forty.

11. Two hundred one million, forty thousand, forty-nine.

12. Nine hundred billion, seventy-five thousand.

13. One billion, ten million, six.

4. Roman System of Notation and Numeration

The Roman notation employs the following seven capital letters to represent numbers:

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

The other numbers are represented by combinations of letters in accordance with the following laws:

I. Repeating a letter repeats its value; thus XX represents 20, CC represents 200.

II. When a letter is followed by one of less value, the values of the letters are added; thus VI represents 6, LX represents 60.

III. When a letter is followed by one of greater value, the value of the smaller is subtracted from that of the greater; thus IV represents 4, XL represents 40.

IV. When a letter is placed between two letters of greater value, the least is subtracted from the sum of the other two; thus XIX represents 19, XXIV represents 24.

V. Placing a bar over a letter multiplies its value by 1000; thus, $\overline{M}$ represents 1,000,000.

Read the following:

I	XI	XXX	C	M
II	XII	XXXV	CC	MC
III	XIII	XL	CI	MCC
IV	XIV	L	CX	MCM
V	XV	LX	CD	MCMXX
VI	XVI	LXX	D	MDCCXC
VII	XVII	LXXV	DC	MCMXVII
VIII	XVIII	LXXX	DCC	$\overline{V}$
IX	XIX	XC	DCCC	$\overline{X}$
X	XX	XCIX	CDD	$\overline{L}$

Write in Roman notation: 31, 45, 67, 81, 95, 110, 125, 275, 510, 687, 995, 1907, 1920, 1492, 2000.

5. U. S. Money

In writing dollars and cents a decimal point must be placed after dollars and before cents. Thus \$7.25 is read: *seven dollars and twenty-five cents.* If cents are written as a decimal part of a dollar, the dollar sign must precede the decimal point. Thus 25 cents is written \$.25 or \$0.25. Cents occupy the first two places at the right of the decimal point; 5 cents is written \$.05 or \$0.05.

Read: \$0.07, \$0.55, \$4.75, \$8.19, \$7.00.

Change to dollars and cents: 35¢, 95¢, 875¢, 1005¢.

Write: five dollars and five cents; eight dollars and ten cents; three hundred dollars and three cents.

6. ADDITION

Henry had 5 thrift stamps and bought 4 more. How many thrift stamps had he then?

The process of uniting two or more numbers into one whole is called **addition.**

The result obtained by adding two or more numbers is called the **sum** or **amount.**

The sign of addition is a perpendicular cross (+). It is read *plus* and indicates that the numbers between which it is placed are to be added.

Read and announce the sum at sight:

1+1=2	9+9=18	18+2=20	17+ 7=24
7+7=14	8+6=14	17+3=20	16+10=26
2+1=3	6+9=15	17+5=22	15+ 7=22
4+6=10	6+6=12	14+8=22	27+ 4=31
1+3=4	5+6=11	19+3=23	16+16=32
2+5=7	8+8=16	17+8=25	12+12=24
2+2=4	5+4=9	16+7=23	15+15=30
3+4=7	7+2=9	17+9=26	14+ 7=21

4+1=	5+3=	15+9=	7+11=
7+1=	4+4=	19+5=	10+ 4=
1+5=	1+8=	13+8=	13+12=
3+6=	8+9=	2+9=	19+ 6=
3+2=	5+5=	12+9=	13+10=
3+3=	6+1=	14+9=	15+ 9=

Oral Drill

1. Add by 5's from 3 to 103; from 7 to 107.
2. Add by 6's from 2 to 104; from 5 to 107.
3. Add by 7's from 5 to 103; from 6 to 104.
4. Add by 8's from 7 to 103; from 5 to 101.
5. Add by 9's from 8 to 107; from 7 to 106.

Is the sum of two or more numbers the same if the order in which they are added is changed?

Test your answer by comparing the sum of:

2+4+6= 4+6+2= 6+4+2=

What is the sum of \$2+\$5?

Can you find the sum of \$2+2 apples? Why?

Only like numbers can be added.

7. Problems

1. A farmer grew 565 bushels of wheat in one field, 384 bushels in another, and 786 bushels in another. How many bushels did he grow in all?

565
384
786
1735

Direction:

Place the numbers so that the figures will form a vertical line, units under units, tens under tens, etc. Begin at the right, look up the column and think only added combinations, thus, 6, 10, 15. Write 5. Add 1 to tens' column. 9, 17, 23. Write 3. Add 2 to hundreds' column. 9, 12, 17. Write 17. The answer is 1735 bu.

The numbers to be added are called **addends.**

2. In one year a man paid for rent $625, for clothes $275, for groceries $427, for telephone $25, for electric lights $21, for insurance $275. How much did he pay in all?

3. A dealer, after selling 24 gallons of gasoline to one customer and 36 gallons to another, had 75 gallons left. How many gallons had he at first?

4. After traveling 196 miles the first day and 216 miles the second day, a tourist was 248 miles from his point of destination. How far from the place of starting to his destination?

5. A has $87, B has $29 more than A, C has $50 more than B. How many dollars have they all?

6. President Wilson was 40 years old in 1896. How old was he 24 years later?

7. A tank has two pipes. The first discharges 89 gallons a minute, the second 13 gallons more a minute than the first. How many gallons will both pipes discharge in a minute?

8. John has $96 in thrift certificates, Henry has $32 more than John, and James has $28 more than Henry. How many dollars in thrift certificates have they all?

9. It is 152 miles from Nashville to Chattanooga, and Atlanta is 136 miles beyond Chattanooga. How far is it from Nashville to Atlanta?

10. Mr. Brown owned four farms. In the first there were 219 acres, in the second 175 A., in the third 97 A., in the fourth 396 A. How many acres in all?

11. Texas has an area of 265,896 square miles; Oklahoma, 70,057 sq. mi.; Louisiana, 48,506 sq. mi.; Arkansas, 53,335 sq. mi.; Mississippi, 46,865 sq. mi.; Alabama 51,998 sq. mi.; Tennessee, 42,022 sq. mi.; Kentucky, 40,958 sq. mi. What is the area of all the above states?

12. The population of Memphis in 1920 was 162,351; Atlanta, 200,616; Charleston, S. C., 67,957; Birmingham, 178,806; Jackson, Miss. 22,817; New Orleans, 387,219. What was the total population of these cities?

13. Raleigh in 1920 had a population of 24,418; Winston-Salem had a population of 23,977 more than Raleigh. What is the population of both cities?

14. Houston in 1920 had a population of 138,276; Dallas had a population of 20,700 more than Houston; San Antonio had a larger population than Dallas by 2,403. Find the total population of the three cities.

8. Practice Exercises

NOTE: In all arithmetical operations two things are to be attained, accuracy and rapidity. Time yourself in the examples below and see how quickly you can solve each.

1.	2.	3.	4.
78975643	84387059	9999999	76964307
84383267	93267854	8888888	67894021
95867931	84326751	7777777	32897654
69032009	54328709	6666666	43209786
58643287	64789326	5555555	58973209

5.	6.	7.	8.
8976321	74286789	98732107	29876509
9654008	54321765	70184567	92768408
8579329	62789460	65432098	89654387
9543287	89765321	59342875	78900543
1987602	54847989	48736875	48678306

9.	10.	11.	12.
89783207	78403927	19867508	68887964
543209	95603879	91312456	987
8764328	32874129	86786549	3201
529	58936878	56789031	84678
98764329	9856721	98594321	986548

13.	14.	15.	16.
88695478	76540872	39876534	38976542
37689542	67504783	93785643	79385679
10987654	32456789	34658739	58937642
89789854	98763210	37856496	98732078
95897863	29843798	87654032	76342796

17.	18.	19.	20.
78901234	43201987	87984318	48888757
98710567	19870346	65438978	84875896
32098763	84329657	56348743	54798768
35678989	97389578	21054328	39856732
84676547	57834687	87643972	28793254

21.	22.	23.	24.
$5865.27	$4768.28	$3997.65	$5786.95
$6878.40	$5486.37	$9378.79	$6374.19
$9843.27	$4321.15	$7246.45	$3987.65
$7653.00	$2976.95	$8439.25	$1796.97

9. Oral Drill

Name sum at sight:

8+ 7=	18+ 7=	6+ 5=	16+ 5=
3+ 2=	13+12=	4+ 5=	4+15=
6+ 9=	16+ 9=	16+19=	8+17=
8+17=	33+33=	23+23=	25+27=
9+ 4=	19+ 4=	19+14=	7+ 7=
17+ 7=	17+17=	6+ 8=	16+ 8=
9+19=	19+19=	6+ 6=	16+ 6=
6+ 8=	16+ 8=	16+18=	25+25=
7+ 7=	17+ 7=	17+17=	38+38=
9+ 9=	19+ 9=	19+19=	50+50=
2+ 5=	12+ 5=	12+15=	40+30=

3+ 5=	13+ 5=	13+15=	60+40=
4+ 4=	4+14=	14+14=	70+30=
6+ 4=	16+ 4=	16+14=	35+45=
6+ 5=	16+ 5=	16+15=	45+45=
6+ 7=	16+ 7=	16+17=	33+33=
7+ 2=	17+ 2=	17+12=	60+35=
7+ 3=	17+ 3=	17+13=	37+37=
7+ 4=	17+ 4=	17+17=	27+27=
7+ 5=	5+17=	17+15=	27+25=
6+ 7=	17+ 6=	17+16=	27+16=
7+ 9=	19+ 7=	19+17=	29+17=
9+ 8=	9+18=	19+18=	29+28=
9+ 3=	19+ 3=	19+13=	29+13=

To test addition, begin at the top of the right hand column, add the columns downward, and see if the results agree. Another test is to begin at the bottom of the right hand column and write the entire sum of each column, placing the right hand figure in each sum one place further to the left, and then add the sums of the columns; then begin at the top of the left hand column and write the entire sum of each column, placing the right-hand figure in each sum one place further to the right, and add the sums of the columns.

Illustration

Begin at the top of left hand column, add and write result thus:

29	89876	
30	58438	
18	79328	
18	94361	
23	322003	
322003		

Begin at bottom of right hand column, add and write result thus:

		23
		18
		18
		30
		29
		322003

Since the results all agree, the work is assumed to be correct.

Since only digits of the same order can be added, units must be written under units, tens under tens, etc., forming vertical columns.

Write in columns, add, and test:

1. 25+7867+407+9+8967+84.
2. 8687+940+83+7+100287+65+9029.
3. 10374+18653+5423+342+34+6.
4. 98876+703+47+8467+87428+10.
5. 78765+8765+765+65+50219+967.
6. 67402+20400+359+64+5227+10824.
7. 57984+1867+3246+34+10+94627.
8. 47857+7865+90+876+16+87983.

10. SUBTRACTION

James had 14 marbles, and gave Thomas 6. How many had he left?

Subtraction is the process of finding the difference between two numbers. The number from which the subtraction is made is called the **minuend.** The number subtracted is called the **subtrahend.**

The result obtained by subtracting the subtrahend from the minuend is called the **remainder** or **difference.**

The sign, (−), of subtraction is called **minus.** It indicates that the number before which it is placed is to be subtracted from the number preceding it.

Thus, \$9−\$5 indicates that \$5 is to be subtracted from **\$9.**

Drill

Subtract by 5's from 101 to 1.
Subtract by 4's from 102 to 2.
Subtract by 3's from 100 to 1.
Subtract by 7's from 100 to 2.
Subtract by 8's from 103 to 7.
Subtract by 9's from 105 to 6.

Subtract by 8's from 100 to 4.

Subtract by 9's from 100 to 1.

Read and give the remainder at sight:

15−7=	17− 8=	17− 9=	16− 7=	18− 9=
16−9=	26−19=	15− 8=	25−18=	13− 8=
23−8=	14− 9=	24−19=	14− 7=	24−17=
13−6=	23−16=	14− 5=	24−15=	30−20=
9−6=	19−16=	25−16=	26−19=	35−25=
11−6=	21− 6=	21−16=	12− 9=	22− 9=
15−6=	25−16=	18− 9=	28−19=	99−50=

Does it change the remainder if you add the same number to the minuend and subtrahend before performing the subtraction? Test your answer.

11. Written Exercises

From 815 subtract 326.

815
326
489

This method of subtraction, which is called the Austrian method, is based on the principle that equal numbers may be added to both minuend and subtrahend without changing the result, and that the sum of the subtrahend and remainder equals the minuend. We must find a number which added to the subtrahend will make the minuend. As 6 is a larger number than 5, there is no number that could be added to 6 to make 5. Hence, we add 10 to 5 which makes 15. Since 9 added to 6 makes 15, write 9 as the units' figure of the remainder. As 1 ten was added to the minuend. we must add 1 ten to the figure 2 in subtrahend which makes 3 tens. As no number can be added to 3 to make 1, we must add 10 tens to 1 ten, which makes 11 tens. The number to be added to 3 tens to make 11 tens is 8 tens, therefore write 8 tens in the remainder. As 10 tens, which equal 1 hundred, were added to the minuend, 1 hundred must be added to the 3 hundreds in the subtrahend, which makes 4 hundreds. As 4 added to 4 makes 8, write 4 for the hundreds' figure in the subtrahend.

815
326
489

In solving examples in subtraction you may do it rapidly, thus: 9+6 equals 15, write 9; 8+3 equals 11, write 8; 4+4 equals 8, write 4.

To check, add the subtrahend and remainder. If the sum is the minuend, the work is assumed to be correct.

2.	876543 538796	**3.**	529865 430767	**4.**	756899 535497
5.	968734 359685	**6.**	643860 281954	**7.**	105760 94327
8.	958432 547524	**9.**	376497 267398	**10.**	267511 128423
11.	128487 103568	**12.**	263564 158473	**13.**	700201 127010
14.	777777 387945	**15.**	183521 64008	**16.**	800000 526432
17.	$5867.90 634.25	**18.**	$7598.76 6753.84	**19.**	$4896.00 3789.25
20.	$95876.10 83957.15	**21.**	$9481.17 893.38	**22.**	$5000.11 500.22

12. Another Method

From 836 subtract 247.

836 = 7 hundred + 12 tens + 16 units
247 = 2 hundred + 4 tens + 7 units
589 = 5 hundred + 8 tens + 9 units = 589.

7 units cannot be subtracted from 6 units; hence, we take 1 ten from the 3 tens and add it to 6 units, which makes 16 units. 7 from 16 leaves 9. As 4 tens cannot be subtracted from 2 tens, we take 1 hundred, which is equal to 10 tens, and add it to 2 tens, making 12 tens. 4 tens from 12 tens leave 8 tens. 2 hundred from 7 hundred leave 5 hundred.

836
247
589

In practice we say: 7 from 16 leaves 9, and 4 from 12 leaves 8, and 2 from 7 leaves 5.

NOTE TO TEACHER: One method should be chosen and used continuously, as the use of several methods is confusing to pupils.

23.	18756 9384	**24.**	49848 27932	**25.**	1607428 984509
26.	38765 15483	**27.**	684901 509382	**28.**	8916302 7847205
29.	48008 37409	**30.**	900804 367845	**31.**	1234569 987406
32.	90009 1110	**33.**	543804 364705	**34.**	9874307 1985405
35.	88307 77418	**36.**	762496 528439	**37.**	2102407 1153508
38.	71516 56495	**39.**	928317 589428	**40.**	2000905 1000806
41.	81307 79208	**42.**	717507 84387	**43.**	7007025 3428016
44.	97405 68308	**45.**	119208 87405	**46.**	842878 320989

13. Problems

1. The area of Alaska is 590,884 sq. mi.; the area of Texas is 265,896 sq. mi. What is the difference in the area of these two countries?

2. John and James have together $257. If James has $167, how much money has John?

3. The area of the earth's surface is 196,900,000 sq. mi. Of this area 139,700,000 sq. mi. are water. How large is the land area?

4. Florida has a population of 968,470; South Dakota has a population of 636,547. How much larger is the population of Florida than that of South Dakota?

5. A has \$50, B has \$105, D has \$500. How much more money has D than A and B together have?

6. A father had \$15,000 in Liberty bonds. He gave his wife \$4,000, his son \$2,500, and his daughter \$2,000. How much had he left?

7. A man bought a farm for \$15,000. He built a house at a cost of \$4,750, a barn for \$1,250, and spent \$7,500 for live stock. He sold the farm, including live stock and improvements, for \$35,000. How much did he make in the transaction?

8. Columbus discovered America in 1492. The Constitution of the United States was adopted in 1789. How long from the discovery of America to the adoption of the Constitution?

9. A milliner sold two hats for \$17.50 each; on one she gained \$7.50, on the other she lost \$7.50. How much more did one hat cost than the other?

10. A boy earned \$17.50 a week. He spent for board \$7.25, for a hat \$5.25, for a shirt \$2.50, for a tie \$1. How much had he left at the end of the week?

11. A boy on being asked how much was his monthly salary replied, "If I were getting \$31.25 more than I now receive, my salary would be \$111.25 a month." What was his salary?

12. A lady lacked \$7.50 of having enough money to buy a \$25 hat. She bought a hat for \$12.50. How much money had she left?

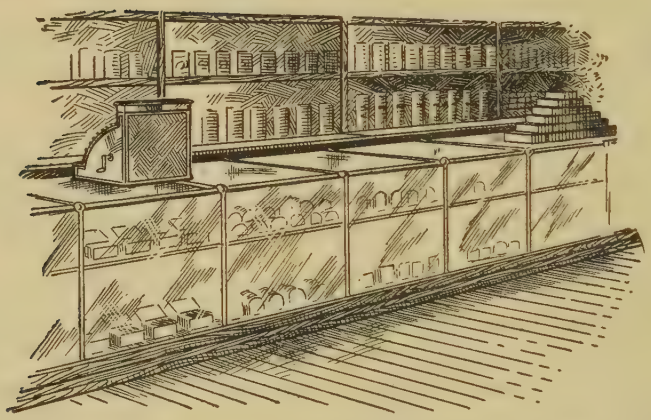
13. James had \$50 and invested in War Savings Stamps \$37.98. How much money had he left?

14. Rufus bought a pig for \$17.50, which he sold for \$25.25. How much did he gain?

15. There were 3,250,475 soldiers engaged in a certain battle; 375,498 were captured or slain. How many were left?

16. George Washington was born in 1732· Woodrow Wilson in 1857. Find the difference in years between the dates.

14. Making Change



In making change it is customary to add change to the amount of the purchase till the successive sums equal the amount of money received.

Illustrative problem:

John purchased a suit of clothes for $18.50. He gave a $20 bill to the salesman, who made the change by adding to the purchase; thus $19, $20.

Purchase	*Received*	*Change*	*Sums*
		$0.50	$19
$18.50	$20	$1.00	$20

15. Exercises—Oral and Written

Make change in each of the following, using as few pieces of money as possible:

	Purchase	*Received*	*Change*	
1.	$0.15	$0.25	How much?	What pieces?
2.	$0.30	$0.50	How much?	What pieces?
3.	$0.55	$1.00	How much?	What pieces?
4.	$0.85	$1.00	How much?	What pieces?
5.	$1.15	$2.00	How much?	What pieces?

6.	$1.45	$2.00	How much?	What pieces?
7.	$2.25	$5.00	How much?	What pieces?
8.	$2.75	$5.00	How much?	What pieces?
9.	$3.15	$5.00	How much?	What pieces?
10.	$5.65	$10.00	How much?	What pieces?
11.	$1.05	$1.25	How much?	What pieces?
12.	$1.15	$2.00	How much?	What pieces?
13.	$7.65	$10.00	How much?	What pieces?
14.	$11.10	$15.00	How much?	What pieces?
15.	$15.25	$20.00	How much?	What pieces?
16.	$17.50	$20.00	How much?	What pieces?
17.	$21.15	$22.00	How much?	What pieces?
18.	$22.50	$25.00	How much?	What pieces?

19. James bought a pair of shoes for $8.50. He gave the merchant a $10 bill. Make the change.

20. Rachel bought a pair of gloves for $2.50, a handkerchief for 50¢, and a pair of hose for $1.50. She gave the clerk a $5 bill. Make the change.

21. Thomas purchased a suit of clothes for $31.50, a pair of shoes for $8.50, 6 collars for $1.50, a shirt for $2.50, and a tie for $1.25. He gave the merchant a $50 bill; what change was due him?

22. Bob bought a tie for 75¢, and 2 shirts at $1.75 each. He gave the merchant a $5 bill. How much change was due him?

23. A farmer bought from a grain dealer 20 bu. seed wheat at $2.25 a bushel, 30 bu. of corn at $1.85 a bushel, 60 bu. of oats at 85¢ a bushel. He gave the dealer a check for $160. How much change was due the farmer?

24. Mrs. Rogers went to the grocery store and made the following purchases: 5 lb. potatoes at 6¢ a pound, 3 lb. breakfast bacon at 55¢ a pound, 2 lb. of sugar at 10¢ a pound, 1 sack of flour at $1.75, 5 cans of tomatoes at 15¢ a can, 2 lb. navy beans at 11¢ a pound, 3 lb. of coffee at 45¢ a pound. She gave the clerk a $10 bill. How much change was due her?

16. MULTIPLICATION

Multiplication is a short method of adding equal numbers.

$$\$5+\$5+\$5+\$5+\$5+\$5=\$30$$

The process may also be written, $6\times\$5=\$30$.

The oblique cross ($\times$) is the sign of multiplication and is read *times* or *multiplied by*.

The number to be multiplied is called the **multiplicand.**

The number showing how many times the multiplicand is taken is called the **multiplier.**

The result obtained by multiplication is called the **product.**

$$4\times 7 \text{ bu.}=28 \text{ bu.}$$

Which is the multiplicand? The multiplier? The product?

What is the name of the product? The name of the multiplicand?

A **concrete** number is one that refers to some particular kind of unit or thing, as \$5, 10 ft., 7 horses.

An **abstract** number is one that does not refer to some particular kind of unit or thing, as 5, 7, 9.

The multiplicand may be either a concrete or an abstract number. The product is always of the same denomination as the multiplicand. The multiplier is always used as an abstract number.

In such statements as the following: *If a carpenter earns \$6 a day, how much will 10 carpenters earn?* it appears that the multiplier, *10 carpenters*, is a concrete number, but when we analyze the problem we see that in multiplying we use it abstractly. Thus we would say, *If 1 carpenter earns \$6 a day, 10 carpenters will earn 10×\$6 or \$60.* We would not say, *10 carpenters times \$6.*

Numbers that are to be multiplied together are called **factors.**

Does the order in which the factors occur affect the product?

Test your answer with the following:

$2\times3\times5=$ $5\times2\times3=$ $3\times2\times5=$ $5\times3\times2=$

17. Oral Exercise

Give at sight the product of the factors in the following:

9× 9=	8 × 7=	7× 9=	12×12=	14× 8=
1.	**2.**	**3.**	**4.**	**5.**
9× 9=	8× 7=	7× 9=	12×12=	14× 8=
20× 9=	80× 7=	31×10=	11×13=	12× 7=
20×20=	30×20=	6× 9=	60× 7=	80× 5=
12× 9=	25×10=	20× 9=	7× 7=	20×20=
40× 4=	50× 5=	15× 6=	30× 7=	19× 5=
40×40=	50×50=	15×10=	8× 9=	16× 5=
20× 5=	60× 6=	15× 8=	90× 8=	35× 2=
10×30=	12×13=	14× 7=	20×25=	100× 9=

18. Oral Exercise

Name at sight two factors that will produce the following:

1.	**2.**	**3.**	**4.**	**5.**	**6.**	**7.**	**8.**
63	54	45	36	49	144	100	150
25	125	32	132	44	96	108	120
64	66	72	80	81	84	90	88
99	105	250	180	160	300	400	500
70	68	65	60	56	42	48	132
16	18	28	82	35	130	900	800
74	76	13	11	7	140	169	600

19. Written Exercises

1. Multiply 637 ft. by 27.

Multiplicand	637 ft.
Multiplier	27
1st partial product	4459 ft. = 7×637 ft.
2nd partial product	12740 ft. = 20×637 ft.
Entire product	17199 ft. = 27×637 ft.

To test multiplication, exchange places with the multiplicand and multiplier and perform the operation again; if the products agree the work is assumed to be correct. Or divide the product by the multiplier and if the quotient is the same as the multiplicand the work is assumed to be correct.

2. How much will 35 dozen eggs cost at 55¢ a dozen?

$0.55
35
275
165
$19.25

Point off in the product two places at the **right** for cents.

3. What will 70 milk cows cost at $85 per head?

$85
70
$5950

4. What will be the cost of 202 Plymouth Rock hens at $1.25 each?

$1.25
202
250
250
$252.50

As any number multiplied by 0 gives 0 for a product, it is unnecessary to multiply by the 0 factor.

Multiply:

5.	805×7690	**6.**	750×3006	**7.**	4002×2587
8.	907×8654	**9.**	837×9625	**10.**	5002×8756
11.	508×7006	**12.**	998×1001	**13.**	8008×5387
14.	423×8758	**15.**	837×4519	**16.**	2687×7843
17.	519×6387	**18.**	750×8064	**19.**	3249×9876
20.	405×6070	**21.**	387×6547	**22.**	8976×5431
23.	890×9007	**24.**	184×4329	**25.**	7777×8888
26.	660×5005	**27.**	999×5555	**28.**	8407×6542

If you multiply 2002 by 5847, which number would be more convenient to use as a multiplier? Why? Test your answer by using 2002 for multiplier and then using 5847 for multiplier. Count your figures and see which is the shorter way.

29. 6006×4444 **30.** 9865×60005 **31.** 87896×90009

32. If 0 is annexed to 3, what number will it make? How much is 10×3?

33. If two 0's are annexed to 3, what number will it make? How much is 100×3?

34. If three 0's are annexed to 3, what number will it make? How much is 1000×3?

35. What is the shortest way to multiply a number by 10, 100, 1000, etc.?

36. Give at sight the following products?

(a) Multiply by 10: 8, 9, 7, 95, 65, 57, 125, 1756.

(b) Multiply by 100: 8, 9, 7, 95, 65, 57, 125, 1756.

(c) Multiply by 1000: 8, 9, 7, 95, 65, 57, 31, 56.

20. Factors

Supply the missing number:

$?\times 3=12$	$?\times 15=30$	$11\times ?=88$	$?\times 6=54$
$8\times ?=56$	$15\times ?=30$	$?\times 8=96$	$7\times ?=63$
$20\times ?=200$	$7\times ?=56$	$12\times ?=132$	$?\times 8=72$
$10\times ?=90$	$?\times 8=56$	$10\times ?=70$	$?\times 5=45$
$7\times ?=84$	$9\times ?=63$	$11\times ?=121$	$?\times 6=42$
$?\times 12=96$	$?\times 9=81$	$?\times 9=99$	$7\times ?=49$

The multiplicand and multiplier are called **factors** of the product. In the above exercise, how did you find the missing factor? If 15 is the product, and the multiplier is 3, what is the multiplicand? If you have the product and either the multiplier or the multiplicand given, how do you find the other factor?

If two factors are the same, as 2×2, 3×3, 4×4, 9×9, etc., the product is called the square of one of the factors. Instead of writing the factors twice, as 2×2, we use a small figure, called **an exponent**, placed at the top and right of the number, thus

2^2, to tell how many times the figure is used as a factor. The expression 2^2 is read *2 squared.* What is the product of 3×3, or 3^2? If the factor is used 3 times as a factor, as $4\times4\times4$, it may be written 4^3. The expression 4^3 is read, *4 raised to the third power*, or *4 cubed.*

Give at sight the following:

$4^2, 5^2, 6^2, 7^2, 8^2, 9^2, 10^2, 11^2, 12^2, 20^2.$
$2^3, 3^3, 4^3, 5^3, 6^3, 10^3.$

Perform the operations indicated:

1. $25^2=25\times25=625.$

2.	35^2	**3.**	59^2	**4.**	87^2	**5.**	98^2	**6.**	25^3
7.	327^2	**8.**	479^2	**9.**	625^2	**10.**	987^2	**11.**	2687^3
12.	5627^2	**13.**	68956^2	**14.**	76548^2	**15.**	256789^3		
16.	35^4	**17.**	86^3	**18.**	99^3	**19.**	100^4		

How do you find the square of a number?
How do you find the cube of a number?

The table below shows the velocity (speed per hour) and force of winds:

Kind of Wind	Mi. an hour	Approximate force in lbs. Per sq. ft.
Brisk wind	22	2
High wind	32	4
Storm	50	10
Hurricane	90	33

1. Find the force against the side of a barn 2400 sq. ft. in area in a high wind; a hurricane.

2. Find the force against the side of a freight car 60 ft. long and 8 ft. high (480 sq. ft. in area) in a storm.

3. Find the force against the side of a street car 25 ft. long and 8 ft. high (200 sq. ft.) in a hurricane.

21. Problems

1. How many days in 52 weeks?

2. At \$50 a month rent for a dwelling, what is the rent for 5 years?

3. What is the cost of 25 dozen eggs at 55¢ a dozen?

4. If a carpenter earns \$6 a day and works an average of 25 days a month, how much will he earn in a year?

5. If a teacher receives \$125 a month and works 9 months in the year, how much will the teacher earn in a year?

6. At 45¢ a pound, find the cost of 25 lbs. of butter.

7. (a) A boy bought a pair of shoes for \$7.50, a hat for \$3.50, and a suit for 3 times as much as the cost of his shoes and hat. How much did they all cost?

(b) He gave the merchant a \$50 bill. Make the change.

8. A farmer sold 25 bu. of potatoes at 65¢ a peck. How much did he receive?

9. At 60¢ an hour, how much can a workman earn in 17 da. of 8 hr.?

10. A city that had a scholastic population of 13,125 appropriated for school purposes \$29.75 for each child. What was the appropriation for schools?

11. A business man lived 6 mi. from his office and made 2 round trips daily. How far did he travel in 25 da.?

12. A newsboy sells 75 papers each day in the week. How many papers does he sell in 52 wk.?

13. Find the cost of building 32 mi. of macadamized road at \$7125 a mile.

14. A farmer sold 6 beef cattle whose average weight was 1150 pounds at 9¢ a pound. How much did he receive for them?

22. Incomplete Problems

Make problems out of the following statements:

1. James is 10 years of age and Thomas is 4 years older than James.

2. A farmer grew 1785 bu. of wheat and sold 987 bu.

3. A man earned $5.00 a day and spent $2.50.

4. Mr. Brown bought 2 farms; for one he paid $12,750, for the other $15,750.

5. There are 52 weeks in a year, and six work-days in a week.

6. It took 5 hours to make a trip on a train that ran 35 miles an hour.

7. A man made 2 trips to market and back each day. He lived 6 mi. from the market.

8. A dairyman carried to market 20 gallons of milk which he sold at 15¢ a quart.

9. A cabman made 6 trips to and from the depot each day. The depot was 2 miles distant.

10. A farmer gathered 25 wagon loads of corn from a field and each wagon load contained 25 bu.

11. In an orchard there were 35 peach trees and 9 times as many apple trees as there were peach trees.

12. A farmer grew 2025 bu. of corn, and 750 bu. more wheat than corn.

13. A man sold two horses for $250 each. On one horse he gained $25 and on the other he lost $35.

14. The population of a city in 1910 was 128,735. In 1920 the population had increased to 175,874.

23. DIVISION

Division is the process of finding how many times one number is contained in another.

The number to be divided is called the **dividend.**

The number by which we divide is called the **divisor.**

The result obtained by dividing is called the **quotient.**

The sign, (÷), of division is read *divided by.* When placed between two numbers, it indicates that the one on the left is divided by the one on the right. Thus, $6 \div 3$ is read, *6 divided by 3.*

The **remainder** is the part of the dividend left when the division is not exact; thus,

$$4\overline{)9} \quad 2\text{—}1 \text{ remainder}$$

1. A father had \$20 which he wished to divide equally among his 4 sons. How much would each receive?

$$4\overline{)\$20} \quad \$5$$

We could also say if the 4 sons receive \$20, one son would receive $\frac{1}{4}$ of \$20 or \$5. Note that in the above the quotient is the same name as the dividend and is only 1 of 4 equal parts of the dividend. This kind of division is called **division by partition.**

2. How many times is 4 ft. contained in 12 ft.?

$$4 \text{ ft.}\overline{)12 \text{ ft.}} \quad 3$$

Note in the above problem that the divisor and the dividend are concrete numbers and the quotient is an abstract number. This kind of division is called **division by measurement.**

Division may be thought of as a short method of subtraction.

Thus: 12 ft.−4 ft.=8 ft.; 8 ft.−4 ft.=4 ft.; and 4 ft. −4 ft.=0.

Division may be indicated in three ways: $4\overline{)12}$ or $12 \div 4$ or $\frac{12}{4}$.

To test division, multiply the divisor by the quotient and add the remainder, if any, to the product. If the result gives the dividend, the work is assumed to be correct.

Give quotient at sight:

63÷ 9,	63÷ 7,	108÷ 9,	84÷ 7,	120÷10,	40÷ 8,
20÷ 5,	200÷ 5,	42÷ 7,	46÷ 8,	250÷ 5,	36÷ 6,
72÷ 8,	72÷ 9,	80÷10,	81÷ 9,	88÷11,	96÷12,
96÷ 8,	28÷ 7,	36÷ 9,	45÷ 5,	48÷ 8,	42÷ 6,
50÷10,	500÷10,	40÷20,	75÷ 5,	84÷12,	49÷ 7,
64÷ 8,	30÷15,	60÷20,	90÷30,	26÷13,	66÷11,
99÷11,	121÷11,	132÷12,	35÷ 7,	144÷12,	180÷ 9.

Supply at sight the missing number:

18÷ 9= ?	?÷12= 8	150÷5 =?	200÷ 5= ?
27÷ ?= 3	96÷ ?=12	60÷ ?=5	?÷ 5=20
?÷ 6= 6	99÷ ?=11	36÷ ?=9	?÷ 9= 7
10÷ ?= 5	?÷ 7=12	?÷ 6=9	300÷50= ?
45÷ 9= ?	?÷12= 7	64÷ ?=8	?÷11= 4
36÷ ?= 4	72÷ ?= 9	96÷ ?=8	?÷15=75
?÷12= 3	?÷ 8=72	?÷ 7=9	?÷12= 7
?÷ 6=36	56÷ ?= 8	84÷ ?=7	?÷ 9= 9
42÷ 7= ?	63÷ ?= 9	?÷ 8=6	48÷ ?= 9
?÷ 6= 7	?÷ 9= 7	35÷ ?=7	84÷ ?= 7
49÷ ?= 7	?÷12=11	99÷ ?=9	?÷ 9= 6
?÷10=12	121÷ ?=11	?÷ 6=9	?÷11= 3
144÷12= ?	66÷ ?= 6	55÷ ?=5	150÷30= ?
?÷ 7= 7	?÷15=60	?÷11=8	?÷15= 4

24. Written Exercises

	a	*b*	*c*	*d*	*e*	*f*
1.	$2\overline{)678}$	$3\overline{)999}$	$4\overline{)416}$	$4\overline{)828}$	$5\overline{)575}$	$3\overline{)969}$
2.	$3\overline{)948}$	$6\overline{)876}$	$9\overline{)981}$	$6\overline{)966}$	$5\overline{)735}$	$9\overline{)927}$

3.	7)714	3)273	4)792	7)742	9)981	6)636
4.	7)784	6)684	5)975	7)980	7)679	9)693
5.	4)816	8)864	4)468	9)729	6)972	3)294

6. Divide 8769 by 9.

a

$$974\tfrac{3}{9}$$
$$9)8769$$

b

$$\begin{array}{r} 974\tfrac{3}{9} \\ 9)8769 \\ 81 \\ \hline 66 \\ 63 \\ \hline 39 \\ 36 \\ \hline 3 \end{array}$$

The process may be outlined as follows: (1) Divide (87÷9); (2) Multiply (9×9); (3) Subtract (87−81); (4) Add (6 hundreds, or 60 tens+6 tens=66 tens). Continue as from the beginning till every figure in the dividend is used.

The method shown in *a* is called *short division*. The method shown in *b* is called *long division*.

What is the difference between the two processes?

Where do you write the first quotient figure?

Divide and test:

7.	896÷12	**8.**	57890÷ 5	**9.**	17891÷ 9
10.	8417÷ 6	**11.**	95687÷11	**12.**	65878÷12
13.	63847÷ 8	**14.**	49786÷ 7	**15.**	87626÷11
16.	97887÷ 9	**17.**	75879÷ 6	**18.**	83737÷ 9
19.	48324÷ 6	**20.**	59863÷ 7	**21.**	85623÷ 4
22.	99818÷ 3	**23.**	6307÷ 8	**24.**	17930÷ 8

25. When the Divisor is a Large Number

1. Divide 1004879 by 246.

$$
\begin{array}{r}
4084\tfrac{215}{246} \\
246\overline{)1004879} \\
\underline{984} \\
2087 \\
\underline{1968} \\
1199 \\
\underline{984} \\
215
\end{array}
$$

The fewest left-hand figures of the dividend that make a number large enough to contain the divisor are 1004, which is called a partial dividend.

How many times is 246 contained in 1004?

When we think of 246 as being nearly 250 and 1004 as just a little more than 1000, and by comparing we know 1000 is 4×250, we conclude that 4 is the first quotient figure; write it over the 4 in the dividend. Multiply the divisor by quotient figure 4 and subtract the product from the partial dividend. Bring down 8 from the dividend and write it with the remainder to form the next partial dividend. 208, the partial dividend, is smaller than the divisor 246, and therefore will not contain it and we write 0 in the quotient. Bring down 7 from the dividend and write it with the remainder for the next partial dividend. The partial dividend 2087 contains the divisor 246, 8 times. Multiply and subtract. Continue the process till every figure in the dividend is used. Judgment is required in ascertaining the correct quotient figure.

If in the trial division the product of the divisor by the quotient is larger than the partial dividend, the quotient figure is too large. If after subtracting the product of the divisor by the quotient figure from the partial dividend the remainder is found to be equal to or larger than the divisor, the quotient figure is too small.

Examples for Practice

	(a)	(b)	(c)	(d)	(e)
2.	86017543 ÷	365,	247,	896,	884, and 796.
3.	901805 ÷	218,	748,	590,	670, and 347.
4.	321891675 ÷	347,	408,	508,	706, and 760.
5.	9546832 ÷	746,	595,	473,	763, and 874.
6.	380069 ÷	296,	564,	875,	963, and 847.
7.	281730600 ÷	260,	874,	396,	281, and 470.
8.	26908804 ÷	760,	987,	483,	543, and 807.
9.	90806734 ÷	826,	508,	967,	385, and 186.
10.	90069547 ÷	563,	603,	754,	586, and 905.

26. Short Method of Division

Divide 60 by 10.

$$1\not{0})6\not{0}\quad 6$$

Omit or cut off the naughts from 60 and 10. Is there any difference between the quotient of $60 \div 10$, and $6 \div 1$?

Give at sight each of the quotients:

$80 \div 10$, $90 \div 10$, $250 \div 10$, $270 \div 10$, $290 \div 10$, $180 \div 10$, $520 \div 10$, $630 \div 10$, $840 \div 10$.

Divide 600 by 100.

$$1\not{0}\not{0})6\not{0}\not{0}\quad 6$$

Cut off two naughts from 600 and from 100. Does the quotient differ from the quotient of 600 divided by 100? What is the shortest way to divide by 100?

Give at sight the quotient of the following numbers when divided by 100:

700, 800, 900, 1200, 5200, 7500, 9500, 8400, 2300, 65700, 934,800, 56700, 6500.

Divide 8000 by 1000.

How many 0's do you cut off to divide by 1000?

Cutting off one naught is equivalent to dividing by 10; cutting off two naughts, by 100, etc.

Is the quotient changed when the dividend and divisor are each divided by 10? by 100? by 1000? by any number? Test your answers.

Give at sight the quotient of each of the following numbers when divided by 1000:

8000, 9000, 95000, 77000, 87000, 93000, 124000, 164000, 876000, 923000, 45000, 19000, 84000, 73000, 65000, 125000, 86000, 77000.

1. Divide 482616 by 6000.

$$6/000)482/616\quad 80\tfrac{2\,6\,1\,6}{6\,0\,0\,0}$$

How was the quotient obtained? How was the remainder found?

2.	$5615 \div 400$	**3.**	$78654 \div 3000$	**4.**	$9287 \div 100$
5.	$7896 \div 600$	**6.**	$18765 \div 900$	**7.**	$489784 \div 12000$
8.	$96879 \div 400$	**9.**	$875687 \div 6000$	**10.**	$1459276 \div 500$
11.	$\$18.25 \div 25$	**12.**	$\$14.25 \div 5$	**13.**	$\$34.65 \div 7$

$$
\begin{array}{r}
.73 \\
25)\overline{\$18.25} \\
175 \\
\hline
75 \\
75 \\
\hline
\end{array}
$$

14.	$\$37.50 \div 125$	**15.**	$\$1.00 \div 5$	**16.**	$\$200 \div 80$
17.	$\$300 \div 15$	**18.**	$\$600.45 \div 15$	**19.**	$\$2.00 \div 10$
20.	$\$13.56 \div 12$	**21.**	$\$32.64 \div 16$	**22.**	$\$14.60 \div 5$
23.	$\$1.00 \div 5$	**24.**	$\$2.00 \div 8$	**25.**	$\$30.00 \div 8$
26.	$\$27.45 \div 15$	**27.**	$\$47.14 \div 15$	**28.**	$82000 \div 1000$
29.	$\$8.10 \div 18$	**30.**	$\$31.25 \div 5$	**31.**	$\$108.60 \div 12$

27. Exercises

Problems numbered with odd numbers are to be solved orally; those with even numbers are to be written.

1. How many tablets at 7¢ each can be bought for 21¢?

2. How many horses at $225 each can be bought for $3375?

3. Find the cost of 2 gal. of milk at 15¢ a quart.

4. How many quart bottles can a dairyman fill from 20 gal. and 3 qt. of milk?

5. If 3 yd. of ribbon cost 24¢, what will 7 yd. cost?

6. If 64 sheep cost $576, what will 100 sheep cost?

7. In his spelling lesson, James missed 6 words out of 25. How many words did he spell?

8. A trader bought 600 head of cattle. He sold 250 and 10 died. How many had he left?

9. A merchant bought 6 dozen eggs at 45¢ a dozen. One dozen spoiled and he sold the remainder at 60¢ a dozen. How much did he gain by the transaction?

10. A broker bought 9 Liberty bonds at $968 each, and sold them for $982 each. How much did he gain in the transaction?

11. How many bushels of potatoes can be bought for $6 when potatoes are selling at $1.50 a bushel?

12. When chestnuts are selling at 10¢ a quart, how many bushels can be purchased for $12.80?

13. I have in mind a number which, when multiplied by 2 and 1 added to the product, gives 9. What is the number?

14. I have in mind a number which, when multiplied by 30 and 25 added to the product, gives 775. What is the number?

15. A boy drove his horse 18 mi. in 3 hr. How far did he drive in 1 hr.?

16. A tourist drove his car 1008 mi. in 48 hr. What was his rate of travel per hour?

17. I have in mind a number from which if 3 be subtracted and the remainder divided by 4 the quotient is 3. What is the number?

18. I have in mind a number from which if 23 be subtracted and the remainder divided by 25 the quotient is 36. What is the number?

19. The divisor is 5, the quotient is 3; what is the dividend?

20. The divisor is 42, the quotient is 63; what is the dividend?

21. A boy bought 5 apples for 20¢. He ate 1 apple and sold the remainder at 5¢ each. How much did he make in the transaction?

22. A farmer raised 1200 bu. of corn. He kept 575 bu. to feed and sold the remainder at $1.05 a bushel. How much did he receive?

23. At 6¢ a pint, find the cost of 2 gal. of milk.

24. At 60¢ a peck, find the cost of 25 bu. of apples.

25. At 40¢ a dozen, how many eggs can be bought for $2.40?

26. At $125 an acre, how many acres of land can be bought for $29500?

27. I bought 3 oranges and received 10¢ change out of a quarter-dollar. What was the price of each orange?

28. I bought 12 pounds of sugar and received $2.36 change out of a $5 bill. What was the cost of a pound of sugar?

29. After giving William 5 marbles and Thomas 12, James had 13 marbles left. How many had he at first?

30. A merchant gave one check to Brown & Co. for $1250, another to Gross & Co. for $2350, and had left in the bank $5275. How much had he in bank before checking?

31. When eggs are selling for 60¢ a dozen, how many eggs can be bought for 20¢?

32. When town lots are selling at $25 a front foot, how many front feet can be bought for $1250?

33. A boy buys papers for 3¢ and sells them for 5¢ each. How much does he gain on 100 papers?

34. Pencils that cost $4.32 a gross are sold at $5 a hundred. How much does the dealer make on 5 gross?

35. James had 25¢. John had 10¢ less. How many cents did they both have?

36. A owned 3 farms. The first farm was worth $3500 less than the second, and the second was worth $2750 less than the third, which was worth $10,000. What was the worth of the 3 farms?

37. A fruit dealer bought 5 bu. apples for \$15. For how much per bushel must he sell them to gain \$10?

38. A man bought 65 A. of land for \$8125. For how much an acre must he sell it to gain \$3120?

39. James took 20 bu. of corn to the market. He sold the corn at \$1.20 per bushel. He bought a hat for \$4.50, a pair of shoes for \$6, and a neck-tie for 75¢. How much of the money received for the corn had he left?

40. Mary had $\frac{1}{8}$ A. of tomatoes. She gathered 20 bu. of tomatoes, which canned 16 quart cans to the bushel. She sold the tomatoes at 15¢ a quart. How much money did she receive for the tomatoes? At the same rate, how much would she have received from an acre of tomatoes?

28. Review Questions

1. What is addition? **2.** What are addends? **3.** What is the sum or amount? **4.** How do you test addition? **5.** What is subtraction?

6. What is the minuend? The subtrahend? The remainder or difference?

7. How do you test subtraction?

8. What is multiplication?

9. What is the multiplicand? The multiplier? The product?

10. What is the effect of multiplying a number by 0?

11. What is an abstract number? A concrete number?

12. What are factors?

13. What kind of number is the multiplier?

14. Define notation; numeration.

15. What is the difference between the Arabic and the Roman systems of notation?

16. How do you test multiplication?

17. What is division?

18. Define the terms: dividend, divisor, quotient, remainder.

19. How do you test division?

20. What is the difference between long division and short division?

21. If you have the product of 2 factors and one factor is given, how is the other found? If you have the product of 3 factors and 2 of them are given, how is the other found?

22. Name and make all the arithmetical signs you have used.

23. If you have the divisor and the quotient given, how do you find the dividend?

24. How do you find the square of a number? The cube of a number?

25. How do you indicate that a number is to be squared? cubed?

29. Symbols of Operations

You have learned the meaning of the signs $+$, $-$, $\times$, $\div$, when used with numbers in single relations: Thus, $4+4$ indicates addition, $8-4$ indicates subtraction, 5×4 indicates multiplication, $8\div4$ indicates division.

$4+8\div2=$ What? If we add $(4+8)$ and divide the sum by 2, the result is 6. If we add 4 to the quotient of $8\div2$, the result is 8. The latter result is correct because the operations indicated by $\times$ or $\div$ take precedence over the operations indicated by $+$ or $-$.

Operations indicated by $\times$ and $\div$ are performed in the order of their occurrence unless otherwise indicated by parenthesis, as, $16+8\times4-12\div6\times3=42$. The signs $+$ and $-$ set off the terms of an expression. Thus, in the expression above the terms are 16, 8×4, $12\div6\times3$.

$4+5\times2=14$. Why not 18?

$16-12\div4=13$ $\qquad$ $8+4\div2=10.$

$8-2\times3=2.$ $\qquad$ $8\times4\div2=16.$

The parenthesis ( ) is used to show that the numbers within it are to be used as one simple quantity; thus $(8+4) \div 2$, means that the sum of 8 and 4, or 12, is to be divided by 2. Hence, $(8+4) \div 2 = 6$.

Give results at sight:

	a	*b*	*c*
1.	$(4+5) \times 2 = ?$	$6+3 \times 4 = ?$	$(6+3) \times 4 = ?$
2.	$8 \times 2+3 = ?$	$9-2 \times 3 = ?$	$16-4 \times 4 = ?$
3.	$5 \times 9-4 = ?$	$8-6 \times 1 = ?$	$(8-6) \times 1 = ?$
4.	$8-16 \div 4 = ?$	$(28-16) \div 4 = ?$	$49-9 \div 3 = ?$
5.	$5+6 \div 3 = ?$	$(5+6) \div 3 = ?$	$40-28 \div 4 = ?$
6.	$9+6 \times 3 = ?$	$(9-6) \times 3 = ?$	$(9-6) \times 3 = ?$
7.	$81 \div 9-5 = ?$	$19-14 \div 7 = ?$	$27+9 \times 3 = ?$
8.	$(8+4) \times (3+2) = ?$	$(5+3) \times (6-4) = ?$	$25 \div 5+3 \times 2 = ?$
9.	$(9-2) \div (14 \div 2) = ?$	$3 \times 5-5 \times 2 = ?$	$49 \div 7+6+3 = ?$
10.	$(5+4-3) \times (7+2-6) = ?$		$(9+8 \div 4)-(9 \div 3-3) = ?$
11.	$(16 \div 8-2)+4 = ?$		$5+4-3 \times 3+2 = ?$
12.	$9+8 \div 4-9 \div 3-3 = ?$		$16 \div 8+2 \times 4-10 \div 5 = ?$

30. Factoring

An *integer*, or *integral number*, is a whole number, as 1, 2, 3. An *even* number is one that can be exactly divided by 2. Name the even numbers from 10 to 20. An *odd* number is one that cannot be divided by 2 without a remainder. Name the odd numbers from 1 to 10.

The product is 48 and one factor is 6. What is the other factor?

Name 2 factors that produce 12, 15, 10, 21, 56, 54, 63, 72, 81, 77, 84, 88, 90, 96, 100, 110, 121, 132, 35.

A *prime* number is one that has no integral factors except itself and one; as 1, 2, 3, 7, 11, 13. Name all the prime numbers between 20 and 30.

A *composite* number is the product of two or more integral factors; as 4 is composed of 2×2, $10 = 2 \times 5$, $21 = 3 \times 7$.

Name all the composite numbers between 30 and 40.

A *prime factor* is a factor that is a prime number. Thus, the prime factors of 6 are 2 and 3.

Name the prime factors of 9, 10, 15, 21.

A number can have only one set of prime factors. $2\times4=8$. 2 and 4 are factors of 8, but not prime factors, since 4 is not a prime number. $8=2\times2\times2$. It may easily be seen that the prime factors of 8 are 2 taken 3 times, as indicated. When 2 is used 3 times as a factor, it may be indicated thus, 2^3. The prime factors of 16 are $2\times2\times2\times2$, which may be written 2^4. The small figure written at the right and a little above the factor to show how many times the factor is used is called an *exponent*.

What does 6^2 represent? 3^3? 5^3? 10^2? 4^5? 8^4?

$2^2\times5^2$ are the prime factors of what number?

$2^2\times3^2$ are the prime factors of what number?

The prime factors of a number are $2^2\times5^2\times3^2$. What is the number?

Name the prime factors of 20, 22, 24, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38 40, 45, 50, 54, 55, 56, 60, 64, 72, 75, 80, 90, 95, 100.

31. Written Exercises

What are the prime factors of 252?

2)252
2)126
3) 63
3) 21
7

DIRECTION: Divide the number by any prime factor; divide the quotient thus obtained in like manner; continue the division until a prime quotient is obtained. The prime quotient and the prime divisors constitute the prime factors. Hence 2, 2, 3, 3, 7, or 2^2 3^2, 7 are the prime factors.

Find the prime factors of:

1.	168	**2.**	268	**3.**	125	**4.**	295	**5.**	296
6.	81	**7.**	250	**8.**	300	**9.**	275	**10.**	363
11.	288	**12.**	256	**13.**	175	**14.**	625	**15.**	189
16.	576	**17.**	375	**18.**	630	**19.**	1280	**20.**	936

When certain integers will divide a number.

It is sometimes difficult to ascertain whether a number contains certain factors; in such cases the following tests are very useful for determining without actual division:

1. A number is divisible by 2 if its last digit is 0, 2, 4, 6, or 8; as 10, 12, 14, 26.

2. A number is divisible by 4 if the number expressed by its last two digits is divisible by 4; as 416, 584, 636.

3. A number is divisible by 3 if the sum of its digits is divisible by 3; as 324, 471, 564, 492.

4. A number is divisible by 5 if its right hand digit is 0 or 5.

5. A number is divisible by 9 if the sum of its digits is divisible by 9; as 918, 774, 162.

6. Which of the following numbers are divisible by 3? 345, 335, 624, 476, 726, 824, 936, 516, 444, 555, 217, 326.

7. Which of the following numbers are divisible by 4? 416, 8276, 918, 728, 515, 864, 1036, 927.

8. Which numbers in examples 6 and 7 are divisible by 2? By 9?

9. Write four numbers divisible by 3; by 9; by 4.

32. GREATEST COMMON DIVISOR

The word *divisor* is frequently used instead of *factor*.

1. A **common divisor** of two or more numbers is a number that exactly divides each of them. What number or numbers will exactly divide 12 and 18? 16 and 20? 18 and 24? 63 and 72? 40 and 20? 36 and 48? 21 and 24? 21 and 35? 45 and 54?

2. The **greatest common divisor** of two or more numbers is the largest number that exactly divides each. Thus, 6 is the greatest common divisor (G. C. D.) of 12 and 18.

3. Name the G. C. D. of 21 and 35; 20 and 50; 25 and 100; 12, 21 and 36; 18, 48 and 54; 18, 36 and 81.

4. If two integers have no common divisor except 1, they are said to be *prime to each other.*

5. What is the G. C. D. of 7 and 11? 3 and 7? 9 and 17? 18 and 29?

Exercises

1. Find the G. C. D. of 64, 96, and 132.

Find the prime factors of each number:

2) 64	2) 96	2) 132
2) 32	2) 48	2) 66
2) 16	2) 24	3) 33
2) 8	2) 12	11
2) 4	2) 6	
2	3	

$64=2^6$. $96=2^5\times3$. $132=2^2\times3\times11$.

The G. C. D. will be the product of all the factors common to each. 2^2 is the only factor found in all the numbers. Therefore, 2^2, or 4, is the G. C. D.

Find the G. C. D. of each of the following:

2. 825, 725. **3.** 56, 232. **4.** 368, 336. **5.** 224, 728.
6. 136, 442, 626. **7.** 32, 48, 60. **8.** 36, 72, 54.
9. What is the G. C. D. of 48, 72, 90?
10. Find the G. C. D. of 48, 56, 104. **11.** 72, 86, 180.
12. 45, 63, 108. **13.** 105, 136, 165. **14.** 28, 72, 656.
15. 13, 65, 78. **16.** 7, 9, 11. **17.** 720, 684, 342.
18. 88, 264, 512. **19.** 75, 225, 500. **20.** 3, 7, 13.

21. A printer buys his card-board stock from the wholesale paper dealer in sheets 24 in. by 30 in., with enough extra in each dimension to make up for the little waste in cutting into small cards. The printer gets orders for various sizes of form cards. Could he cut the large sheets into cards 5 in. by 9 in. without any loss? 4 in. by 6 in.? Find other sizes.

33. LEAST COMMON MULTIPLE

What numbers will contain 3? How many are there? What numbers will contain 4? How many are there?

What numbers will contain 3 and 4?

All numbers that will contain 3 are called **multiples** of 3. All numbers that will contain 4 are called multiples of 4.

Common multiples of two or more numbers are numbers that contain each of them exactly; thus, 12, 24, 36, etc., are common multiples of 3 and 4; 36, 72, 144, etc., are common multiples of 2, 3, 4, 6, 9, 12 and 18. What is the least number that will contain 3 and 4? 7 and 9? 5 and 10? 8 and 12? 6 and 9?

The **least common multiple** (L. C. M.) of two or more numbers is the least number that exactly contains each of them; thus, 36 is the L. C. M. of 9 and 12.

Name at sight the least common multiple of 6 and 8; 6 and 10; 5 and 12; 9 and 12; 6 and 7; 6, 8, and 12; 4, 6, and 8; 3, 6, and 9.

34. Written Exercises

1. Find the L. C. M. of 8, 12, and 18.

2)	8	12	18
2)	4	6	9
3)	2	3	9
	2	1	3

Divide the numbers by any prime factor common to two or more of them. Continue the division of the resulting quotients until no two of them have a common factor. The divisors and remaining quotients are factors of the L. C. M.

$2\times2\times3\times2\times1\times3=72$ (L. C. M.)

2. Find the L. C. M. of 12, 15, 36, 66.

3)	12	15	36	66
2)		5	12	22
2)		5	6	11
		5	3	11

As 12 is an exact divisor of 36, strike out 12, since the L. C. M. of all four numbers, including 36, is a multiple of any factor of 36.

L.C.M. $=3\times2\times2\times5\times3\times11=1980$.

Second Method

Find the L. C. M. of 12, 15, 36, 64.

$12 = 2 \times 2 \times 3 = 2^2 \times 3$

$15 = 3 \times 5$

$36 = 2 \times 2 \times 3 \times 3 = 2^2 \times 3^2$

$64 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 = 2^6$

L.C.M. $= 2^6 \times 3^2 \times 5 =$

$64 \times 9 \times 5 = 2880$

Resolve each number into its prime factors. Then, the L. C. M. will be the product of all the different prime factors, each used as many times as the greatest number of times it appears in any one set of factors. In this example there are three different prime factors in the four sets of factors, namely, 2, 3, 5. To find the L. C. M., 2 must be used 6 times as a factor in order to contain 2^6 in 64; 3, two times, in order to contain 3^2 in 36; 5, one time, in order to contain 5 in 15. Hence, to find a number large enough to contain 2^6, 3^2, and 5, we must have $2^6 \times 3^2 \times 5 = 64 \times 9 \times 5 = 2880$.

Find the L. C. M. of: **3.** 18, 28, 36. **4.** 15, 18, 24.

5. 16, 20, 24. **6.** 18, 24, 30. **7.** 6, 10, 12. **8.** 18, 20, 30.

9. 12, 36, 72, 120. **10.** 8, 12, 18, 28. **11.** 12, 18, 27, 30.

12. 9, 12, 16, 28. **13.** 16, 21, 49. **14.** 3, 5, 7.

15. 7, 17, 68. **16.** 20, 48, 72. **17.** 15, 24, 30.

35. Review Problems

1. What is the least common multiple (L. C. M.) of 24¢ and 36¢?

2. James earns 24¢ an hour and Thomas earns 16¢ an hour. They wish to work until each has the same amount of money. How many hours will each work?

3. A large basket contains apples and pears. Two smaller baskets will hold 30 apples and 20 pears, respectively. Find the least number of apples and pears the large basket could contain in order to fill exactly either of the smaller baskets a certain number of times.

4. Find the least sum of money that will buy an exact number of articles at a cost of either \$6 or \$10 for each article.

5. Three men travel 15, 21 and 35 miles a day, respectively, in going the same distance. Find the least distance which they can travel in an exact number of days.

6. What is the shortest rope that can be cut into exact pieces either 9 ft., 15 ft. or 24 ft. long?

7. What is the length of the longest concrete frame that could be used in molding blocks for two stretches of walk 56 ft. long and 184 ft. long in order that there may be an exact number of blocks and no loss?

8. A fruit merchant with 48,312 apples and 24,552 oranges wishes to pack them in equal number in separate boxes so as to put the largest number possible in each box. How many will be placed in each box?

9. What is the smallest sum of money that can be counted exactly with either $5, $10, or $20 bills?

10. What is the least number that can be divided by 4 or 3 and have a remainder of 1?

11. Two persons take steps of 32 in. and 36 in. If they start in step, how far will they walk before they are in step again?

12. A farmer has two fields, one of which contains 54 A. and the other 87 A. He wishes to divide them into an exact number of lots as large as possible. Find the size of each lot.

13. What is the smallest number that can be divided by 5 or 6 and leave a remainder of 4?

14. Write three numbers that are prime to each other.

15. A printer has orders from time to time for different sizes of card forms, as follows: 3 in. by 5 in., 6 in. by 8 in., and 8 in. by 10 in. In what sizes should he buy his card-board material so as to have no loss in cutting?

36. CANCELLATION

Cancellation is the process of striking out or cancelling equal factors from both dividend and divisor in order to shorten the operation.

Cancelling does not affect the quotient since the dividend and divisor may be multiplied or divided by the same number without changing the quotient. $\frac{16}{4}=?$ $\frac{8}{2}=?$ $\frac{32}{8}=?$ $\frac{64}{16}=?$

Divide $6\times5\times9$ by 8×3.

$$\frac{\overset{3}{\cancel{6}}\times5\times\overset{3}{\cancel{9}}}{\underset{4}{\cancel{8}}\times\underset{1}{\cancel{3}}}=\frac{45}{4}=11\tfrac{1}{4}.$$

2 is a common factor of 6 and 8; divide both terms by the common factor. After cancelling all factors common to both divisor and dividend, divide the product of the factors left in the dividend by the product of the factors left in the divisor.

Written Exercise

Divide:

1. $2\times8\times9\times10$ by $3\times4\times5$.
2. $5\times10\times21\times48$ by $5\times7\times24$.
3. $16\times8\times9\times34$ by $8\times3\times4\times17$.
4. $132\times81\times63\times15$ by $12\times99\times9\times5$.
5. $46\times63\times8\times4$ by $23\times21\times16$.
6. $50\times40\times75$ by $25\times20\times15$.
7. $24\times28\times36\times42$ by $14\times12\times9\times21$.
8. $39\times19\times54\times60$ by $13\times38\times27\times30$.
9. $10\times12\times14\times28$ by $5\times24\times7\times14$.
10. $48\times36\times24$ by 1728.
11. $\dfrac{160\times150\times300}{80\times75\times100}$
12. (a) $\dfrac{204\times8}{51\times4}$ (b) $\dfrac{144}{2\times8\times3}$ (c) $\dfrac{366}{90}$

13. A farmer exchanged 4 bu. of corn at 76¢ a bushel for sugar at 10¢ a pound. How many pounds of sugar did he receive?

14. If 8 men working 8 hours a day dig a ditch in 40 days, how long will it take 10 men working 10 hours a day?

15. A housewife bought 10 lb. of coffee at 40¢ a pound, and paid for it with eggs at 50¢ a dozen. How many dozen did it take?

16. A dairyman exchanged his dairy consisting of 65 cows valued at $95 each for a farm valued at $75 an acre. How many acres in the farm?

17. A farmer exchanged 40 bu. of corn at 75¢ a bushel for flour at $10 a barrel. How many barrels of flour did he receive?

18. A truck farmer sold 15 crates of berries containing 16 qt. each at 20¢ a quart; with the money he bought a carpet at $1.25 a yd. How many yards did he receive?

19. Divide the product of $23\times27\times84$ by $9\times28\times46$.

37. Drill Exercise

Remember that when several operations occur in the same expression, the operations indicated by $\times$ and $\div$ are performed before operations indicated by $+$ and $-$, unless parentheses change the order.

Name result:

1.	$20-5\times3$	**2.**	$20\div5+3$	**3.**	$20-8\div2$
4.	$24\div3+3$	**5.**	$12\times8+4$	**6.**	$8+4\times2$
7.	$10+3\times2$	**8.**	$44\div4-4$	**9.**	$12\div6+7$
10.	$7+2\times3$	**11.**	$16\div4+4$	**12.**	$16+4\div4$
13.	$36\div6+3$	**14.**	$36+6\div3$	**15.**	$36-6\div3$
16.	$18\div9+3$	**17.**	$18\div9-1$	**18.**	$18-9\div3$
19.	$16\div8\times2$				

20. $16\div8\times2+5-3+5$ **21.** $8\times4-8\div4+10-5$

22. $14-7+2+9\div3\times(5-3)$ **23.** $16\div8+8-6+8$

24. $21\div7+7-5+3-4+8$ **25.** $17-4\times2+3-6+4$

26. Which is larger, $12 \div 12 + 12$ or $12 - 12 \div 12$?
27. What is the sum of $12 \div 6 + 2$ and $4 \div (4 \div 2) + 5$?
28. What is the sum of $55 \div 5 + 4$ and $55 - 5 \times 10$?
29. How many times is $36 \div 9 + 4$ contained in $48 + 2 \times 8$?
30. $8 \div 4 + 6 \div 2 + 9 \times 3$ **31.** $21 \div 7 + 16 \div 8 + 5 \times 2$
32. $(8 \times 7 + 4) \div 10 - 5$ **33.** $(18 \div 9 + 4) - (16 \div 8 - 1)$
34. $(9 \times 8 + 3) \div 15 + 7 - 6 \div 2$ **35.** $4 \times 12 + 7 \div (10 \div 2)$
36. $9 \div 3 + 5 \times 2 - 2 \times 6$
37. $40 + 8 + 4 \times 3 + 6 = ?$ **38.** $24 - 8 + 16 \div 4 + 7 = ?$
39. $(48 + 8) \div (2 \times 4) + 3 + 2 \times 5 = ?$
40. $(39 - 7) \div (8 \times 2) + (8 - 4) = ?$
41. $(8 \times 4 + 3) \div (15 \div 3) = ?$
42. $(8 \times 7 - 6) \div (15 - 10 \div 5) = ?$
43. $8 + 7 - 9 + 3 + 5 \times 3 = ?$
44. $(27 \div 9 + 3) \times 2 \times 6 = ?$

38 Drill in Addition and Subtraction

20	8	7	9	13	5	4	23	11
6								17
18								6
9								10
1								15
5								4
8								3
7								11
14								19
25	16	9	4	39	6	8	22	8

Begin with any number in the margin and add till 100 or some other number specified by the teacher is reached.
Begin with 100 or any specified number and subtract the successive numbers.
Have a contest for speed and accuracy.

39. Test Questions

1. If you have the multiplicand and multiplier given, how do you find the product?

2. If all the factors of a number are given, how is the number found?

3. What is an integral number?

4. What is the only even number that is prime?

5. Name five odd numbers that are prime.

6. Name five odd numbers that are composite.

7. When are two numbers prime to each other?

8. What is the L. C. M. of two or more prime numbers?

9. How can you tell whether a number is divisible by 2? by 4? by 3? by 5? by 9?

10. Could you write a number that is divisible by 2, 3, 4, 5 and 9?

11. What is the difference between a common multiple of two or more numbers and the L. C. M. of those numbers?

12. What is the difference between a common divisor of two or more numbers and the G. C D. of those numbers?

13. If you knew the cost of a farm and the price paid an acre, how would you find the number of acres?

14. If you knew the number of yards Mary bought for her dress and the price per yard, how would you find the cost of her dress?

15. If a man owed two accounts and wished to pay each in bills of the same value, how would you find the size of the bills?

16. What is the least debt that can be paid in bills of either \$5, \$10, \$20 or \$50 denominations?

17. What is the use of the parenthesis?

18. If you knew the cost of $\frac{1}{2}$ dozen eggs, how would you find the cost of 2 dozen?

19. If you had the subtrahend and remainder given, how would you find the minuend?

20. If you knew the cost of $2\frac{1}{2}$ yd. of cloth, how would you find the cost of 4 yd.?

21. Is the product of two prime numbers a prime number? Illustrate.

22. If you knew the cost of a number of oranges and the price of one orange, how would you find the number of oranges?

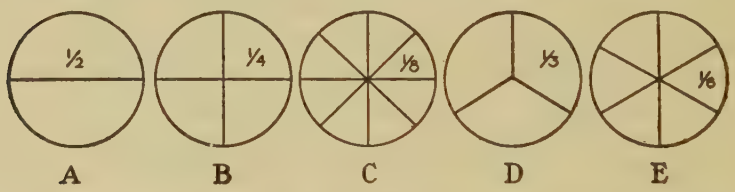
40. FRACTIONS

Before anything can be measured, we must have a unit of measure. What is the length of your book? What is the unit of measure?

What is the unit of measure in each of the following? \$5; 7 bu.; 8 yds.; 3 hrs.; 10 *units*.

Note that the number and its unit of measure must be of the same denomination.

A unit is a quantity used for comparing or measuring another quantity of the same kind; as, 1 bu. for 7 bu.; 1 day for 10 days.



Into how many equal parts is circle A divided? Circle B? Circle C? Circle D? Circle E?

One of the equal parts into which an integral unit is divided is a *fractional unit*. Name the fractional units in circle A; B; C; D; E.

A fraction is one or more fractional units, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{7}{8}$, $\frac{2}{3}$, $\frac{5}{6}$.

Fractions may be expressed in the form of division in which the numerator takes the place of the dividend and the denominator takes the place of the divisor. Thus, $3 \div 2$ may be written $\frac{3}{2}$.

The *denominator* names the size of the fractional unit as compared with the integral unit. The word *denominate* means to name.

The *numerator* tells the number of fractional units taken; thus, the fraction $\frac{7}{8}$ tells that a unit has been divided into 8 equal parts and that 7 of the fractional units are taken.

1. In the fraction $\frac{6}{7}$:

What is the 7 called? What does it show?

What is the 6 called? What does it show?

What does the fraction $\frac{6}{7}$ show?

The numerator and the denominator are the terms of the fraction.

2. Read aloud and tell what the terms of each of the fractions show: $\$\frac{3}{4}$, $\frac{7}{8}$ ft., $\frac{5}{9}$ mi., $\frac{15}{16}$ hr., $\frac{10}{11}$, $\frac{14}{15}$, $\frac{49}{50}$, $\frac{7}{8}$ bu.

Which of the above fractions indicates the largest fractional unit? The smallest?

In which is the largest number of fractional units taken? The smallest number of fractional units taken?

41. Changing to Higher Terms

By comparing circles, we can see that

$\frac{1}{2}=\frac{2}{4}=\frac{4}{8}$; $\frac{1}{4}=\frac{2}{8}$; $\frac{3}{4}=\frac{6}{8}$; $\frac{1}{3}=\frac{2}{6}$; $\frac{2}{3}=\frac{4}{6}$.

Hence, both terms of a fraction may be multiplied by the same number without changing the value of the fraction.

1. Change $\frac{3}{7}$ to twenty-eighths (28ths): $\frac{3}{7}=\frac{3\times4}{7\times4}=\frac{12}{28}$

To get the required denominator 28, we must multiply the denominator 7 by 4. If we multiply the denominator by 4, the numerator must also be multiplied by 4, otherwise the value of the fraction would be changed.

2. Change to 8ths: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{4}{4}$.

3. Change to 18ths: $\frac{1}{2}$, $\frac{1}{8}$, $\frac{5}{6}$, $\frac{8}{9}$, $\frac{2}{3}$, $\frac{1}{6}$, $\frac{7}{9}$.

4. Change to 24ths: $\frac{1}{2}$, $\frac{1}{8}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{8}$, $\frac{5}{8}$, $\frac{11}{12}$.

5. Change to 30ths: $\frac{1}{2}$, $\frac{1}{8}$, $\frac{5}{6}$, $\frac{9}{10}$, $\frac{14}{15}$, $\frac{3}{5}$, $\frac{2}{3}$.

6. Change to 36ths: $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{9}$, $\frac{11}{12}$, $\frac{17}{18}$, $\frac{8}{9}$.

7. Change to 20ths: $\frac{1}{2}$, $\frac{3}{4}$, $\frac{4}{5}$, $\frac{9}{10}$, $\frac{3}{5}$, $\frac{7}{10}$, $\frac{2}{5}$.

8. How do you change fractions from lower to higher terms?

42. Changing to Lowest Terms

$\$\frac{4}{8} = \$\frac{1}{2}$. $\frac{9}{12}$ ft. $= \frac{3}{4}$ ft. $\frac{9}{15}$ yd. $= \frac{3}{5}$ yd.

1. Change $\frac{18}{24}$ to its lowest terms:

$$\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{9 \div 3}{12 \div 3} = \frac{3}{4}$$

Divide both terms by any common factor and continue the division until numerator and denominator are prime to each other. The reduction can be made more quickly by dividing both terms by their G. C. D. Thus:

$$\frac{18}{24} = \frac{18 \div 6}{24 \div 6} = \frac{3}{4}$$

Change to lowest terms:

2. $\frac{2}{4}, \frac{4}{8}, \frac{3}{9}, \frac{6}{9}, \frac{4}{10}, \frac{8}{12}, \frac{13}{15}, \frac{10}{15}, \frac{14}{18}, \frac{18}{24}$.

3. $\frac{12}{15}, \frac{15}{18}, \frac{18}{20}, \frac{18}{21}, \frac{20}{24}, \frac{21}{24}, \frac{24}{28}, \frac{25}{30}, \frac{28}{35}, \frac{30}{32}$.

4. $\frac{22}{88}, \frac{11}{44}, \frac{10}{22}, \frac{24}{32}, \frac{24}{27}, \frac{18}{27}, \frac{16}{18}, \frac{35}{42}, \frac{42}{48}, \frac{49}{56}$.

5. $\frac{32}{56}, \frac{30}{80}, \frac{56}{88}, \frac{34}{68}, \frac{72}{84}, \frac{66}{88}, \frac{77}{99}, \frac{84}{96}, \frac{90}{100}, \frac{76}{95}$.

6. $\frac{110}{125}$ **7.** $\frac{248}{576}$ **8.** $\frac{512}{624}$ **9.** $\frac{121}{363}$ **10.** $\frac{144}{1728}$ **11.** $\frac{262}{917}$

12. $\frac{150}{225}$ **13.** $\frac{302}{755}$ **14.** $\frac{423}{987}$ **15.** $\frac{90}{198}$ **16.** $\frac{267}{899}$ **17.** $\frac{1050}{8150}$

18. $\frac{25}{80}$ **19.** $\frac{24}{28}$ **20.** $\frac{24}{26}$ **21.** $\frac{56}{64}$ **22.** $\frac{10}{12}$ **23.** $\frac{7}{21}$

24. $\frac{15}{21}$ **25.** $\frac{16}{28}$ **26.** $\frac{12}{28}$ **27.** $\frac{45}{68}$ **28.** $\frac{63}{81}$ **29.** $\frac{72}{81}$

30. $\frac{60}{96}$ **31.** $\frac{108}{120}$ **32.** $\frac{36}{144}$ **33.** $\frac{76}{95}$ **34.** $\frac{85}{95}$ **35.** $\frac{116}{144}$

36. $\frac{210}{820}$ **37.** $\frac{85}{105}$ **38.** $\frac{38}{95}$ **39.** $\frac{38}{57}$ **40.** $\frac{77}{88}$ **41.** $\frac{17}{51}$

How can you tell when a fraction is in its lowest terms?

What effect does it have upon the fraction to divide numerator and denominator by the same number?

43. Changing Improper Fractions to Whole or Mixed Numbers

An **improper** fraction is one whose value is equal to or greater than 1; as $\frac{2}{2}$, $\frac{9}{7}$, $\frac{5}{5}$.

A **proper** fraction is one whose value is less than 1, as $\frac{2}{3}$, $\frac{5}{6}$.

Fractions whose numerator and denominator are both expressed are called **common fractions.**

A number composed of a whole number and a fraction is called a **mixed number,** as $2\frac{1}{2}$ bu., $3\frac{1}{2}$ yd.

1. Change $\frac{15}{4}$ to a mixed number.

1 = 4 fourths and in $\frac{15}{4}$ there are as many times 1 as 4 fourths is contained in 15 fourths, or $3\frac{3}{4}$.

Divide the numerator by the denominator and write the remainder over the denominator.

Change to whole or mixed numbers:

1. $\frac{168}{3}$	**2.** $\frac{8414}{7}$	**3.** $\frac{116}{29}$	**4.** $\frac{418}{15}$	**5.** $\frac{801}{89}$
6. $\frac{1001}{40}$	**7.** $\frac{513}{24}$	**8.** $\frac{3801}{91}$	**9.** $\frac{8651}{108}$	**10.** $\frac{756}{28}$
11. $\frac{848}{26}$	**12.** $\frac{87}{4}$	**13.** $\frac{29}{7}$	**14.** $\frac{26}{5}$	**15.** $\frac{45}{8}$
16. $\frac{1284}{96}$	**17.** $\frac{98}{7}$	**18.** $\$\frac{53}{4}$	**19.** $\frac{25}{6}$ lb.	**20.** $\frac{137}{6}$

44. Changing Whole or Mixed Numbers to Improper Fractions

1. Change 4 to a fraction with 4 for a denominator.

Solution: $1 = \frac{4}{4}$

$4 = 4 \times \frac{4}{4} = \frac{16}{4}$

2. Change 5 to a fraction whose denominator is 6.

3. Change to thirds: 1, 4, 5, 6, 7, 8.

4. Change to sixths: 1, 5, 7, 8, 9, 10.

5. Change to fifteenths: 1, 2, 5, 7, 8, 10.

6. Change $\$2\frac{1}{2}$ to an improper fraction.

$\$1 = \$\frac{2}{2}$.
$\$2 = \$\frac{4}{2}$; $\$\frac{4}{2} + \$\frac{1}{2} = \$\frac{5}{2}$.
$\$2\frac{1}{2} = \$\frac{5}{2}$.

Multiply the denominator of the fraction by the whole number and add to the product the numerator of the fraction, writing the sum over the denominator of the fraction.

7.	$2\frac{3}{7}$	**8.**	$7\frac{5}{7}$	**9.**	$12\frac{2}{8}$	**10.**	$12\frac{5}{6}$
11.	$25\frac{1}{2}$ yd.	**12.**	$4\frac{1}{8}$	**13.**	$57\frac{1}{4}$	**14.**	$67\frac{3}{5}$
15.	$16\frac{2}{3}$	**16.**	$20\frac{4}{7}$	**17.**	$37\frac{1}{2}$	**18.**	$33\frac{1}{3}$
19.	$66\frac{2}{8}$	**20.**	$69\frac{3}{4}$	**21.**	$201\frac{1}{2}$	**22.**	$87\frac{1}{2}$
23.	$86\frac{1}{5}$	**24.**	$35\frac{3}{5}$	**25.**	$125\frac{7}{12}$	**26.**	$97\frac{2}{8}$
27.	$77\frac{3}{7}$	**28.**	$10\frac{1}{3}$	**29.**	$187\frac{9}{10}$	**30.**	$88\frac{8}{9}$

45. Changing to Fractions with Least Common Denominator (L. C. D.)

A **common denominator** of two or more fractions is a common multiple of all the denominators; thus, 12 is a common denominator of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{6}$.

The **least common denominator** of two or more fractions is the least common multiple of all the denominators of the fractions; thus, 6 is the L. C. D. of $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{6}$.

Fractions that have the same denominator are similar fractions.

1. Reduce $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$ to similar fractions having the least common denominator.

$\frac{2}{3} \times \frac{4}{4} = \frac{8}{12}$
$\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}$
$\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}$

What is the least common multiple of 3, 4, and 6? What did you do to both terms of each fraction to change the fractions to twelfths?

Change to similar fractions with least common denominator:

2. $\frac{1}{4}, \frac{1}{2}, \frac{1}{8}$ **3.** $\frac{1}{2}, \frac{1}{4}, \frac{3}{5}$ **4.** $\frac{2}{8}, \frac{3}{5}, \frac{5}{6}$

5. $\frac{1}{8}, \frac{7}{8}, \frac{11}{12}$
6. $\frac{2}{8}, \frac{5}{6}, \frac{7}{9}$
7. $\frac{1}{2}, \frac{3}{8}, \frac{7}{16}$
8. $\frac{1}{6}, \frac{4}{9}, \frac{7}{12}$
9. $\frac{3}{10}, \frac{1}{15}, \frac{29}{30}$
10. $\frac{5}{7}, \frac{2}{8}, \frac{6}{7}$
11. $\frac{8}{9}, \frac{17}{18}, \frac{5}{6}$
12. $\frac{1}{4}, \frac{3}{11}, \frac{5}{22}$
13. $\frac{2}{5}, \frac{9}{10}, \frac{14}{15}$
14. $\frac{1}{2}, \frac{2}{8}, \frac{5}{6}, \frac{8}{9}$
15. $\frac{2}{8}, \frac{3}{7}, \frac{13}{14}, \frac{3}{21}$
16. $\frac{1}{4}, \frac{1}{8}, \frac{1}{3}, \frac{3}{16}, \frac{5}{32}$
17. $\frac{5}{6}, \frac{7}{10}, \frac{4}{5}, \frac{7}{15}$
18. $\frac{3}{7}, \frac{1}{8}, \frac{3}{14}, \frac{10}{21}$
19. $\frac{1}{8}, \frac{1}{6}, \frac{2}{9}, \frac{5}{18}, \frac{7}{12}$
20. $\frac{1}{4}, \frac{3}{11}, \frac{5}{22}, \frac{3}{44}$
21. $\frac{2}{7}, \frac{5}{12}, \frac{1}{3}, \frac{1}{4}$
22. $\frac{1}{2}, \frac{3}{4}, \frac{7}{8}, \frac{15}{16}, \frac{29}{32}$
23. $2\frac{1}{2}, 3\frac{1}{3}, 5\frac{1}{4}$
24. $\frac{7}{8}, \frac{5}{12}, \frac{5}{6}, 1\frac{1}{2}$
25. $\frac{8}{9}, \frac{17}{18}, 5\frac{1}{6}, 2\frac{2}{3}$
26. $\frac{1}{2}, \frac{9}{10}, \frac{19}{20}, \frac{4}{5}$
27. $37\frac{1}{2}, 3\frac{1}{4}, 3\frac{1}{8}, \frac{3}{4}$
28. $\frac{9}{10}, 4\frac{1}{15}, \frac{29}{30}, \frac{5}{6}$
29. $9\frac{1}{2}, 10\frac{2}{8}, 12\frac{1}{6}, 15\frac{1}{4}$.

46. Addition

1. Count by $\frac{1}{3}$'s to 6. Thus: $\frac{1}{3}, \frac{2}{3}, 1, 1\frac{1}{3}, 1\frac{2}{3}, 2$, etc.
2. Count by $\frac{1}{2}$'s to 8.
3. Count by $\frac{1}{4}$'s to 7.
4. Count by $1\frac{1}{2}$'s to 12.
5. Count by $1\frac{1}{3}$'s to 18.
6. Count by $2\frac{1}{2}$'s to 50.
7. Count by $3\frac{1}{2}$'s to 21.

We have learned that only like integral units can be added.

Before fractions can be added, they must be parts of like integral units and must also have fractional units of the same size.

8. What change must be made before $\$\frac{1}{2}$ and $\$\frac{1}{4}$ can be added? The units are *alike*, but they are not of the same size. $\$\frac{1}{2} + \$\frac{1}{4} = \$\frac{2}{4} + \$\frac{1}{4} = \$\frac{3}{4}$.

9. Add: $\frac{7}{8} + \frac{3}{4} + \frac{1}{16}$.

$\frac{7}{8} + \frac{3}{4} + \frac{1}{16} = \frac{14}{16} + \frac{12}{16} + \frac{3}{16} = \frac{29}{16} = 1\frac{13}{16}$

To add fractions: Reduce the fractions to similar fractions, add the numerators, and place the sum over the common denominator. Always change the fraction in the answer to its simplest form.

10. $\frac{5}{6} + \frac{7}{12}$
11. $\frac{4}{5} + \frac{11}{15}$
12. $\frac{3}{7} + \frac{9}{14}$

13. $\frac{7}{8}+\frac{15}{16}$ **14.** $\frac{1}{8}+\frac{3}{7}$ **15.** $\frac{8}{9}+\frac{2}{8}$

16. $\frac{15}{17}+\frac{19}{34}$ **17.** $\frac{3}{5}+\frac{5}{6}$ **18.** $\frac{3}{4}+\frac{5}{6}+\frac{11}{12}$

19. $\frac{7}{8}+\frac{3}{4}+\frac{15}{16}$ **20.** $\frac{5}{9}+\frac{2}{8}+\frac{17}{18}$

21. $\frac{1}{5}+\frac{9}{10}+\frac{13}{15}$ **22.** $\frac{3}{5}+\frac{1}{4}+\frac{5}{8}$ **23.** $\frac{1}{6}+\frac{1}{9}+\frac{5}{18}$

24. $\frac{3}{14}+\frac{2}{7}+\frac{6}{35}+\frac{8}{21}$ **25.** $\frac{1}{6}+\frac{2}{9}+\frac{2}{3}+\frac{17}{18}$

26. $\frac{5}{8}+\frac{5}{12}+\frac{15}{32}+\frac{13}{16}$ **27.** $\frac{1}{3}+\frac{3}{23}+\frac{5}{69}+\frac{7}{28}$

28. $\frac{1}{7}+\frac{2}{9}+\frac{8}{21}+\frac{1}{3}$ **29.** $\frac{1}{8}+\frac{4}{9}+\frac{5}{6}+\frac{11}{12}$

To add mixed numbers, add the sum of the fractions to the sum of the integers.

30. Find the sum of $5\frac{1}{2}$, $6\frac{7}{8}$, $9\frac{5}{12}$, $6\frac{3}{16}$.

48 = L. C. D.

$5\frac{1}{2} = 5+\frac{1}{2} = 5+\frac{24}{48}$

$6\frac{7}{8} = 6+\frac{7}{8} = 6+\frac{42}{48}$

$9\frac{5}{12} = 9+\frac{5}{12} = 9+\frac{20}{48}$

$6\frac{5}{16} = 6+\frac{5}{16} = 6+\frac{15}{48}$

$26+\frac{101}{48} = 26+2\frac{5}{48} = 28\frac{5}{48}$

Find the sum of:

31. $4\frac{5}{8}$, $21\frac{5}{6}$, $84\frac{1}{8}$ **32.** $105\frac{3}{7}$, $287\frac{1}{8}$, $365\frac{2}{8}$

33. $17\frac{3}{4}$, $19\frac{5}{6}$, $18\frac{7}{8}$ **34.** $82\frac{2}{3}$, $87\frac{1}{8}$, $18\frac{5}{16}$

35. $175\frac{3}{5}$, $871\frac{9}{10}$, $43\frac{2}{15}$ **36.** $57\frac{5}{21}$, $87\frac{8}{63}$, $55\frac{4}{9}$, $88\frac{3}{7}$

37. $28\frac{1}{7}$, $43\frac{1}{6}$, $95\frac{17}{42}$ **38.** $91\frac{8}{9}$, $89\frac{7}{8}$, $66\frac{7}{12}$, $84\frac{5}{24}$

39. $65\frac{5}{8}$, $84\frac{3}{14}$, $66\frac{5}{7}$ **40.** $86\frac{3}{4}$, $17\frac{5}{8}$, $37\frac{1}{2}$, $63\frac{2}{3}$

41. $63\frac{3}{4}$, $36\frac{4}{5}$, $48\frac{5}{6}$ **42.** $97\frac{15}{16}$, $84\frac{29}{32}$, $61\frac{47}{64}$, $105\frac{1}{2}$

47. Oral Drill

1.	$\frac{3}{4}+\frac{1}{2}$	$\frac{2}{8}+\frac{1}{4}$	$\frac{1}{2}+\frac{1}{8}$	$\frac{1}{8}+\frac{1}{6}$
2.	$\frac{1}{8}+\frac{2}{9}$	$\frac{1}{8}+\frac{4}{9}$	$\frac{1}{8}+\frac{1}{9}$	$\frac{1}{6}+\frac{1}{2}$
3.	$\frac{1}{8}+\frac{1}{4}$	$\frac{2}{8}+\frac{1}{4}$	$\frac{3}{4}+\frac{1}{8}$	$\frac{1}{2}+\frac{1}{4}$
4.	$\frac{1}{6}+\frac{1}{8}$	$\frac{1}{8}+\frac{1}{4}$	$\frac{7}{8}+\frac{1}{16}$	$\frac{1}{8}+\frac{5}{6}$
5.	$\frac{1}{2}+\frac{1}{8}$	$\frac{1}{8}+\frac{1}{5}$	$\frac{5}{6}+\frac{1}{2}$	$\frac{7}{9}+\frac{1}{3}$
6.	$\frac{9}{10}+\frac{1}{5}$	$\frac{3}{5}+\frac{7}{10}$	$\frac{5}{8}+\frac{7}{12}$	$\frac{1}{7}+\frac{4}{9}$

48. Problems

1. A boy picked $2\frac{1}{2}$ bu. beans before noon, and $2\frac{1}{3}$ bu. after noon. How many bushels did he pick in a day?

2. A cow gave $29\frac{7}{8}$ lb. milk on Monday, $28\frac{3}{4}$ lb. on Tuesday, $30\frac{1}{3}$ lb. on Wednesday, $29\frac{5}{6}$ lb. on Thursday. How much milk did she produce in the 4 days?

3. A farmer sold $437\frac{3}{8}$ bu. of corn and had left $525\frac{3}{4}$ bu. How many bushels had he before selling?

4. James has $\$5\frac{1}{2}$; Henry has $\$2\frac{3}{4}$ more than James. How much money have they both?

5. A merchant sold to one customer $37\frac{1}{2}$ gal. of gasoline, to another $7\frac{1}{4}$ gal., and to another $5\frac{3}{4}$ gal. How many gallons did he sell in all?

6. A farmer has $17\frac{1}{2}$ A. in one field, in another $5\frac{7}{8}$ A. more than in the first. How many acres in both fields?

7. A pair of shoes cost $\$8\frac{1}{2}$, a coat $\$17\frac{1}{2}$, and a hat $\$4\frac{1}{4}$. What is the total bill?

8. James is $12\frac{5}{6}$ years old and his brother Henry is $2\frac{1}{2}$ years older. What is the sum of their ages?

9. Ellen attended school $17\frac{1}{2}$ days last month. Susan attended 2 days more than Ellen. How many days did they both attend?

10. A man works 10 hrs. a day and sleeps 8 hrs. What fraction of a day does he work? Sleep?

11. Harry grew $67\frac{1}{2}$ bu. of corn on an acre, and Thomas grew $12\frac{3}{4}$ bu. more than Harry. How many bushels did they both grow?

12. A tourist traveled $16\frac{3}{4}$ mi. the first hour, $17\frac{1}{2}$ mi. the second hour, $20\frac{7}{8}$ mi. the third hour, and $19\frac{5}{6}$ mi. the fourth hour. How far did he travel in the four hours?

13. A boy bought a reader for $\$\frac{1}{4}$ and an arithmetic for $\$\frac{2}{5}$. What part of a dollar did they both cost?

14. A merchant had three remnants of silk. In the first were $5\frac{3}{4}$ yd., in the second $6\frac{3}{5}$ yd., and in the third $7\frac{2}{3}$ yd. How many yards were in the three remnants?

15. Add 1 to each term of the fraction $\frac{2}{3}$ and see if the value of the fraction is changed. Subtract 1 from each term. State what change in value, if any.

16. A merchant bought a knife for $\$\frac{3}{5}$. He sold it at a gain of $\$\frac{1}{4}$. For what part of a dollar did he sell the knife?

17. A merchant paid $\$\frac{9}{25}$ a yd. for cloth, which he sold at a gain of $\$\frac{1}{5}$ a yd. For how much did he sell a yard?

18. A post is set 30 in. in the ground and is $5\frac{3}{4}$ ft. above ground. What is the length of the post?

19. A man sold three piles of wood, the first containing $5\frac{3}{4}$ cords, the second $7\frac{2}{3}$ cords, and the third $9\frac{7}{8}$ cords. How many cords did he sell?

20. A rectangular field was $45\frac{1}{2}$ rods long and $37\frac{1}{4}$ rods wide. What was the distance around the field?

21. One side of a triangle is $5\frac{1}{4}$ in., another $7\frac{1}{2}$ in. and the other $9\frac{2}{3}$ in. What is the perimeter of the triangle?

22. If your school room is $30\frac{1}{2}$ ft. long and $24\frac{1}{3}$ ft. wide, what is its perimeter?

23. It is $37\frac{1}{2}$ ft. from the ground to the first limb on a tree, and $25\frac{7}{8}$ ft. from the first limb to the top of the tree. What is the height of the tree?

A $7\frac{1}{2}$ B $6\frac{2}{3}$ C $8\frac{7}{8}$ D

24. It is $7\frac{1}{2}$ miles from A to B, $6\frac{2}{3}$ mi. from B to C, and $8\frac{7}{8}$ mi. from C to D. How far is it from A to D?

25. A dealer bought coal at $\$4\frac{1}{4}$ a ton. He sold it at a profit of $\$1\frac{7}{8}$ a ton. For how much did he sell a ton?

26. What is the cost of a suit at $\$39\frac{3}{4}$ and a pair of shoes at $\$9\frac{1}{4}$?

49. Subtraction

$\frac{7}{\text{oranges}} - \frac{4}{\text{oranges}} = \frac{3}{\text{oranges}}$. Similarly, $\frac{7}{8} - \frac{4}{8} = \frac{3}{8}$.

Before fractions can be subtracted, the integral units must be like and the fractional units the same size.

1. From $\frac{7}{8}$ subtract $\frac{2}{3}$. 24 = L. C. D.

$\frac{7}{8} = \frac{21}{24}$

$\frac{2}{3} = \frac{16}{24}$. $\frac{21}{24} - \frac{16}{24} = \frac{5}{24}$

What is the first step in the subtraction of fractions?

After fractions are made similar, subtract numerator of subtrahend from numerator of minuend; place the result over the denominator and reduce to simplest form.

Oral Drill

Name remainder:

1.	$\frac{1}{2} - \frac{1}{4}$	$\frac{2}{3} - \frac{1}{6}$	$\frac{3}{4} - \frac{5}{8}$	$\frac{5}{6} - \frac{11}{12}$
	$\frac{5}{6} - \frac{1}{3}$	$\frac{8}{9} - \frac{2}{3}$	$\frac{8}{9} - \frac{5}{6}$	$\frac{11}{12} - \frac{1}{3}$
2.	$\frac{2}{3} - \frac{1}{2}$	$\frac{3}{4} - \frac{1}{3}$	$\frac{5}{8} - \frac{1}{4}$	$\frac{9}{10} - \frac{1}{5}$
	$\frac{9}{10} - \frac{1}{2}$	$\frac{7}{10} - \frac{2}{5}$	$\frac{29}{36} - \frac{7}{9}$	$\frac{23}{36} - \frac{4}{9}$
3.	$\frac{11}{15} - \frac{1}{3}$	$\frac{13}{15} - \frac{4}{5}$	$\frac{8}{15} - \frac{1}{5}$	$\frac{17}{18} - \frac{8}{9}$
	$\frac{13}{18} - \frac{2}{9}$	$\frac{11}{18} - \frac{4}{9}$	$\frac{37}{40} - \frac{6}{10}$	$\frac{37}{42} - \frac{6}{7}$
4.	$\frac{7}{18} - \frac{1}{3}$	$\frac{11}{20} - \frac{1}{2}$	$\frac{19}{20} - \frac{4}{4}$	$\frac{17}{20} - \frac{3}{4}$
	$\frac{3}{20} - \frac{1}{10}$	$\frac{7}{20} - \frac{1}{4}$	$\frac{11}{18} - \frac{17}{36}$	$\frac{29}{40} - \frac{7}{10}$
5.	$\frac{7}{24} - \frac{1}{4}$	$\frac{5}{24} - \frac{1}{8}$	$\frac{11}{24} - \frac{1}{3}$	$\frac{13}{24} - \frac{1}{2}$
	$\frac{17}{24} - \frac{2}{3}$	$\frac{23}{24} - \frac{7}{8}$	$\frac{8}{9} - \frac{1}{3}$	$\frac{43}{49} - \frac{6}{7}$
6.	$\frac{6}{25} - \frac{1}{5}$	$\frac{23}{25} - \frac{4}{5}$	$\frac{23}{27} - \frac{7}{9}$	$\frac{19}{27} - \frac{2}{3}$
	$\frac{25}{27} - \frac{8}{9}$	$\frac{26}{27} - \frac{5}{9}$	$\frac{37}{42} - \frac{5}{6}$	$\frac{17}{36} - \frac{5}{18}$
7.	$\frac{5}{6} - \frac{4}{5}$	$\frac{26}{30} - \frac{5}{6}$	$\frac{29}{30} - \frac{9}{10}$	$\frac{9}{10} - \frac{4}{5}$
	$\frac{19}{30} - \frac{1}{3}$	$\frac{17}{30} - \frac{4}{5}$	$\frac{9}{10} - \frac{3}{5}$	$\frac{8}{15} - \frac{1}{5}$
8.	$\frac{7}{16} - \frac{1}{4}$	$\frac{9}{16} - \frac{1}{2}$	$\frac{13}{16} - \frac{3}{4}$	$\frac{19}{32} - \frac{3}{8}$
	$\frac{21}{32} - \frac{5}{8}$	$\frac{27}{32} - \frac{11}{16}$	$\frac{7}{12} - \frac{1}{6}$	$\frac{9}{10} - \frac{4}{5}$

50. Written Work

Solve:

1.	$\frac{7}{8}-\frac{2}{8}$	2.	$\frac{5}{6}-\frac{3}{4}$	3.	$\frac{7}{9}-\frac{1}{6}$	4.	$\frac{7}{8}-\frac{7}{12}$
5.	$\frac{17}{18}-\frac{2}{8}$	6.	$\frac{4}{5}-\frac{2}{10}$	7.	$\frac{21}{25}-\frac{1}{4}$	8.	$\frac{49}{50}-\frac{4}{5}$
9.	$\frac{28}{33}-\frac{6}{11}$	10.	$\frac{5}{7}-\frac{27}{56}$	11.	$\frac{77}{84}-\frac{11}{12}$		
12.	$\frac{89}{96}-\frac{7}{8}$	13.	$\frac{9}{10}-\frac{7}{12}$	14.	$\frac{13}{15}-\frac{7}{12}$		
15.	$\frac{17}{18}-\frac{5}{12}$	16.	$\frac{29}{42}-\frac{4}{7}$	17.	$\frac{2}{3}-\frac{3}{5}$		
18.	$\frac{11}{20}-\frac{8}{15}$	19.	$\frac{16}{21}-\frac{8}{15}$	20.	$\frac{35}{36}-\frac{5}{27}$		
21.	$\frac{12}{35}-\frac{5}{21}$	22.	$\frac{35}{88}-\frac{3}{11}$	23.	$\frac{19}{21}-\frac{17}{36}$		
24.	$\frac{33}{35}-\frac{31}{42}$	25.	$\frac{53}{56}-\frac{7}{8}$	26.	$\frac{49}{63}-\frac{3}{7}$		

27. From $8\frac{5}{9}$ subtract $5\frac{2}{3}$.

L. C. D. = 9

$8\frac{5}{9}=8\frac{5}{9}=7\frac{14}{9}$

$5\frac{2}{3}=5\frac{6}{9}=5\frac{6}{9}$

$2\frac{8}{9}$

In mixed numbers the integers and fractions are subtracted separately. The fraction $\frac{6}{9}$ in the subtrahend is larger than the fraction $\frac{5}{9}$ in the minuend. Hence, we take 1 integer from 8, which is equal to $\frac{9}{9}$, and add it to the $\frac{5}{9}$, which makes the minuend assume the form of $7\frac{14}{9}$.

Solve:

28.	$15\frac{3}{8}-3\frac{1}{2}$	29.	$16\frac{1}{4}-7\frac{5}{12}$
30.	$12\frac{3}{4}-11\frac{7}{8}$	31.	$10\frac{1}{9}-3\frac{1}{6}$
32.	$14\frac{2}{9}-9\frac{5}{8}$	33.	$8\frac{3}{7}-6\frac{4}{5}$
34.	$15\frac{1}{10}-11\frac{1}{2}$	35.	$12\frac{2}{8}-7\frac{3}{5}$
36.	$19\frac{4}{11}-6\frac{3}{7}$	37.	$13\frac{3}{4}-3\frac{4}{5}$
38.	$12\frac{3}{8}-7\frac{4}{5}$	39.	$15\frac{1}{2}-7\frac{1}{3}$
40.	$45\frac{3}{4}-36\frac{2}{8}$	41.	$23\frac{9}{24}-14\frac{11}{12}$
42.	$36\frac{5}{18}-25\frac{11}{15}$	43.	$19\frac{6}{7}-15\frac{5}{63}$
44.	$12\frac{29}{40}-6\frac{3}{10}$	45.	$30\frac{3}{5}-18\frac{1}{7}$
46.	$31\frac{2}{3}-29\frac{3}{4}$	47.	$24\frac{15}{16}-11\frac{2}{3}$
48.	$84\frac{7}{8}-29\frac{2}{3}$	49.	$29\frac{11}{12}-19\frac{1}{2}$
50.	$19\frac{1}{4}-12\frac{1}{2}$	51.	$35\frac{1}{7}-25\frac{1}{6}$

51. Problems

Solve odd-numbered problems orally:

1. The sum of three numbers is 10. The smallest number is $1\frac{2}{3}$. The largest number is $4\frac{1}{3}$ more than the smallest. Find the other number.

2. A, B, and C have $\$562\frac{1}{2}$. A has $\$175\frac{1}{4}$ and B lacks $\$37\frac{1}{2}$ of having as much money as A. How much money has C?

3. The remainder is $\frac{1}{6}$, and the minuend $\frac{2}{3}$. What is the subtrahend?

4. The minuend is $597\frac{15}{16}$ and the subtrahend is $239\frac{1}{2}$. What is the remainder?

5. After spending $\frac{1}{2}$ and $\frac{1}{8}$ of my money, what part of my money have I left?

6. From a tank containing 90 gallons, there were drawn $18\frac{1}{2}$ gal., $16\frac{2}{8}$ gal., $25\frac{4}{7}$ gal., $11\frac{2}{8}$ gal. How many gallons were left in the tank?

7. Ellen canned $\frac{1}{4}$ of her tomatoes, sold $\frac{3}{5}$ of them, and the family consumed the remainder. What part of her tomatoes did the family consume?

8. (a) What is the effect of adding 1 to both terms of $\frac{7}{8}$? (b) Of adding 1 to both terms of any proper fraction? (c) Of subtracting 1 from both terms of a proper fraction?

9. If a man can do a piece of work in 36 da., what part of the work can he do in 15 days?

10. If a man travels $19\frac{1}{4}$ mi. the first hour, $3\frac{1}{2}$ mi. less the second hr., and $3\frac{1}{5}$ mi. more the third hour than the first, how far did he travel in three hours?

11. At a school examination $\frac{1}{20}$ were absent, $\frac{1}{10}$ failed, and the remainder passed. What part of the school passed?

12. James is $16\frac{1}{3}$ years old and Thomas is $2\frac{1}{2}$ years younger. How old is Thomas?

13. If $\frac{1}{5}$ of a pole is in the mud, $\frac{1}{3}$ in the water and the remainder above the water, what part of the pole is above the water?

14. A lady purchased a hat for $12\frac{1}{5}$, a coat for $25\frac{1}{2}$, a handkerchief for $\$\frac{7}{10}$. She gave the merchant a $50 bill. Make the change.

15. If I spent $\$\frac{3}{5}$ for eggs and $\$\frac{3}{4}$ for sugar, how much did I spend for both?

16. A box of goods weighed $567\frac{1}{8}$ lb. If the box weighed $25\frac{3}{4}$ lb., what was the net weight of the goods?

17. A boy had 35¢ and spent 10¢ for a writing tablet. What part of his money did he spend? What part had he left?

18. When coffee sold for $42\frac{1}{2}$¢ a pound, the dealer made $12\frac{1}{2}$¢ a pound. What was the cost?

19. A merchant bought oranges at $\$\frac{2}{5}$ a dozen and sold them at $\$\frac{1}{2}$ a dozen. What part of a dollar did he make on each dozen?

20. Ellen studied her lesson $1\frac{1}{4}$ hr. Susan studied hers $1\frac{1}{2}$ hr. How much longer did Susan study her lesson than Ellen?

21. James earned $\$\frac{1}{4}$ an hour, and Thomas earned $\$\frac{1}{5}$ an hour. How much more did James earn per hour than Thomas?

22. James walked $2\frac{1}{2}$ miles while Henry walked $3\frac{1}{3}$ miles. How much farther did Henry walk than James?

23. A man started on a journey of 75 miles. After he had traveled $27\frac{3}{4}$ miles, how much farther had he to go?

24. Mr. Brown owned a farm containing 180 A. He gave his son $87\frac{1}{2}$ A. How many acres had he left?

25. Ellen spent $\$\frac{3}{4}$ for a book. Susan spent $\$\frac{1}{2}$ for a box of paper. How much more did Ellen spend than Susan?

26. What length must be added to a line $\frac{5}{12}$ ft. long to make it $\frac{1}{2}$ ft.?

27. What must be added to $2\frac{1}{2}$ to make it $3\frac{2}{3}$?

28. Ellen spelled $\frac{9}{10}$ of the words of her lesson and Susan spelled only $\frac{3}{4}$ of them. How many more words did Ellen spell than Susan if there were 40 words in the lesson?

52. Multiplication

$6 \times \$5 = \$30$. 6×5 sevenths $= 30$ sevenths.

5 sevenths is another way of expressing the fraction $\frac{5}{7}$.

Hence, $6 \times \frac{5}{7} = \frac{30}{7} = 4\frac{2}{7}$.

Multiply the number of fractional units by the multiplier for the same reason that the number of \$'s is multiplied by the multiplier.

Solve:

1. $7 \times \frac{1}{5}$	**2.** $8 \times \frac{7}{9}$	**3.** $4 \times \frac{5}{6}$
4. $9 \times \frac{9}{10}$	**5.** $10 \times \frac{7}{11}$	**6.** $2 \times \frac{11}{12}$
7. $12 \times \frac{2}{5}$	**8.** $11 \times \frac{7}{8}$	**9.** $6 \times \frac{3}{7}$
10. $9 \times \frac{3}{4}$	**11.** $7 \times \frac{13}{15}$	**12.** $8 \times \frac{7}{11}$
13. $15 \times \frac{3}{4}$	**14.** $18 \times \frac{15}{16}$	**15.** $25 \times \frac{3}{4}$
16. $21 \times \frac{8}{9}$		

17. $\overset{4}{\cancel{40}} \times \frac{49}{\underset{5}{\cancel{50}}} = \frac{196}{5} = 39\frac{1}{5}$.

Since the product of the numerator by the whole number is divided by the denominator, employ cancellation wherever a common factor may be found.

18. $10 \times \frac{13}{15}$	**19.** $9 \times \frac{16}{18}$	**20.** $11 \times \frac{21}{2}$
21. $15 \times \frac{24}{25}$	**22.** $29 \times \frac{4}{9}$	**23.** $35 \times \frac{69}{70}$
24. $40 \times \frac{5}{11}$	**25.** $40 \times \frac{39}{40}$	**26.** $47 \times \frac{3}{4}$
27. $49 \times \frac{5}{56}$	**28.** $35 \times \frac{5}{7}$	**29.** $17 \times \frac{31}{84}$
30. $18 \times \frac{6}{7}$	**31.** $9 \times \frac{3}{11}$	**32.** $84 \times \frac{3}{7}$
33. $33 \times \frac{7}{11}$	**34.** $43 \times \frac{7}{8}$	**35.** $54 \times \frac{7}{9}$
36. $63 \times \frac{5}{7}$	**37.** $21 \times \frac{7}{11}$	**38.** $91 \times \frac{2}{8}$
39. $68 \times \frac{4}{5}$	**40.** $77 \times \frac{5}{11}$	**41.** $35 \times 1\frac{7}{8}$

42. $\frac{3}{7} \times 21$. Solution: $\frac{3}{\underset{1}{\cancel{7}}} \times \overset{3}{\cancel{21}} = 9$.

When the multiplier is a fraction, the sign $\times$ is equivalent to the word *of*.

43. $\frac{4}{9}$ of 7. Solution: $\frac{1}{9}$ of $7=\frac{7}{9}$.
$\frac{4}{9}$ of $7=\frac{2\,8}{9}=3\frac{1}{9}$

44. $\frac{5}{\cancel{8}}$ of $\frac{\overset{2}{\cancel{16}}}{17}=\frac{10}{17}$ $\quad$ **45.** $\frac{7}{1\,1}$ of $\frac{2\,2}{2\,8}$

46.	$\frac{8}{9}\times\frac{1\,5}{1\,6}$	**47.**	$\frac{7}{1\,0}\times 35$	**48.**	$\frac{1\,1}{1\,2}$ of 36
49.	$\frac{1\,3}{1\,5}$ of $\frac{2\,5}{2\,6}$	**50.**	$\frac{3\,9}{4\,0}$ of $\frac{8}{1\,3}$	**51.**	$\frac{7}{1\,2}$ of 80
52.	$\frac{8}{1\,1}$ of 30	**53.**	$\frac{3}{7}$ of \$35	**54.**	$\frac{7}{1\,2}$ of 84 ft.
55.	$\frac{7}{5\,4}\times 24$	**56.**	$\frac{1\,5}{1\,7}\times 35$	**57.**	$\frac{3\,7}{4\,2}\times 126$
58.	$\frac{7\,1}{7\,7}\times 121$	**59.**	$\frac{3}{7}$ of \$216	**60.**	$\frac{5}{9}$ of 36 yd.
61.	$\frac{3}{8}$ of 64 bu.	**62.**	$\frac{1\,0}{1\,1}$ of \$55.	**63.**	$\frac{5}{6}$ of 12 mo.

64. Show that, in the fraction $\frac{8}{9}$, if you multiply the numerator by 3 or divide the denominator by 3 the value of the resulting fraction is the same.

65. What two ways are there of multiplying a fraction?

53. Multiplying a Whole Number by a Mixed Number

1. Find the cost of $22\frac{5}{7}$ tons of hay at \$28 a ton.

\$28
$22\frac{5}{7}=20+2+\frac{5}{7}$
$20=\$28\times\frac{5}{7}$
$56=\$28\times 2$
$560=\$28\times 20$
\$636

\$28
$22\frac{5}{7}$
20
56
56
\$636

2. How far will a train travel in $16\frac{2}{3}$ hr., if it averages 39 mi. an hour?

3. What is the cost of $17\frac{1}{2}$ T. of coal at \$5.60 a ton?

4. Find the cost of $15\frac{2}{3}$ dozen eggs at 54¢ a dozen.

5. What is the value of $33\frac{3}{4}$ bu. of corn at \$1.20 a bushel?

6. When wheat was selling at \$1.25 a bushel, what was the value of $15\frac{2}{3}$ bu.?

7. If it costs 57¢ a cu. yd. to haul crushed stone, how much will it cost to haul $23\frac{5}{9}$ cu. yd.?

8. Find the cost of $112\frac{3}{4}$ yd. of lace at 30¢ a yard.

9. When coffee sold for 39¢ a pound, find the cost of $29\frac{1}{2}$ lb.

10. If farm lands are worth \$65 an acre, what will be the cost of $87\frac{1}{2}$ A.?

54. Multiplying a Mixed Number by a Whole Number

1. What will 7 yd. of ribbon cost at $22\frac{1}{2}$¢ a yd.?

$$
\begin{array}{r}
.22\frac{1}{2} \\
7 \\
\hline
3\frac{1}{2} = 7\times\frac{1}{2} \\
154 \quad = 7\times 22 \\
\hline
\$1.57\frac{1}{2}
\end{array}
$$

As we have no $\frac{1}{2}$¢ piece of money, we count $\frac{1}{2}$¢ as 1¢ in the answer which is \$1.58. If the fraction of a cent is less than $\frac{1}{2}$ it is not counted.

2. If a man earns $\$7\frac{1}{4}$ a day, how much will he earn in 7 da.?

3. There are $16\frac{1}{2}$ ft. in a rod. How many feet in 25 rd.?

4. There are $30\frac{1}{4}$ sq. yd. in a square rod. How many square yards in 84 sq. rd.?

5. There are $365\frac{1}{4}$ da. in a common year. How many days in 25 yr.?

6. A farmer planted $25\frac{3}{4}$ A. in corn and 6 times as much in cotton. How many acres did he plant in cotton?

7. There are $31\frac{1}{2}$ gal. in a barrel. How many gallons in 19 barrels?

8. It took $6\frac{1}{4}$ yd. to make Mary a dress. How many yards will it take to make 5 dresses for Mary?

9. A farmer plowed $3\frac{1}{2}$ A. a day with his tractor. How many acres can he plow in 9 days?

10. A carpenter receives $\$1\frac{1}{5}$ an hour. How much will he receive in 26 da. if he works 8 hr. a day?

11. A boy rode his bicycle an average of $17\frac{3}{8}$ mi. a day. How far will he ride in 17 da.?

Exercise

Find the product:

1.	$105\frac{3}{4} \times 8$	**2.**	$18\frac{1}{2} \times 9$	**3.**	$36\frac{6}{10} \times 26$
4.	$217 \times 608\frac{3}{7}$	**5.**	$486\frac{3}{4} \times 98$	**6.**	$84 \times 48\frac{7}{12}$
7.	$96\frac{3}{4} \times 72$	**8.**	$248 \times 10\frac{3}{7}$	**9.**	$9867\frac{1}{3}\frac{2}{3} \times 429$
10.	$110\frac{7}{10} \times 19$	**11.**	$497\frac{3}{4} \times 36$	**12.**	$48\frac{8}{9} \times 81$
13.	$763 \times 97\frac{3}{4}$	**14.**	$207\frac{3}{5} \times 75$	**15.**	$967\frac{3}{4} \times 29$
16.	$777 \times 55\frac{5}{7}$	**17.**	$999 \times 77\frac{3}{4}$	**18.**	$832\frac{1}{2} \times 99$
19.	$555\frac{5}{9} \times 99$	**20.**	$712 \times 48\frac{4}{5}$	**21.**	$964\frac{3}{4} \times 96$
22.	$737\frac{7}{10} \times 907$	**23.**	$222\frac{2}{9} \times 97$	**24.**	$884 \times 36\frac{5}{6}$
25.	$676 \times 87\frac{3}{8}$	**26.**	$157 \times 999\frac{1}{8}$	**27.**	$1001\frac{2}{8} \times 66$
28.	$296\frac{3}{4} \times 417$	**29.**	$375 \times 89\frac{3}{5}$	**30.**	$767 \times 676\frac{3}{4}$
31.	$837 \times 79\frac{8}{9}$	**32.**	$625 \times 65\frac{1}{2}$	**33.**	$837 \times 376\frac{9}{10}$

55. Finding Fractional Parts of a Fraction

1. What is $\frac{1}{5}$ of \$10? $\frac{2}{5}$ of \$10?

2. What is $\frac{1}{5}$ of 10 tenths? $\frac{2}{5}$ of 10 tenths?

3. What is $\frac{2}{3}$ of \$9? What is $\frac{2}{3}$ of 9 tenths $(\frac{9}{10})$?

Analysis: $\frac{1}{3}$ of \$9 = \$3 $\qquad$ $\frac{1}{3}$ of $\frac{9}{10} = \frac{3}{10}$

$\frac{2}{3}$ of \$9 = 2 × \$3 or \$6. $\qquad$ $\frac{2}{3}$ of $\frac{9}{10} = 2 \times \frac{3}{10}$ or $\frac{6}{10} = \frac{3}{5}$.

Solve orally:

4.	$\frac{1}{8}$ of $\frac{3}{4}$	**5.**	$\frac{3}{4}$ of $\frac{8}{9}$	**6.**	$\frac{2}{8}$ of $\frac{9}{11}$
7.	$\frac{5}{9}$ of $\frac{18}{19}$	**8.**	$\frac{5}{6}$ of $\frac{11}{12}$	**9.**	$\frac{3}{10}$ of $\frac{20}{21}$
10.	$\frac{3}{5}$ of $\frac{10}{11}$	**11.**	$\frac{7}{8}$ of $\frac{8}{9}$	**12.**	$\frac{7}{8}$ of $\frac{24}{25}$
13.	$\frac{4}{5}$ of $\frac{15}{16}$	**14.**	$\frac{5}{7}$ of $\frac{7}{9}$	**15.**	$\frac{6}{7}$ of $\frac{14}{15}$
16.	$\frac{9}{10}$ of $\frac{30}{81}$	**17.**	$\frac{3}{11}$ of $\frac{19}{22}$	**18.**	$\frac{3}{7}$ of $\frac{7}{9}$
19.	$\frac{2}{11}$ of $\frac{22}{28}$	**20.**	$\frac{5}{7}$ of $\frac{28}{29}$	**21.**	$\frac{3}{7}$ of $\frac{14}{17}$
22.	$\frac{5}{18}$ of $\frac{26}{27}$	**23.**	$\frac{7}{9}$ of $\frac{9}{11}$	**24.**	$\frac{14}{15}$ of $\frac{30}{81}$
25.	$\frac{7}{15}$ of $\frac{45}{46}$	**26.**	$\frac{7}{8}$ of $\frac{32}{88}$	**27.**	$\frac{7}{12}$ of $\frac{36}{87}$
28.	$\frac{5}{18}$ of $\frac{39}{40}$	**29.**	$\frac{5}{6}$ of $\frac{1}{2}$	**30.**	$\frac{2}{8}$ of $\frac{5}{6}$
31.	$\frac{7}{8}$ of $\frac{8}{11}$.	**32.**	$\frac{5}{6}$ of $\frac{18}{19}$	**33.**	$\frac{11}{12}$ of $\frac{24}{25}$

56. Written Work

Find the product:

1. $\frac{3}{4}$ of $\frac{5}{6}=\frac{3\times5}{4\times6}=\frac{15}{24}=\frac{5}{8}$.

A fractional part of a fraction is the product found by multiplying the numerators for the numerator of the product and the denominators for the denominator of the product.

A fraction of a fraction is called a **compound** fraction; thus. $\frac{2}{8}$ of $\frac{3}{4}$ is a compound fraction.

2. $\frac{\cancel{7}}{\cancel{9}_{1}}$ of $\frac{\cancel{18}^{2}}{19}=\frac{14}{19}$

Employ cancellation and strike out all common factors found in numerators and denominators.

3.	$\frac{5}{8}\times\frac{16}{27}$	**4.**	$\frac{3}{4}\times\frac{9}{10}$	**5.**	$\frac{10}{11}\times\frac{3}{5}$
6.	$\frac{7}{9}\times\frac{3}{7}$	**7.**	$\frac{4}{5}\times\frac{7}{12}$	**8.**	$\frac{3}{4}\times\frac{5}{7}$
9.	$\frac{5}{8}\times\frac{9}{10}$	**10.**	$\frac{11}{15}\times\frac{6}{7}$	**11.**	$\frac{7}{9}\times\frac{18}{19}$
12.	$\frac{10}{11}\times\frac{17}{80}$	**13.**	$\frac{8}{17}\times\frac{34}{85}$	**14.**	$\frac{10}{11}\times\frac{33}{85}$
15.	$\frac{3}{4}\times\frac{20}{21}$				

16. $\frac{25}{26}\times\frac{13}{15}$ 17. $\frac{29}{30}\times\frac{5}{6}$ 18. $\frac{42}{48}\times\frac{86}{87}$

19. $\frac{27}{29}\times\frac{58}{81}$ 20. $\frac{64}{65}\times\frac{13}{16}$ 21. $\frac{81}{84}\times\frac{7}{9}$

22. $\frac{71}{72}\times\frac{144}{218}$ 23. $\frac{8}{15}\times\frac{15}{16}$ 24. $\frac{8}{9}\times\frac{6}{7}$

25. $\frac{4}{5}\times\frac{9}{10}$ 26. $\frac{7}{11}\times\frac{33}{49}$

27. $5\frac{1}{2}\times6\frac{2}{3}=\frac{11}{2}\times\frac{20}{3}=\frac{220}{6}=36\frac{2}{3}$.

To find the product of two or more mixed numbers, reduce mixed numbers to improper fractions and proceed as in multiplication of fractions.

28. $8\frac{3}{4}\times10\frac{3}{4}$ 29. $23\frac{9}{11}\times19\frac{4}{7}$

30. $6\frac{3}{4}\times17\frac{4}{5}$ 31. $44\frac{9}{14}\times 7\frac{7}{25}$

32. $6\frac{1}{2}\times 4\frac{1}{26}$ 33. $6\frac{2}{3}\times 4\frac{1}{5}$

34. $6\frac{7}{8}\times 2\frac{3}{4}$ 35. $78\frac{1}{2}\times62\frac{1}{8}$

36. $5\frac{1}{2}\times 5\frac{1}{2}$ 37. $6\frac{2}{3}\times 1\frac{19}{20}$

38. $6\frac{3}{5}\times 6\frac{2}{3}$ 39. $94\frac{1}{2}\times 1\frac{8}{17}$

57. Problems

1. What is the perimeter of a square, each side being $7\frac{7}{8}$ in.?

2. What will be the cost of $8\frac{3}{4}$ T. of timothy hay at $\$22\frac{3}{4}$ a ton?

3. If a workman earns $\$5\frac{1}{2}$ a day, how much will he earn in $5\frac{1}{2}$ days?

4. What is the cost of $5\frac{1}{2}$ lb. roast beef at $27\frac{1}{2}¢$ a pound?

5. A farmer in 1922 grew $24\frac{1}{2}$ bu. of corn to the acre. By better cultivation in 1923 he increased his yield per acre $9\frac{3}{4}$ bu. How much corn did he produce in 1923 on a field containing $20\frac{1}{2}$ A.?

6. A girl bought $5\frac{1}{4}$ yards for a dress at $80¢$ a yard. The trimmings cost her \$1.50, and buttons $75¢$. Find the total cost.

7. A boy grew 75 bu. of corn on 1 A. He sold 50 bu. What part of the corn did he sell? What part had he left?

8. If a family uses $3\frac{1}{2}$ lb. of butter a week, what is the cost at 55¢ a pound?

9. A farmer had \$3200. He spent $\frac{3}{4}$ of his money for live stock, and $\frac{1}{2}$ the remainder for farming implements. How much money had he left?

10. If a boy's average monthly earnings are $\$9\frac{3}{4}$, how much will he earn in 12 months?

11. If James walks $3\frac{1}{2}$ mi. an hour, how far will he walk in $5\frac{1}{2}$ hrs. at the same rate?

12. If $29\frac{7}{8}$ T. of coal are carried in 1 car, how many tons are carried in 28 cars?

13. Find the cost of $5\frac{3}{4}$ lb. steak at 25¢ a pound.

14. A boy spent $\frac{1}{8}$ of his money for a book, and $\frac{1}{5}$ for a magazine, and had 70¢ left. How much money had he before spending any?

15. Multiply the sum of $\frac{2}{3}$ and $\frac{5}{6}$ by the difference between $2\frac{1}{2}$ and $1\frac{1}{3}$.

58. Division

6 ft. $\div 3 = ?$ 6 sevenths $\div 3 = ?$

Just as 6 ft. $\div 3 = 2$ ft., so 6 sevenths $\div 3 = 2$ sevenths. That is, $\frac{6}{7} \div 3 = \frac{2}{7}$.

Consider: $\frac{6}{7} \div 5$.

Since we cannot divide the *number* of sevenths by 5 exactly, we must reduce $\frac{6}{7}$ to higher terms so that the numerator will contain 5 an exact number of times.

$\frac{6}{7} \div 5 = \frac{30}{35} \div 5 = \frac{6}{35}$; or we may obtain the same result by multiplying $\frac{6}{7}$ by the reciprocal of 5; thus, $\frac{6}{7} \times \frac{1}{5} = \frac{6}{35}$.

The reciprocal of a number is 1 divided by that number.

The reciprocal of a fraction is simply the fraction inverted. The reciprocal of $\frac{2}{5}$ is $\frac{5}{2}$; the reciprocal of $\frac{1}{2}$ is $\frac{2}{1}$ or 2.

From these illustrations it is seen that a fraction is divided by an integral number either by dividing the numerator or by multiplying the denominator by the integer.

Solve:

1.	$\frac{6}{11} \div 2$	**2.**	$\frac{12}{19} \div 4$	**3.**	$\frac{10}{17} \div 5$
4.	$\frac{8}{11} \div 8$	**5.**	$\frac{3}{5} \div 5$	**6.**	$\frac{2}{3} \div 9$
7.	$\frac{14}{19} \div 14$	**8.**	$\frac{1}{5} \div 8$	**9.**	$\frac{30}{81} \div 5$
10.	$\frac{20}{81} \div 6$	**11.**	$\frac{48}{49} \div 16$	**12.**	$\frac{5}{12} \div 7$
13.	$\frac{3}{5} \div 6$	**14.**	$\frac{7}{15} \div 3$	**15.**	$\frac{18}{19} \div 12$
16.	$\frac{8}{13} \div 16$	**17.**	$\frac{4}{25} \div 8$	**18.**	$\frac{3}{20} \div 9$
19.	$\frac{15}{16} \div 10$	**20.**	$\frac{5}{12} \div 20$		

21. $8 \div \frac{4}{5} = \frac{8}{1} \div \frac{4}{5} = \frac{40}{5} \div \frac{4}{5} = 10$. The same result is obtained by multiplying the dividend by the divisor inverted; thus, $\frac{8}{1} \div \frac{4}{5} = \frac{\overset{2}{\cancel{8}}}{1} \times \frac{5}{\cancel{4}_{1}} = 10$.

22.	$16 \div \frac{8}{9}$	**23.**	$9 \div \frac{5}{7}$	**24.**	$18 \div \frac{9}{16}$
25.	$21 \div \frac{3}{7}$	**26.**	$25 \div \frac{5}{6}$	**27.**	$28 \div \frac{7}{8}$
28.	$36 \div \frac{4}{9}$	**29.**	$65 \div \frac{13}{14}$	**30.**	$\frac{5}{12} \div 6$
31.	$\frac{14}{17} \div 6$	**32.**	$\frac{42}{43} \div 7$	**33.**	$44 \div \frac{11}{12}$

34. $\frac{6}{25} \div \frac{8}{15} = \frac{\overset{3}{\cancel{6}}}{\underset{5}{\cancel{25}}} \times \frac{\overset{3}{\cancel{15}}}{\underset{4}{\cancel{8}}} = \frac{9}{20}$

35.	$\frac{5}{18} \div \frac{25}{26}$	**36.**	$\frac{41}{42} \div \frac{7}{21}$	**37.**	$\frac{13}{14} \div \frac{13}{15}$
38.	$\frac{3}{14} \div \frac{5}{7}$	**39.**	$\frac{5}{98} \div \frac{10}{27}$	**40.**	$\frac{8}{15} \div \frac{9}{10}$
41.	$\frac{21}{32} \div \frac{9}{16}$	**42.**	$55 \div \frac{11}{4}$	**43.**	$\frac{19}{38} \div \frac{6}{19}$
44.	$\frac{4}{17} \div \frac{11}{21}$	**45.**	$\frac{11}{24} \div \frac{11}{12}$	**46.**	$\frac{23}{169} \div \frac{11}{26}$
47.	$\frac{7}{11} \div 3$	**48.**	$3 \div \frac{7}{11}$	**49.**	$\frac{5}{12} \div \frac{5}{6}$
50.	$65 \div \frac{5}{18}$	**51.**	$\frac{7}{12} \div \frac{3}{4}$	**52.**	$26 \div \frac{2}{3}$

53. $3\frac{3}{4} \div 1\frac{1}{2} = \frac{15}{4} \div \frac{3}{2} = \frac{\overset{5}{\cancel{15}}}{\underset{2}{\cancel{4}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{3}}} = \frac{5}{2} = 2\frac{1}{2}.$

54. $5\frac{5}{6} \div 1\frac{2}{3}$ **55.** $1\frac{2}{5} \div 21$ **56.** $\frac{4}{11} \div 1\frac{3}{8}$

57. $2\frac{3}{4} \div 1\frac{5}{8}$ **58.** $5\frac{1}{2} \div 6\frac{1}{2}$ **59.** $4\frac{1}{4} \div \frac{17}{18}$

60. $\frac{17}{18} \div 4\frac{1}{4}$ **61.** $15\frac{1}{4} \div \frac{31}{82}$ **62.** $7\frac{1}{2} \div \frac{5}{7}$

63. $\frac{5}{7} \div 7\frac{1}{2}$ **64.** $11\frac{1}{2} \div \frac{23}{24}$ **65.** $6\frac{2}{3} \div 16\frac{2}{3}$

59. Review Problems

1. How long will it take to earn \$60, if a man earns $\$7\frac{1}{2}$ a day?

2. How many yards of cloth at $\$\frac{3}{4}$ a yard can be bought for $\$9\frac{1}{4}$?

3. From a purse containing \$10, Mr. Smith pays accounts equivalent to $\$2\frac{1}{4}$, $\$1\frac{1}{2}$, $\$2\frac{3}{10}$, and $\$3\frac{1}{20}$. How much has he left?

4. The three joints of a fishing rod measure separately $40\frac{1}{2}$ in., $40\frac{3}{8}$ in., and $38\frac{1}{4}$ in. When the rod is put together $5\frac{1}{2}$ inches are lost by overlapping of the joints. What is the length of the rod?

5. James gave Thomas $\frac{1}{2}$ of his chestnuts, and Henry $\frac{1}{8}$, and then had 10 chestnuts left. How many had he at first?

6. A man left $\frac{1}{8}$ of his estate to his widow, $\frac{1}{4}$ to his son, and $\frac{1}{5}$ to his daughter. He left the remainder, amounting to \$26,000, to charity. What was the value of the whole estate?

7. If $77\frac{3}{4}$ tons of coal are carried in equal loads in 15 trucks, what is the load carried in each?

8. If there are $31\frac{1}{2}$ gal. in a barrel, how many barrels will it take to hold 252 gal.?

9. There are 31 posts placed at equal distances apart in a straight line. The distance from the first to the last is $240\frac{1}{4}$ ft. What is the distance from one post to the next?

10. What is the income of A who receives $\frac{7}{15}$ of the profits of a business that pays a net dividend of $6,000 annually?

11. A merchant bought $6\frac{1}{2}$ dozen eggs at 40¢ a dozen. He lost by breakage $\frac{3}{4}$ dozen and he sold the remainder for 62¢ more than total cost. What was the selling price per dozen?

12. Multiply the reciprocal of $\frac{2}{5}$ by the reciprocal of $\frac{5}{7}$.

13. Divide the reciprocal of $\frac{1}{6}$ by the reciprocal of $\frac{1}{3}$.

14. Add $\frac{2}{3}$ to the reciprocal of 3.

15. Find the difference between the reciprocals of $\frac{3}{5}$ and $\frac{5}{6}$.

16. A boy sold $\frac{2}{5}$ of his corn and had 30 bu. left. How many bushels had he at first?

17. If oranges sell at 50¢ a dozen, how many oranges can be bought for $1.25?

18. If $\frac{1}{4}$ dozen eggs cost 10¢, how many eggs can be bought for 90¢?

19. A newsboy bought papers for 3¢ each and sold them for 5¢. The cost of the paper was what part of the selling price?

20. A farmer bought a cow for $90 and sold her for $100. The cost price was what part of the selling price?

21. A young man bought a farm for $6,000, live stock for $2,000. The cost of the live stock was what part of the cost of the farm?

22. If $\frac{7}{8}$ of a yard of cloth cost 28¢, what will $\frac{3}{4}$ of a yard cost?

Solution: If $\frac{7}{8}$ yd. cost 28¢, $\frac{1}{8}$ yd. will cost $\frac{1}{7}$ of 28¢, or 4¢. If $\frac{1}{8}$ yd. cost 4¢, $\frac{8}{8}$ or 1 yd. will cost $8\times4¢=32¢$. If 1 yd. cost 32¢, $\frac{3}{4}$ yd. will cost $\frac{3}{4}$ of $32¢=24¢$.

23. If $\frac{3}{5}$ of a bu. of wheat cost $1.35, what is the cost of $\frac{1}{8}$ bu.?

24. There are 12 da. in $\frac{3}{7}$ of the month of February. How many days in $\frac{1}{4}$ of the month?

25. There are 24 da. in $\frac{4}{5}$ of each of the following months, April, June, Sept. and Nov. How many days in $\frac{5}{6}$ of each of the above named months?

26. James has 15¢, and $\frac{1}{8}$ of his money is $\frac{1}{8}$ of Henry's. How much money has Henry?

27. If $\frac{3}{4}$ yd. of silk cost 90¢, what will $2\frac{1}{4}$ yd. cost?

28. If $2\frac{1}{2}$ bu. of corn sell for \$3.75, what will $5\frac{1}{2}$ bu. sell for?

29. A merchant bought 5 bu. apples at \$2.50 a bu. He lost by decay $1\frac{1}{4}$ bu., and sold the remainder for \$2.50 more than total cost. At how much a peck did he sell the apples?

30. If a boy walks $2\frac{1}{2}$ ft. at each step, how many steps will he take in going the distance of 380 ft.?

31. If a girl walks 2 ft. and 2 in. each step, how many steps will she take in walking $925\frac{1}{6}$ ft.?

32. Ellen and Lucy were equal partners in operating a lemonade stand. Their expenses averaged each day 75¢, and they took in \$3.25. How much did each make in a day?

33. Multiply the G. C. D. of 24 and 36 by the L. C. M. of 10 and 15.

34. If corn is produced at a cost of 95¢ a bushel and is sold for \$1.75 a bushel, what is the profit on 1000 bu. of corn?

35. A farmer cultivated 66 A. in corn and $\frac{2}{3}$ as many acres in wheat. How many acres did he cultivate in all?

36. If $5\frac{1}{2}$ yd. of cloth cost \$3.63, what will $6\frac{1}{4}$ yd. cost?

37. If $\frac{3}{8}$ of a bushel of clover seed will sow 3 A., how many bushels of seed will it take to sow 32 A.?

38. The length and breadth of a room appear on measurement with a foot-rule to be 24 feet and 20 feet respectively, but it is found that each foot of the rule is $\frac{1}{24}$ foot too short. What are the true dimensions of the room?

60. Finding the Part One Number is of Another

1. What part of a year is 1 mo.? 2 mo.? 3 mo.? 4 mo.? 5 mo.? 6 mo.? 7 mo.? 8 mo.? 9 mo.? 10 mo.? 11 mo.? 12 mo.?

2. What part of 30 is 1? 2? 3? 4? 5? 6? 8? 10? 12? 15? 16? 18? 19? 20? 21? 22? 24? 25? 26? 27?

3. What part of 50 is 1? 5? 10? 20? 25? 30? 35? 40? 45?

4. What part of a dozen is 11? 10? 9? 8? 6? 4? 3? 2? 1?

5. What part of 60 is a dozen?

6. What part of 100¢ is 5¢? 10¢? 15¢? 20¢? 25¢? 30¢? 35¢? 40¢? 45¢? 50¢? 60¢? 65¢? 70¢? 75¢ 80¢? 90¢? 100¢?

7. What part of a gross (12 doz. or 144) is 12? 24? 48? 36? 64? 72? 84? 96? 120? 132? 144?

8. A merchant had in stock 600 lb. of sugar. He sold on Monday 100 lb., on Tuesday 50 lb., on Wednesday 150 lb., on Thursday 75 lb., on Friday 60 lb., on Saturday 120 lb. What part of his sugar did he sell each day? What part remained unsold at the end of the week?

9. A girl spelled 45 words out of 50. What part of the words did she spell?

10. A pupil goes to school 5 days out of a whole week. What part of a week is he in school?

11. John was absent from school 4 da. out of 20 da. What part of the time did he miss?

12. If a man saves 20¢ out of every dollar he earns, what part of his earnings does he save?

13. A lady had \$60. She spent $\frac{3}{5}$ of her money for a dress and \$12 for a hat. What part of her money had she left?

14. When a pair of shoes that cost \$3 is sold for \$5, how much is the gain? What part of the cost is the gain? What part of the selling price is the gain?

15. A girl had 60¢. She spent $\frac{2}{3}$ of it for a book, and 10¢ for a magazine. What part of her money had she left?

16. There were 100 gallons of gasoline in a tank; $\frac{3}{5}$ of it was sold and 5 gal. leaked out. What part of the gasoline was left in the tank?

17. What part of 1728 is 144?

Solution: $\frac{144}{1728} = \frac{1}{12}$.

18. What part of 986 is 784?

19. What part of 1225 is 375?

20. What part of 832 is 168?

21. What part of a dollar is a dime?

22. What part of a century is 25 yrs.?

23. What part of 1276 is 492?

24. What part of $\frac{5}{7}$ is $\frac{2}{3}$? $\frac{2}{3} \div \frac{5}{7} = \frac{2}{3} \times \frac{7}{5} = \frac{14}{15}$.

To find what part one fraction is of another, divide the one representing the part by the one representing the whole.

25. What part of: $\frac{15}{16}$ is $\frac{3}{4}$?

26. $\frac{23}{25}$ is $\frac{13}{15}$?

27. $\frac{49}{50}$ is $\frac{2}{5}$?

28. $\frac{15}{17}$ is $\frac{5}{7}$?

29. $\frac{27}{28}$ is $\frac{8}{9}$?

30. $\frac{60}{63}$ is $\frac{10}{21}$?

31. $\frac{19}{20}$ is $\frac{4}{5}$?

32. $\frac{17}{18}$ is $\frac{1}{6}$?

33. $\frac{14}{15}$ is $\frac{7}{9}$?

34. $\frac{6}{7}$ is $\frac{12}{13}$?

35. The width of a field is $37\frac{1}{2}$ rods, and the length is 60 rods. What part of the length is the width?

36. Ellen bought a book for 75¢, which she afterwards sold for 60¢. What part of the cost did she lose?

61. Compound Fractions

If I cut an apple into 2 equal parts, 1 part is $\frac{1}{2}$ of 1 apple. If I cut $\frac{1}{2}$ apple into two equal parts, 1 part is $\frac{1}{2}$ of $\frac{1}{2}$ apple or $\frac{1}{4}$ of the whole apple.

If I take $\frac{1}{4}$ apple and divide it into 2 equal parts, 1 part is $\frac{1}{2}$ of $\frac{1}{4}$ apple, or $\frac{1}{8}$ of the whole apple.

Such expressions as $\frac{1}{2}$ of $\frac{1}{2}$, and $\frac{1}{2}$ of $\frac{1}{2}$ of $\frac{1}{2}$, are called compound fractions.

A **compound fraction** expresses a fractional part of a fraction.

A single fraction, as $\frac{2}{5}$, is called a **simple fraction.**

Compound fractions are converted into simple fractions by multiplying the numerators together for a new numerator and the denominators together for a new denominator. Use cancellation wherever possible.

Reduce to simple fractions:

1. $\frac{1}{\cancel{2}_{1}} \text{ of } \frac{\cancel{2}^{1}}{\cancel{3}_{1}} \text{ of } \frac{\cancel{3}^{1}}{\cancel{4}} = \frac{1}{4}$

2. $\frac{3}{4}$ of $\frac{5}{6}$ of $\frac{6}{7}$

3. $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{15}$

4. $\frac{9}{10}$ of $\frac{10}{11}$ of $\frac{11}{18}$

5. $\frac{3}{8}$ of $\frac{16}{17}$ of $\frac{2}{8}$

6. $\frac{1}{2}$ of $\frac{15}{16}$ of $\frac{20}{21}$

7. $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{9}{10}$

8. $\frac{24}{25}$ of $\frac{5}{8}$ of $\frac{7}{12}$

9. $\frac{49}{50}$ of $\frac{11}{14}$ of $\frac{10}{11}$

10. $\frac{30}{81}$ of $\frac{62}{63}$ of $\frac{7}{10}$

11. $\frac{24}{25} \times \frac{25}{26} \times \frac{13}{24}$

12. $\frac{39}{40} \times \frac{10}{13} \times \frac{1}{2}$

13. $\frac{7}{9}$ of $\frac{9}{10}$ of $\frac{10}{11}$

14. A man owns $\frac{7}{12}$ of a ship and sells $\frac{1}{3}$ of his share. What part of the ship does he sell? What part has he left?

15. A farmer planted $\frac{3}{4}$ of a field in corn; $\frac{2}{3}$ of the land in corn overflowed during high water. What part of the field was overflowed?

62. Finding the Whole When a Part is Given

1. Find the number of which 244 is $\frac{4}{11}$.

$\frac{4}{11}$ of the number = 244

$\frac{1}{11}$ of the number = $\frac{1}{4}$ of 244 or 61

$\frac{11}{11}$ of the number = 11×61 = 671.

If 244 is $\frac{4}{11}$ of a number, $\frac{1}{11}$ of the number is $\frac{1}{4}$ of 244 or 61. If $\frac{1}{11}$ of the number is 61, $\frac{11}{11}$ of it, or the whole, is 11×61 or 671.

Or, $244 \div \frac{4}{11} = \frac{\overset{61}{\cancel{244}}}{\underset{1}{\cancel{4}}} \times \frac{11}{\cancel{4}} = 671$

Divide the part of the whole given by its fractional part of the whole.

2. Find the number of which $\frac{9}{10}$ is $\frac{3}{7}$.

$\frac{3}{7}$ of the number = $\frac{9}{10}$

$\frac{1}{7}$ of the number = $\frac{1}{3}$ of $\frac{9}{10}$ or $\frac{8}{10}$

$\frac{7}{7}$ of the number = $7 \times \frac{8}{10} = \frac{21}{10} = 2\frac{1}{10}$.

If $\frac{9}{10}$ is $\frac{3}{7}$ of the number, $\frac{1}{7}$ of the number is $\frac{1}{3}$ of $\frac{9}{10}$ or $\frac{8}{10}$. If $\frac{1}{7}$ of a number is $\frac{8}{10}$, $\frac{7}{7}$ of a number is $7 \times \frac{8}{10} = \frac{21}{10} = 2\frac{1}{10}$.

Or, $\frac{9}{10} \div \frac{3}{7} = \frac{9}{10} \times \frac{7}{3} = \frac{63}{30} = 2\frac{1}{10}$.

Find the number of which:

3. 96 is $\frac{8}{9}$

4. $\frac{4}{5}$ is $\frac{2}{7}$

5. 124 is $\frac{4}{5}$

6. $3\frac{1}{5}$ is $\frac{2}{3}$

7. 872 is $\frac{8}{11}$

8. $3\frac{1}{3}$ is $\frac{5}{6}$

9. $\frac{9}{10}$ is $\frac{3}{11}$

10. 65 is $\frac{5}{13}$

11. $2\frac{1}{2}$ is $\frac{10}{11}$

12. 875 is $\frac{5}{9}$

13. $\frac{16}{17}$ is $\frac{4}{7}$

14. 95 is $\frac{19}{20}$

15. $\frac{1}{2}$ is $\frac{1}{8}$

16. $\frac{2}{7}$ is $\frac{2}{3}$

17. $5\frac{1}{2}$ is $\frac{11}{12}$

18. $6\frac{2}{3}$ is $\frac{4}{5}$

19. 240 is $\frac{3}{8}$

20. $10\frac{1}{2}$ is $\frac{3}{7}$

21. $18\frac{3}{4}$ is $\frac{3}{5}$

22. 144 is $\frac{9}{11}$

23. 672 is $\frac{6}{7}$

24. $21\frac{2}{7}$ is $\frac{7}{9}$

25. $5\frac{5}{6}$ is $\frac{5}{7}$

26. $87\frac{1}{2}$ is $\frac{25}{7}$

27. If 75 miles is $\frac{5}{7}$ of the distance to a place, what is the whole distance?

28. If $\$\frac{1}{2}$ is $\frac{5}{7}$ of my money, how much money have I?

29. If 125 pages are $\frac{5}{13}$ of the number of pages in the book, how many pages in the book?

30. Lucy paid 60¢ for $\frac{3}{4}$ yd. of cloth. What was the cost of 1 yd.?

31. The value of a lot is $\frac{2}{7}$ of the value of the house. What is the value of the house if the lot is worth $1200?

32. When corn was selling at $1.45 a bushel, the price of a bushel of corn was $\frac{5}{9}$ of the price of a bushel of wheat. What was the price of a bushel of wheat?

33. A baseball team won $\frac{5}{8}$ of its games. If it lost 30 games, how many did it win?

34. A man sold a house and lot for $4400, and by so doing lost $\frac{1}{5}$ of the cost price. What was the cost price?

35. A man bought $\frac{7}{8}$ peck of apples for 35¢. What was the cost per bushel?

36. If $\frac{3}{7}$ of my age is 12 yr., how old am I?

37. If the son's age is $\frac{3}{7}$ of the father's, how old is the father if the son is 21 years of age?

38. If $\$1\frac{9}{10}$ is $\frac{3}{7}$ of my money, how much money have I?

39. James spelled correctly 24 words and missed $\frac{1}{5}$ of the words given. How many words were given?

40. A merchant sold a piece of goods for $3.60 and lost $\frac{1}{7}$ of the cost. What was the cost?

41. James missed $\frac{1}{6}$ of the problems in his lesson and solved 15. How many problems in his lesson?

42. Ellen spent $\frac{3}{10}$ of her money for a tablet and had 35¢ left. How much money had she at first?

43. Lucy had 12 pieces of candy, which was $1\frac{1}{2}$ times or $\frac{3}{2}$ times as many pieces as Sarah had. How many pieces had Sarah?

44. Mr. Long had 360 A. in his farm, which was $1\frac{1}{5}$ times as many acres as Mr. Allen had. How many acres had Mr. Allen?

45. There are 49 students in the 5th grade, which is $1\frac{2}{5}$ times as many as are in the 7th grade. How many pupils in the 7th grade?

46. Mary is 18 years old. Her age is $\frac{3}{7}$ of her mother's. How old is her mother?

47. $\frac{2}{3}$ of 15 is $\frac{2}{5}$ of what number?

48. I sold a horse for $280, which was $1\frac{1}{3}$ times the cost. How much did the horse cost?

49. There are 18 girls in a class, and the number of girls is $1\frac{1}{2}$ times the number of boys. How many pupils in all in the class?

50. $\frac{3}{7}$ of 28 marbles is $\frac{4}{11}$ of how many marbles?

51. Harvey missed 2 words out of 25. What part of the words did he spell?

52. What part of $\$\frac{3}{5}$ is 20¢?

53. What part of $2\frac{1}{2}$ is $1\frac{1}{2}$?

54. A girl planted 120 tomato plants; 12 of them died. What part of them lived?

55. Rupert has 20 chestnuts, which is $1\frac{1}{4}$ times as many as Robert has. How many chestnuts have they both?

56. A tourist started on a journey of 350 miles. When he was within 50 miles of the end of his journey, what part of the distance had he traveled?

57. A successful candidate for governor received 120,000 votes; his competitor received $\frac{2}{3}$ as many votes. How many votes did his competitor receive?

58. $\frac{3}{5}$ of 60 ft. is what part of 16 yd.?

59. $\frac{7}{9}$ of a number is $\frac{4}{7}$ of 49. What is the number?

60. John's age is $\frac{3}{4}$ of William's, and William's age is $\frac{4}{5}$ of Henry's. If Henry is 20 years old, what is the age of John and William?

63. To Find a Number When the Number Plus a Fractional Part of it is Given

1. Ellen has $\frac{3}{7}$ more money than I. If she has \$1.40, how much money have I?

Hint: Since there are $\frac{7}{7}$ of my money and Ellen has $\frac{3}{7}$ more, Ellen has $\frac{7}{7}+\frac{3}{7}$ or $\frac{10}{7}$ of my money. $\frac{10}{7}$ of my money = \$1.40.

2. My shoes cost \$12. This was $\frac{1}{2}$ more than my trousers cost. What was the cost of my trousers?

3. Lucy paid 8¢ for a pencil. This was $\frac{1}{3}$ more than Sarah paid for a tablet. What did Sarah pay for her tablet?

4. Jennie paid 10¢ for a tablet. This was $\frac{3}{7}$ more than Mary paid for a pencil. How much did Mary pay for a pencil?

5. Henry paid 35¢ for a book. This was $\frac{2}{5}$ more than William paid for his book. How much did William pay for his book?

6. John paid for a second-hand bicycle \$18, which was $\frac{2}{7}$ more than it was worth. How much was it worth?

7. 15 is $\frac{1}{4}$ more than what number?

8. 37975 is $\frac{2}{8}$ more than what number?

9. A farmer by good cultivation increased his yield $\frac{3}{7}$, and then grew 40 bu. of corn to the acre. How much did he grow before he improved his cultivation?

10. A farmer sold his farm for \$17,500, which was $\frac{2}{5}$ more than it cost him. How much did it cost him?

11. The population of a certain city is 78,000, which is $\frac{1}{9}$ more than it was ten years ago. What was the population ten years ago?

12. Ann was asked to tell her age. She replied if her age was increased by its $\frac{1}{2}$ and $\frac{1}{8}$, she would be 33 years old. How old was Ann?

13. John said \$60 was $\frac{3}{7}$ more money than the amount he had. How much money had John?

14. A dairyman sold his herd of cattle for \$3750, which was $\frac{2}{8}$ more than the herd cost. How much did the herd cost?

64. Mixed Exercises

1. We have already learned that mathematical signs are used to indicate operations and to express relationships. The signs $\times$ and $\div$ are regarded as expressing a closer relationship than the signs $+$ and $-$. Thus, we associate together and treat as one quantity or term expressions or numbers connected by the signs $\times$ and $\div$. The signs of $+$ and $-$ stand between the terms. These operations are to be performed after the operations indicated by $\times$ and $\div$. In case of doubt as to correct association of numbers use the signs of aggregation, called ( ) parenthesis, [ ] bracket, or —— vinculum. These are used to inclose quantities that are to be treated as one, or for the purpose of changing the order of operations from that indicated by the signs of $+$, $-$, $\times$, and $\div$.

Example: $8\div4+6\times3$. The correct order is to add the product of 6×3 to the quotient of $8\div4$. The result is 20. The vinculums or brackets, while not necessary, make the order unmistakable, as $\overline{8\div4}+\overline{6\times3}=20$.

2. $[5+4]-[8\div2]=5$.

3. $8\times4-5\times6=?$ 4. $9\div3\times8\div2=?$ 5. $8-6\div2=?$

6. $\frac{1}{2}-\frac{1}{4}\times\frac{1}{2}=\frac{3}{8}$ 7. $\frac{1}{2}\times\frac{10}{11}-\frac{1}{2}$ of $\frac{5}{22}=?$

8. $\frac{3}{4}$ of $\frac{2}{3}\div\frac{1}{2}\times\frac{5}{6}=?$ 9. $5+4\div4-1=?$

10. $8-3\times2+3=?$ 11. $7-3+2\times6=?$

12. $7\times3-15\div3=?$ 13. $16\div8-6\div3=?$

Signs of aggregations in the exercises that follow change the order of operations indicated by $+$, $-$, $\times$, and $\div$.

14. $(20\div5)\div(14\div7)$ 15. $\frac{3}{5}-\frac{1}{8}+\frac{9}{10}-\frac{1}{5}$

16. $\frac{3}{7}\times\frac{2}{3}+\frac{8}{11}\div2$ 17. $\frac{22}{24}\div\frac{46}{48}+\frac{1}{5}+\frac{1}{3}$

18. $\frac{5}{9}$ of $\frac{3}{7}-\frac{1}{5}$ of $\frac{5}{7}$ 19. $\frac{2}{8}$ of $\frac{3}{5}-\frac{1}{7}$ of $\frac{14}{15}$

20. $\frac{2}{5}\times\frac{3}{4}-\frac{1}{3}\times\frac{1}{5}$ 21. $\overline{(8+9-1)}\div\overline{(9-7+2)}$

22. $\overline{8+7+5}\div\overline{8+6-4}$ 23. $9+8-7\times8+6-4$

24. $\overline{7+3+5}\div\overline{9-3-3}$ 25. $(\frac{1}{2}+\frac{1}{8}-\frac{1}{4})\div(\frac{1}{8}+\frac{2}{8}-\frac{1}{4})$

26. $(\frac{5}{6}-\frac{1}{8}+\frac{1}{4})\times(\frac{3}{5}\times\frac{1}{5}-\frac{1}{10})$ 27. $(\frac{22}{68}-\frac{1}{7}+\frac{1}{9})\div(\frac{2}{8}+\frac{1}{2}\div\frac{3}{1})$

65. Complex Fractions

A fraction that has a fraction in one or both of its terms is called a **complex fraction.**

Thus, $\frac{\frac{1}{2}}{\frac{1}{4}}$, $\frac{2\frac{1}{2}}{5\frac{1}{3}}$, $\frac{\frac{7}{8}}{5}$, $\frac{8}{\frac{2}{3}}$ are complex fractions.

In reading complex fractions, read the numerator as the dividend and the denominator as the divisor; thus, $\frac{\frac{1}{2}}{\frac{1}{4}}$ is read one-half divided by one-fourth. A fraction may be defined as an indicated operation in division.

Complex fractions are simplified by (a) using the numerator for the dividend and the denominator for the divisor; thus, $\frac{1}{2} \div \frac{1}{4} = \frac{1}{2} \times \frac{4}{1} = 2$; or (b) multiplying numerator and denominator each by the L. C. M. of their denominators; thus, $\frac{\frac{1}{2} \times 4}{\frac{1}{4} \times 4} = \frac{2}{1}$

(a) $\frac{2\frac{1}{2}}{5\frac{1}{3}} = 2\frac{1}{2} \div 5\frac{1}{3} = \frac{5}{2} \div \frac{16}{3} = \frac{5}{2} \times \frac{3}{16} = \frac{15}{32}$.

(b) $\frac{2\frac{1}{2} \times 6}{5\frac{1}{3} \times 6} = \frac{15}{32}$ Is the value of the complex fraction changed in multiplying its numerator and denominator each by the same number? Why?

Simplify: [Use either (a) or (b)].

1. $\frac{\frac{3}{4}}{\frac{8}{9}}$ **2.** $\frac{8}{9\frac{1}{2}}$ **3.** $\frac{9\frac{1}{8}}{8}$ **4.** $\frac{\frac{8}{9} \text{ of } \frac{3}{4}}{\frac{3}{5} \text{ of } \frac{10}{11}}$

Suggestion: Simplify numerator and denominator first; then simplify by (a) or (b).

5. $\frac{\frac{1}{2}+\frac{1}{8}}{7-6\frac{1}{2}}$ **6.** $\frac{\frac{3}{4}-\frac{1}{6}}{\frac{1}{2} \div \frac{1}{8}}$ **7.** $\frac{\frac{5}{6} \text{ of } 3\frac{3}{7}}{\frac{8}{9}+\frac{1}{3}}$

(See note p. 79)

NOTE: To use (b) in (5), multiply each term in numerator and denominator by 6; thus, $\frac{(\frac{1}{2}+\frac{1}{3})\times 6}{(7-6\frac{1}{2})\times 6}=\frac{3+2}{42-39}=\frac{5}{3}=1\frac{2}{3}$.

8. $\dfrac{\frac{2}{3}\text{ of }\frac{6}{7}-\frac{1}{7}}{\frac{6}{18}+\frac{5}{26}}$ **9.** $\dfrac{\frac{5}{11}-\frac{3}{22}}{\frac{17}{18}-\frac{1}{3}}$ **10.** $\dfrac{\frac{7}{8}-\frac{3}{5}}{\frac{7}{10}-\frac{1}{8}}$

11. $\dfrac{8\div 2+4}{\frac{1}{2}}$ **12.** $\dfrac{7\times 3-6}{\frac{2}{3}+\frac{1}{4}}$ **13.** $\dfrac{\frac{7}{9}-\frac{5}{18}}{(21\div 3)\times 2}$

14. $\dfrac{\frac{9}{10}+\frac{1}{4}}{\frac{6}{7}}$ **15.** $\dfrac{\frac{7}{11}}{\frac{1}{2}+\frac{1}{22}}$ **16.** $\dfrac{\frac{7}{8}\div\frac{3}{4}}{\frac{19}{21}\div\frac{3}{7}}$

17. $\dfrac{\frac{3}{5}\times\frac{2}{3}+\frac{1}{8}}{\frac{5}{6}\times\frac{3}{4}-\frac{1}{8}}$ **18.** $\dfrac{2\frac{1}{2}+3\frac{1}{2}}{5\frac{1}{4}-3\frac{1}{2}}$ **19.** $\dfrac{27\frac{1}{2}\div 1\frac{3}{8}}{65-13\frac{1}{2}}$

66. Review Problems

1. Kate gave Ellen $\frac{1}{2}$ of her apples and Lucy $\frac{1}{4}$ of them. What part of her apples had she left?

2. Thomas gave Henry $\frac{1}{5}$ of his chestnuts, and William $\frac{1}{2}$ of the remainder. What part of his chestnuts did he give away?

3. Bessie gave Susan $\frac{1}{4}$ of her candy, and Minnie $\frac{3}{5}$ of the remainder. What part of her candy did she give Minnie? What part had she left?

4. Robert sold $\frac{3}{7}$ of his corn and had 40 bushels left. How many bushels had he at first?

5. James was asked his age and replied if he were 1 year younger his age would be $\frac{2}{5}$ of 20 yr. How old was James?

6. Harriet said if she were one year older her age would be $\frac{1}{3}$ of 30 yr. What was Harriet's age?

7. What number is 3 less than $\frac{2}{3}$ of 12?

8. What number is 25 less than $\frac{2}{3}$ of 99?

9. If an auto is driven $17\frac{2}{3}$ mi. an hour, how far will it be driven in $5\frac{1}{2}$ hr.?

10. A piece of cloth contained $30\frac{1}{2}$ yd.; from it one piece was cut containing $9\frac{1}{4}$ yd., another containing $15\frac{1}{2}$ yd. How much was left?

11. A carpenter's wages were $\$8\frac{3}{4}$ a day, and his daily expenses were \$5. How long will it take him to save \$45?

12. A man owned $\frac{3}{8}$ of the stock of a bank; he sold $\frac{1}{4}$ of his interest for \$96,000. What was the whole stock of the bank worth?

13. B owns $\frac{3}{4}$ of a farm. He sells to C $\frac{3}{5}$ of his share and to D $\frac{1}{2}$ of the remainder for \$2800. What was the value of the farm?

14. Find the cost of $12\frac{1}{3}$ cords of wood at $\$4\frac{1}{4}$ a cord.

15. If $\frac{7}{15}$ of a piece of rope is cut off and then $\frac{3}{8}$ of the remainder, what fraction of the whole is left?

16. (a) My age is $\frac{3}{4}$ of my uncle's who is 36 years old. What is my age?

(b) My age is $\frac{9}{5}$ of my nephew's. How old is my nephew if I am 54 yr. of age?

17. A's age is $\frac{5}{4}$ of B's and B's age is $\frac{4}{3}$ of C's. If C is 15, find the ages of A and B.

18. How many times is $\frac{4}{11}$ of $4\frac{1}{8}$ contained in 15?

19. By what fraction must $4\frac{7}{8}$ be multiplied to give the product $4\frac{1}{2}$?

20. By what mixed number must $4\frac{7}{12}$ be divided to give the quotient $1\frac{1}{8}$?

21. What number increased by $\frac{1}{3}$ of itself gives 20?

22. What number falls short of 20 by $\frac{1}{4}$ of itself?

23. If $\frac{1}{3}$ of a number exceeds its $\frac{1}{4}$ part by 2, what is the number?

24. If the dividend is $15\frac{3}{4}$ and the quotient is $3\frac{7}{8}$, find the divisor.

25. If the sum of two fractions is $\frac{19}{20}$ and one of the fractions is $\frac{1}{4}$, what is the other fraction?

26. Subtract 1 from each term of the fraction $\frac{4}{5}$ and tell whether the resulting fraction is increased or diminished and by how much.

27. Add 1 to each term of the fraction $\frac{4}{5}$ and tell whether the resulting fraction is increased or diminished and by how much.

28. Subtract 1 from each term of the fraction $\frac{6}{5}$ and tell whether the resulting fraction is increased or diminished and by how much.

29. Add 1 to each term of the fraction $\frac{6}{5}$ and tell whether the resulting fraction is increased or diminished and by how much.

30. A man owned $\frac{3}{5}$ of a farm and sold $\frac{2}{3}$ of his interest for \$15000. What was the whole farm worth?

31. A man spends $\frac{4}{7}$ of his income and saves \$2100. What was his income?

32. If in an orchard $\frac{1}{5}$ of the trees are pear trees, $\frac{1}{4}$ apple trees, $\frac{2}{5}$ peach trees, $\frac{1}{10}$ plum trees, and the remaining 20 are cherry trees, how many trees are there of each kind, and how many trees are there in the whole orchard?

33. A man owned $\frac{1}{5}$ interest in a saw mill. He sold $\frac{1}{2}$ of $\frac{1}{4}$ of his interest. What part had he left?

34. A man owned $\frac{1}{2}$ interest in a dairy. He sold $\frac{3}{5}$ of $\frac{1}{8}$ of his interest for \$4860. At that rate, what was the whole dairy worth?

35. John and James bought a ball and bat for \$1.25. John paid $1\frac{1}{2}$ times as much as James. How much did each pay?

36. Smith and Brown each bought automobiles. Smith's automobile cost $3\frac{1}{3}$ times as much as Brown's. If both cost \$2600, find the cost of each.

37. Find the cost of a suit of clothes if $\frac{3}{4}$ of $\frac{4}{5}$ of it cost \$30.

38. A merchant invested \$6000 in men's clothing. By selling at $\frac{1}{5}$ profit on the cost, he gained \$7.50 on each suit. How many suits did he buy?

39. A man sold a horse for \$300 and gained $\frac{3}{7}$ of the cost. What was the cost?

40. It required $\frac{3}{4}$ yd. of ribbon to tie each diploma. If 30 yd. were needed to supply the class, how many members in the class?

41. How many pieces 15 in. long can be cut from 35 yd. of ribbon?

42. Find the cost of 9 bu. of wheat if 10 bu. cost \$22.50?

43. Find the cost of $\frac{3}{4}$ yd. of cloth if $\frac{3}{5}$ yd. cost 60¢.

44. If $6\frac{3}{4}$ bu. apples cost \$10.80, what will $9\frac{1}{2}$ bu. cost?

45. Find the cost of $\frac{1}{2}$ of $\frac{3}{4}$ yd. of silk, if $\frac{2}{3}$ of $\frac{3}{5}$ yd. cost 64¢?

46. If $\frac{3}{4}$ lb. of butter costs 45¢, what is the cost of $8\frac{1}{8}$ lb.?

47. If $\frac{7}{12}$ of a man's monthly salary is \$210, what is his monthly salary?

48. When William was asked his age, he replied if he were $2\frac{1}{2}$ times as old as he is he would be 50. What was his age?

49. If to a number you add its $\frac{1}{2}$, its $\frac{1}{8}$, and its $\frac{1}{4}$, the sum will be 50. What is the number?

50. If I earn \$2½ in 3½ hr., how much will I earn in 8 hr.?

51. 33 is $\frac{3}{7}$ of what number?

52. $\frac{3}{5}$ is $\frac{3}{7}$ of what number?

53. Find the perimeter of a room 12½ ft. by 16⅔ ft.

54. If a passenger train makes $37\frac{3}{8}$ mi. an hour, how long will it take to travel a distance of 282 mi.?

55. A man bought a farm of 150 A. for \$12000. He sold $\frac{1}{2}$ of the farm at a gain of \$20 per acre, $\frac{1}{8}$ of it at a loss of \$10 per acre, and the remainder at a gain of \$5 per acre. How much did he gain in the transaction?

56. A merchant bought $7\frac{1}{2}$ bu. of apples for \$20. If $\frac{1}{8}$ of the apples decayed and he sold the remainder at \$4.50 a bushel, how much did he gain or lose by the transaction?

57. Find the cost of attending college for 1 yr. if tuition costs \$120, board $\$4\frac{1}{4}$ a week for 36 weeks, books $\$12\frac{1}{2}$, and incidentals $\$57\frac{1}{2}$.

58. A man bought an automobile for \$1656. He received a reduction of $\frac{1}{20}$ of this price for cash payment. How many War Savings Stamps at \$4.14 each could he purchase with the saving made by the reduction?

59. A man bought 200 War Savings Stamps at $\$4\frac{3}{25}$ each. The stamps have a maturity value of \$5 each. How much profit will he make on his stamps when they mature?

60. A man planned to buy 20 War Savings Stamps during the year at an average price of \$4.17. On Christmas day he discovered he was short $\frac{1}{3}$ of the required amount of money. How much money did he lack?

61. Henry wanted to buy a \$25 bicycle. He had all the money but $\frac{4}{5}$ of the amount. How much had he?

62. If James had $\frac{1}{5}$ more money than he now has, he would have \$1.50. How much money has he?

63. Margaret spent $\frac{1}{3}$ of her money for cakes and had 20¢ left. How much money had she before spending any?

64. If \$5 is $\frac{2}{5}$ of the amount of money Laura has in the bank, how much money has she in the bank?

65. A passenger train and a freight train left the station at the same time and traveled on parallel tracks in the same direction. If the passenger train made $31\frac{1}{2}$ mi. an hour and the freight train made $19\frac{1}{2}$ mi. an hour, how far were they apart in 5 hr.?

66. A tourist traveled at the rate of $16\frac{2}{3}$ mi. an hour. How long would it take him to travel 100 mi.?

67. Helen spent 60¢ for a book; Sarah gave $\frac{3}{4}$ as much for her book as Helen gave for hers, and Mary gave $\frac{2}{3}$ as much for her book as Sarah gave for hers. What was the cost of Sarah's book? Mary's book?

68. A man sold a horse for \$120, which was $\frac{4}{5}$ of the cost. What was the cost of the horse?

67. Review Questions

1. What is an integral number?

2. What is a fraction? Give two definitions and illustrate each.

3. Define the following: numerator, denominator, terms of a fraction, proper fraction, improper fraction, mixed number, common denominator, simple fraction, compound fraction, complex fraction. Give an example of each.

4. What is the reciprocal of a fraction? Give an example.

5. What is necessary before you can add or subtract common fractions? Illustrate.

6. If you have the sum of two fractions and one of them given, how is the other found? Illustrate.

7. How do you find the product of two fractions?

8. If you have the product and one factor given, how is the other found? Illustrate.

9. Why may you multiply or divide both numerator and denominator by the same number without changing the value of the fraction?

10. What are two ways of dividing a fraction by a whole number? Illustrate.

11. What are two ways of dividing one fraction by another? Illustrate.

12. How do you reduce complex fractions to simple fractions? Illustrate two ways.

13. If you know the cost of $2\frac{1}{2}$ articles, how would you find the cost of $3\frac{1}{3}$ articles? Illustrate.

14. If you know a fractional part of a number, how would you find a different fractional part of the same number? Illustrate.

15. When several numbers are connected by different symbols of operation, which symbols indicate the closest association?

16. What is the use of the parenthesis, the bracket, or the vinculum?

17. How do you find what fractional part one number is of another? Illustrate.

18. What is the effect of adding or subtracting the same number to both terms of a proper fraction? Illustrate your answer.

19. What is the effect of adding or subtracting the same number to both terms of an improper fraction? Illustrate your answer.

20. What is the effect of multiplying or dividing both terms of a fraction by the same number? Illustrate.

21. When is a fraction expressed in its lowest terms?

22. What is a compound fraction? Give an example.

23. If you know how much $\frac{1}{2}$ of $\frac{1}{3}$ of a pound of sugar is worth, how would you find the cost of $\frac{1}{4}$ of $\frac{1}{5}$ of a pound?

24. If you know the time it will take John and James when working alone to plow a field, how would you find the time it would take them to do it when working together?

25. If the numerators of two fractions are the same and the denominators are different, which fraction is the larger? Why?

26. If the denominators of two fractions are the same and the numerators are different, which fraction is the larger? Why?

27. If you know the number of hours in a day and the number of hours you sleep, how would you find the fraction of a day you sleep?

28. If you know how many pupils are in school and that $\frac{4}{9}$ of them are boys, how would you find the number of girls?

29. If you know the time it will take three girls separately to do a piece of work, how would you find the time it would take when all worked together?

30. A tank $\frac{1}{2}$ full of water contains 75 gal. How would you find the number of gallons if the tank is $\frac{3}{5}$ full?

68. ALIQUOT PARTS

An **aliquot part** of a number is a number that will exactly divide it; thus, 2 is an aliquot part of 8; 20 is an aliquot part of 100.

By using the aliquot parts of \$1, or 100¢, many operations may be very much shortened. What is the cost of 24 yd. of cloth at $33\frac{1}{3}$¢ a yd.? At \$1 a yd. 24 yd. would cost \$24, but since $33\frac{1}{3}$¢ is $\frac{1}{3}$ of \$1 it can be seen that taking $\frac{1}{3}$ of the cost at \$1 per yd. will give the cost at $\$\frac{1}{3}$ a yd. $\frac{1}{3}$ of \$24 = \$8 cost at $33\frac{1}{3}$¢ or $\$\frac{1}{3}$ per yd.

2 ¢ = $\frac{1}{50}$ of \$1	10 ¢ = ?	$2\frac{1}{2}$¢ = ?
$12\frac{1}{2}$¢ = ?	$3\frac{1}{3}$¢ = ?	$16\frac{2}{3}$¢ = ?
4 ¢ = ?	20 ¢ = ?	5 ¢ = ?
25 ¢ = ?	$6\frac{1}{4}$¢ = ?	$33\frac{1}{3}$¢ = ?
$8\frac{1}{3}$¢ = ?	50 ¢ = ?	$12\frac{1}{2}$¢ = ?

Oral Problems.

1. What is the cost of 20 qt. of milk at $12\frac{1}{2}$¢ a quart?
2. What is the cost of 20 pk. of potatoes at 75¢ a peck?
3. Find the cost of 48 gal. of gasoline at 25¢ a gallon.
4. Find the cost of 12 doz. eggs at 50¢ a dozen.
5. Find the cost of 20 pt. of milk at $8\frac{1}{3}$¢ a pint.
6. What is the cost of 25 yd. of ribbon at 25¢ a yard?
7. What is the cost of 20 apples at 4¢ each?
8. Find the cost of 80 oranges at $2\frac{1}{2}$¢ apiece.
9. Find the cost of 24 lb. of sugar at 20¢ a pound.
10. What is the cost of 75 yd. cloth at $66\frac{2}{3}$¢ a yard.
11. Find the cost of $6\frac{1}{2}$ dozen ears of corn at 20¢ a dozen.
12. Find the cost of $24\frac{4}{5}$ yd. of cloth at 75¢ a yard.
13. What is the cost of $60\frac{1}{2}$ dozen eggs at 50¢ a dozen?
14. Find the cost of $7\frac{1}{2}$ yd. of cloth at $33\frac{1}{3}$¢ a yard.
15. Count by $2\frac{1}{2}$'s to 100; $3\frac{1}{3}$'s to 100; 4's to 100; $6\frac{2}{3}$'s to 100; $8\frac{1}{3}$'s to 100; $12\frac{1}{2}$'s to 100; $16\frac{2}{3}$'s to 100; 20's to 100; 25's to 100; $33\frac{1}{3}$'s to 100; 50's to 250.

Finding Cost When the Number Required is an Aliquot Part of the Number Given

What part of 12 is 6? 4? 3? 2? 8? 9? 10?

69. Oral Problems

1. If 12 dozen eggs cost \$4.80, what will 6 dozen cost.

Since 6 dozen is $\frac{1}{2}$ of 12 dozen, the cost of 6 dozen is $\frac{1}{2}$ the cost of 12 dozen. $\frac{1}{2}$ of \$4.80 is \$2.40.

2. If 12 bu. of corn cost \$15, find the cost of 6 bu.; 4 bu.; 8 bu.; 3 bu.

3. If 12 yards of silk cost \$18 find the cost of 4 yd.; 3 yd.; 2 yd.; 8 yd.; 10 yd.

4. If 12 pigs are worth \$72, find the cost of 6 pigs; 4 pigs; 3 pigs; 2 pigs; 8 pigs; 9 pigs; 10 pigs.

5. If 12 dozen oranges cost \$3.60, find the cost of 2 doz.; 10 doz.; 8 doz.; 9 doz.; 3 doz.; 4 doz.

6. What part of 14 is 7? What part of 14 is 2? 4? 6? 8? 10? 12?

7. What part of 16 is 2? 4? 6? 8? 10? 12? 14?

8. If 16 lb. of tea cost \$12, find the cost of 2 lb.; 4 lb.; 6 lb.; 8 lb.; 10 lb.; 12 lb.

9. If 14 lambs are sold for \$84, find the cost of 4 lambs; 6 lambs; 7 lambs; 10 lambs; 12 lambs.

10. What part of 18 is 2? 3? 4? 6? 8? 9? 10? 12? 15?

11. If 18 yards of worsted cost \$7.20, find the cost of 2 yd.; 3 yd.; 6 yd.; 9 yd.; 10 yd.; 12 yd.

12. What part of 20 is 2? 4? 5? 8? 10? 12? 15? 16?

13. If 20 books cost \$24, find the cost of 2 books; 4 books; 5 books; 10 books; 12 books.

14. If 30 yd. of muslin cost \$4.50, what will 2 yd. cost? 3 yd.? 5 yd.? 6 yd.? 10 yd.? 12 yd.? 15 yd.? 18 yd.? 20 yd.? 24 yd.? 25 yd.?

70. RATIO

The **ratio** of one number to another is the quotient obtained by dividing the first number by the second. Thus, the ratio of 4 to 9 is obtained by dividing 4 by 9, or $\frac{4}{9}$. The ratio of 10 to 5 is $\frac{10}{5} = 2$.

Ratio is written by placing the sign (:) between the first term and the second term. Thus, the ratio of 8 to 4 is written 8 : 4 and is read, *the ratio of 8 to 4.*

The first term of a ratio is called the *antecedent.*

The second term of a ratio is called the *consequent.*

Unlike concrete numbers cannot have a ratio, as 8 men to 2 bu.

What is the ratio of:

1.	8 : 4	**2.**	9 : 3	**3.**	12 : 6
4.	$7\frac{1}{2} : 2\frac{1}{2}$	**5.**	$9 : 4\frac{1}{2}$	**6.**	$5 : 2\frac{1}{2}$
7.	$8\frac{2}{3} : 4\frac{1}{3}$	**8.**	$20 : 2\frac{1}{2}$	**9.**	$100 : 12\frac{1}{2}$
10.	$37\frac{1}{2} : 12\frac{1}{2}$	**11.**	$25 : 8\frac{1}{3}$	**12.**	$13\frac{1}{2} : 41\frac{1}{2}$
13.	$16\frac{2}{3} : 5\frac{1}{3}$	**14.**	100 : 25	**15.**	$66 : 5\frac{1}{3}$
16.	$33 : 5\frac{1}{2}$	**17.**	$44 : 5\frac{1}{2}$	**18.**	22 : 3
19.	$2\frac{1}{2} : 3\frac{1}{3}$	**20.**	$3\frac{1}{7} : 5\frac{1}{11}$	**21.**	\$6 : \$3
22.	\$3 : \$6	**23.**	18 yd. : 6 yd.	**24.**	63 ft. : 9 ft.
25.	200 : 5	**26.**	120 da. : 10 da.		
27.	81 mi. : 9 mi.	**28.**	288 : 24	**29.**	21 yd. : 7 ft.

Numbers cannot be compared or have a ratio unless expressed in units of like denominations. Either 21 yd. : $2\frac{1}{3}$ yd., or 63 ft. : 7 ft.

30.	\$5 : 5¢	**31.**	3 da. : 18 hr.	**32.**	6 bu. : 3 pk.
33.	7 gal. : 3 qt.	**34.**	15 oz. : 3 lb.	**35.**	2 T. : 1600 lb.
36.	$\frac{1}{2} : \frac{1}{8}$	**37.**	$\frac{2}{3} : 1\frac{1}{2}$	**38.**	$1\frac{1}{2} : 1\frac{1}{4}$
39.	$7\frac{1}{2} : 15$	**40.**	4500 : 6000	**41.**	1008 : 144
42.	$33\frac{1}{3} : 100$	**43.**	560 : 28	**44.**	956 : 239

71. Problems

The ratio principle may be applied to shorten operations.

1. If 8 yd. of cloth cost \$10, what will 32 yd. cost?

Since 32 yd. is 4 times 8 yd., the cost of 32 yd. is 4 times the cost of 8 yd. Therefore, cost of 32 yd. is $4 \times \$10 = \$40$.

2. If 8 bu. of wheat costs \$18, what will 28 bu. cost?

Hint: 28 bu. is how many times 8 bu.?

3. If 7 lb. butter cost \$4.20, what will 21 lb. cost?

4. If 6 bu. of potatoes are worth \$14, what are 21 bu. worth?

5. If 10 bu. of corn are worth \$18, what are 45 bu. worth?

6. If $2\frac{1}{2}$ yd. of cloth are worth \$6.25, what are $37\frac{1}{2}$ yd. worth?

7. If $12\frac{1}{2}$ dozen oranges are worth \$5, what are 50 dozen worth?

8. If 25 bu. of oats are worth \$20, what are $6\frac{1}{4}$ bu. worth?

Hint: What part is $6\frac{1}{4}$ of 25?

9. If 33 yd. of gingham cost \$15.60, what will be the cost of 11 yd.? 22 yd.?

10. If 6 doz. bats cost \$30, what will 2 doz. cost?

11. If $16\frac{2}{3}$ tons of coal cost \$85, what will 50 tons cost?

12. If 14 pigs cost \$100, what will 35 pigs cost?

13. If 9 qt. of milk cost \$1.80, what will 3 qt. cost?

14. If a train travels 360 mi. in 8 hr., how far will it travel in 4 hr.?

15. If $\frac{1}{4}$ bu. potatoes cost 40¢, what will $\frac{1}{2}$ bu. cost?

16. If 100 A. of land are worth \$8000, what are 25 A. worth?

17. If $\frac{2}{3}$ of 24 yd. of silk cost \$12, what will $\frac{1}{2}$ of 16 yd. cost?

18. If 3 horses are worth \$500, what are 12 horses worth?

19. If 3 qt. of milk cost 27¢, what will 9 qt. cost?

20. If 2 bu. of corn are worth \$3.50, what are 10 bu. worth?

21. If $2\frac{1}{2}$ yd. of gingham are worth \$1.25, what are 10 yd. worth?

22. If $7\frac{1}{3}$ doz. eggs cost \$4.50, what will 22 doz. cost?

72. Finding Cost When the Number Required is a Multiple of the Number Given

1. If 3 dozen eggs cost \$1.50, what will 12 dozen cost?

2. If 12 bu. corn cost \$18, what will 36 bu. cost?

3. If 5 lb. sugar cost 90¢, what will 25 lb. cost?

4. If 15 lambs sell for \$90, what will 75 lambs sell for?

5. If 30 A. of land sell for \$3600, what will 90 A. sell for?

6. If a man receives \$2.50 for 3 hr. labor, how much will he receive for 18 hr. labor?

7. If 4 yd. silk cost \$5, what will 24 yd. of silk cost?

8. If 3 oranges cost 20¢, what will be the cost of 12 oranges? 15 oranges? 24 oranges?

9. If 6 histories cost \$9, what will 42 histories cost?

10. If 4 cans of jam cost \$1.20, what will be the cost of 20 cans? 12 cans? 24 cans?

11. If 6 barrels of flour cost \$60, what will be the cost of 30 barrels? 48 barrels?

12. If 7 yd. of ribbon cost \$2.80, what will 21 yd. cost?

13. If 8 lb. of beans cost \$1, what will 48 lb. cost? 24 lb.? 80 lb.? 96 lb.?

14. If it takes 35 yd. to make 3 rugs, how many yards will it take to make 18 rugs?

15. If there are 45 da. in $1\frac{1}{2}$ months, how many days in $4\frac{1}{2}$ months?

16. If there are 960 rods in 3 miles, how many rods in 12 miles?

17. If a man received \$20 for 8 da. work, how much would he receive for 32 da. work?

18. If the product of 6 times a number is 54, what is the product of 18 times the number?

19. If James sold 6 bu. of oats for \$5.00, how much were 30 bu. worth?

73. PROBLEMS WITH INSUFFICIENT DATA

Before problems can be solved, certain necessary relations must be known. For instance, if I say, *Mr. Henson sold a horse for $300; how much did he gain or lose?* there is no way of finding gain or loss except by the relation the cost price bears to the selling price. The cost price is unknown. Hence, the gain or loss cannot be determined.

State why the following cannot be solved; supply necessary data and solve:

1. A merchant sold a box of potatoes for $10. What was the selling price per bushel?

2. Ellen's pencil was 3 times the length of Lucy's pencil. Find the length of Ellen's pencil.

3. James took 1200 steps in walking from school home. How far did he live from school?

4. How many cubic yards of crushed stone will be required to build 1 mile of road 24 ft. in width?

5. Nora studied her lessons 3 times as long as Margaret studied hers. How long did Nora study her lesson?

6. How long will it take a servant to put a pile of coal into the cellar if he puts it in at the rate of 30 bu. an hour?

7. How many bushels of coal in the pile if he puts it all in with 800 shovelfuls?

8. How many bushels will a box hold that is 5 ft. in length and 3 ft. in width?

9. How many acres in a rectangular field 80 rods in length?

10. How many yards of linoleum 1 yd. wide will be required to cover a hallway 200 yards in length?

11. How long will it take to sail from New York to Calcutta at the rate of 21 miles an hour?

12. How long will it take an aviator to go from Washington to Los Angeles?

13. What is the area of a triangle whose base is 10 ft.?

14. What is the area of a rectangle one of whose parallel sides is 10 ft.?

15. John sold his melons for \$7.50 and bought books with the money. How many books did he buy?

16. James is the heaviest boy in the class and Thomas is the lightest. What is the difference in their weight?

17. If Robert reads 75 pages of a story book in an hour, how long will it take him to read the book?

18. If a hotel uses 25 lb. of sugar a day, how long will the sugar last?

19. If a bricklayer earns 75¢ an hour, how much does he earn for the whole time he works?

20. If John can do a piece of work in 5 da., how long will it take John and James together to do it?

21. If Ethel has 40¢ more than Flora, how much money have they both?

22. If Edward's money is $\frac{3}{5}$ of Russell's, how much money has each?

23. A passenger train traveled from New York to Chicago at an average rate of 47 mi. an hour. How long did it take to make the trip?

24. Ellen spent \$3 for gingham to make her a dress. How many yards did it take?

25. If the altitude of a triangle is 8 ft., what is its area?

26. If James walks $7\frac{1}{2}$ mi., at what rate did he travel per hour?

74. ESTIMATING

1. What is the sum of 645 and 367?

NOTE: You can tell at a glance that the sum of the hundreds is 9 hundred, and you will observe that 45 is a little less than 50 and 67 is a little more than 50; hence, the sum of these numbers is approximately 100. Your estimate would probably be about 1000. By addition, you can test your estimate and ascertain just how much you have overestimated or underestimated the number. Bear in mind there is a wide difference between estimating and guessing. You might guess at what was under a box, turned upside down, and miss your guess entirely; but if you look at the size of the box and estimate how much it will hold, your estimate will be in units representing the capacity of the box, although your estimate may be far too large or too small.

Estimate first and then test your estimate by performing the operation:

1. $668+330$	**4.** $287+397$	**7.** $2500-1350$
2. $290+595$	**5.** $29\frac{1}{2}-13\frac{1}{8}$	**8.** 675×99
3. $666+430$	**6.** $837-225$	**9.** $800\div 70$

10. Estimate the perimeter of your school room. Measure and test your estimate.

11. Estimate the length of your blackboard. Measure and test your estimate.

12. Estimate the circumference of the nearest tree to your school house. Measure and test your estimate.

13. Estimate the distance to the nearest tree from your school house. Measure and test your estimate.

14. Estimate the perimeter of your school yard. Measure and test your estimate.

15. Estimate the length of your arm. Measure and test your estimate.

16. How far can you throw a base ball? Measure and test your estimate.

17. Estimate the number of pages you can read in a story book in one hour. Time yourself and test your estimate.

NOTE: The teacher will find it a profitable exercise to give occasional tests in estimating. It stimulates observation and calls into exercise one's judgment.

75. RECEIPTS

$25. BIRMINGHAM, ALA., Feb. 1, 1923.

Received of THOMAS NELSON

————Twenty-five ————$\frac{X}{100}$———— Dollars

for rent of house at No. 24 Beacon St. from Jan. 1, 1923, to Feb. 1, 1923.

JAMES A. BLAND.

The above is the customary form of receipt used in business transactions.

Every receipt shows the following:

The place and date, who pays the money, how much, for what, and to whom the money was paid.

Why should one take a receipt when he pays money?

Why should receipts be preserved?

Write a receipt for a sum of money you have received in payment of a debt due you from your teacher.

Write a receipt for a debt you have paid your teacher.

Make out receipts, using your post-office and these data:

1. Arthur Roy paid the Rhodes-Mahoney Furniture Co. $27.50 for furniture.

2. Henry Lee paid Harry Gordon for his bicycle $17.50.

3. John Roe sold John Randolph seed corn and received cash $1.75.

4. James Brown received from Henry Lansing $200 for the sale of a horse.

5. What six things are shown on a receipt?

76. BILLS

Jackson, Miss., Feb. 1, 1920.

Mr. John Wright,
24 Davis St.

Bought of Brown & Co.
29 Main St.

1920				
Jan. 1	5 doz. eggs	60¢	$ 3	00
Jan. 7	$2\frac{1}{2}$ lb. steak	35¢		88
Jan. 10	1 pk. potatoes	80¢		80
Jan. 20	3 lb. sugar	22¢		66
Jan. 25	6 lb. lard	27¢	1	62
			$6	96

Received payment Feb. 15, 1920.

Brown & Co.,
per Henson.

When, by whom, and from whom were the purchases made?

What was the cost of each item and the amount of the bill?

What things are shown on every bill?

What words show that the above bill is receipted, or paid?

Did the seller or a clerk receipt the bill?

The one who buys is called the *debtor*, because he owes something.

The one who sells is called the *creditor*, because something is owed him.

In the above bill, who is the debtor? the creditor?

Complete these bills and receipt them:

1. New Orleans, Jan. 31, 1920.

Mr. Herbert Mason bought of Henry Monroe:

6 lb. coffee	@	40¢
12 lb. sugar	@	$16\frac{1}{2}$¢
6 lb. rice	@	$12\frac{1}{2}$¢
8 lb. prunes	@	$11\frac{1}{2}$¢
$12\frac{1}{2}$ lb. butter	@	50¢

2. Nashville, Tenn., Jan. 10, 1921.

Mr. John Smith bought of Hill & Co.:

$22\frac{1}{2}$ yd. of calico	@	$22\frac{1}{2}$¢
25 yd. of muslin	@	$37\frac{1}{2}$¢
3 pair gloves	@	$2.75
25 yd. sheeting	@	$47\frac{1}{2}$¢
15 yd. ribbon	@	$37\frac{1}{2}$¢

3. Montgomery, Ala., April 30, 1923.

Mr. James Sanford bought of Henderson & Co.:

8 chairs	@	$4.75
11 cots	@	3.65
7 tables	@	11.75
16 yd. carpet	@	1.65
20 mattresses	@	10.50

4. Make out and receipt a bill for goods you have purchased from your own merchant.

77. DECIMALS

You are familiar with the form of a common fraction, such as $\frac{3}{10}, \frac{75}{100}, \frac{125}{1000}$. There is a more convenient way of writing a fraction whose denominator is 10 or some power of 10. Only the numerator is written and the denominator is indicated by a period, called a *decimal point*, placed so there are as many figures to the right as there are naughts in the denominator. For example, $\frac{3}{10} = .3$, $\frac{75}{100} = .75$, $\frac{125}{1000} = .125$.

Fractions whose denominators are indicated by the position of a decimal point are called *decimal fractions*, or *decimals*.

A **decimal fraction,** then, is a fraction whose numerator only is written and whose denominator is ten or some power of ten, such as 100, 1,000, 10,000, etc., and is indicated by the position of a decimal point.

78. Reading Decimals

The numerator of a decimal fraction is read as a whole number. The denominator is read by calling a one place decimal *tenths*, as .5, *five tenths*; a two place decimal *hundredths*, as .05, *five hundredths;* a three place decimal *thousandths*, as .005, *five thousandths;* and so on.

In the table below, the places to the right of the decimal point are the decimal places. The places to the left are the places in any integer.

In decimals as in whole numbers, the place value increases from right to left ten fold for each place, as in 22.22. It can readily be seen that 2 tenths are 10 times 2 hundredths, and 2 units are 10 times 2 tenths, and 2 tens are 10 times 2 units. Hence, decimals and whole numbers increase and decrease by the same scale. The word decimal is derived from a Latin word which means ten.

Units	Decimal point	Tenths	Hundredths	Thousandths	Ten-thousandths	Hundred-thousandths	Millionths	Ten-millionths	Hundred-millionths	Billionths
	.	2	3	0	6					

Read the number, 2306, as a whole number, *two thousand three hundred six.* This is the numerator. Note the position of the last figure to the right of the decimal point. It is in ten-thousandths' place. This determines the denominator; the decimal is then read, *two thousand three hundred six ten-thousandths.* The right hand place of the decimal always names the denominator.

If there is a whole number and a decimal, the point separates the whole number from the decimal. The point is read *and;* as 75.45, seventy-five *and* forty-five hundredths. Numbers composed of a whole number and a decimal are *mixed decimals.*

Read the following:

1.	.15	.25	1.25	.325	.1325	8.25
2.	.234	9.05	.005	64.25	.50	9.812
3.	4.8967	54.0125	3.1416	.7854	7.3	
4.	6.534128	4.8665	2150.42		1001.0001	
5.	1.0101	.00095	.400	.095	95.0095	
6.	6.1001	2.0001	2.01	.9763	8.567	
7.	10.001	20.20	3.003	3.300	9.656	
8.	8.00065	7.163	.0801	.5768	6.579	

Mixed *decimals* may be read in either of two ways. Thus, 3.5 may be read, three *and* five-tenths, or thirty-five tenths.

Read in two ways:

9.	5.6	9.9	1.25	12.5	7.868
10.	845.6	100.1	2.007	20.07	200.7
11.	9.365	2.22	1.001	10.010	8.357
12.	100000.1	10000.01	100.001	10.0001	1.000001
13.	1000.25	834.1	65.301	8.7643	854.39

Whole numbers may be written in decimal form without changing their value.

Thus, 2 may be written 2.0 or 2.00 or 2.000, etc., and may be read either *2* or *20 tenths* or *200 hundredths* or *2000 thousandths*.

Read in two ways:

14.	1.0	25.00	75.0	37.00	1.000
15.	9.8	137.0	1.000	10.0000	5.0000

79. Writing Decimals

To write a decimal fraction, write the numerator and indicate the denominator by pointing off as many decimal places as there are naughts in the denominator of the corresponding common fraction. If the numerator does not have as many figures as there are naughts in the denominator, fill in the necessary number of naughts between the point and the figures of the numerator, as $\frac{3}{100} = .03$.

Write the following as decimals:

1.	$\frac{5}{10}$	**5.**	$\frac{50}{100}$	**9.**	$\frac{284}{10000}$
2.	$\frac{25}{100}$	**6.**	$\frac{65}{100}$	**10.**	$\frac{48}{100000}$
3.	$\frac{175}{1000}$	**7.**	$\frac{7}{100}$	**11.**	$\frac{95}{10}$
4.	$\frac{1325}{10000}$	**8.**	$\frac{5}{1000}$	**12.**	$\frac{3258}{100}$

13. Five tenths; eight hundredths; nine thousandths.

14. Nine and one tenth; seventy-five and five tenths.

15. Nineteen thousandths; eighty-five ten-thousandths.

16. Thirty-two hundredths; one hundredth.

17. One and five thousandths; fifteen hundred-thousandths.

18. Seven hundred twenty-five thousandths.

19. Seven hundred and twenty-five thousandths; seventy-five tenths.

20. One thousandth; one millionth; one hundredth; ten tenths.

21. One thousand one billionths; one hundred five millionths.

22. Ten thousand twenty-five hundredths; one million one millionths.

23. Thirty thousand five thousandths; ten thousand twenty-five tenths.

24. Ninety thousand and one thousandth.

25. One million millionths; three hundred hundredths.

80. Reduction of Common Fractions to Decimals

Only the common fraction whose denominator is a factor of 10 or some power of 10 may be reduced to a *pure decimal.*

$$\frac{4}{5}=\frac{4\times 2}{5\times 2}=\frac{8}{10}=.8; \qquad \frac{3}{4}=\frac{3\times 25}{4\times 25}=\frac{75}{100}=.75$$

$$\frac{7}{8}=\frac{7\times 125}{8\times 125}=\frac{875}{1000}=.875.$$

Decimals in which the numerator is a whole number are *simple* or *pure decimals.*

There are some fractions whose denominators are not factors of 10 or some power of 10, and whose numerators are not whole

numbers when the denominators are reduced to 10 or some power of 10.

Thus, $\frac{2}{8}=\frac{2\times 3\frac{1}{8}}{3\times 3\frac{1}{8}}=\frac{6\frac{2}{8}}{10}=.6\frac{2}{8}$

$$\frac{3}{7}=\frac{3\times 1\frac{3}{7}}{7\times 1\frac{3}{7}}=\frac{4\frac{2}{7}}{10}=.4\frac{2}{7}$$

Or, $\frac{3}{7}=\frac{3\times 14\frac{2}{7}}{7\times 14\frac{2}{7}}=\frac{42\frac{6}{7}}{100}=.42\frac{6}{7}$

These are called *complex decimals.*

A common fraction is an indicated division; hence, a common fraction may be changed to a decimal by dividing the numerator by the denominator:

$$\frac{3}{4}=\begin{array}{r} .75 \\ \hline 4)3.00 \\ 28 \\ \hline 20 \\ 20 \\ \hline \end{array}$$

In this solution, the numerator is written in the form of a decimal; thus, 3.00, which is read *300 hundredths.* 300 hundredths $\div 4 = 75$ hundredths $= .75$.

To reduce a common fraction to a decimal: Annex naughts to the numerator, divide by the denominator and point off in the quotient as many decimal places as there are naughts annexed to the numerator. If the division can be made exact, the quotient is a pure decimal; if the division cannot be made exact, the quotient becomes a *repeating decimal* if the division is carried far enough, and may be expressed as a complex decimal.

Reduce these common fractions to pure decimals or to repeating decimals:

	(a)	(b)	(c)	(d)	(e)
1.	$\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{10}$	$\frac{1}{5}$	$\frac{3}{14}$
2.	$\frac{1}{4}$	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{5}{18}$	$\frac{7}{20}$
3.	$\frac{2}{8}$	$\frac{1}{9}$	$\frac{5}{6}$	$\frac{5}{12}$	$\frac{1}{16}$
4.	$\frac{3}{4}$	$\frac{9}{15}$	$\frac{1}{20}$	$\frac{75}{100}$	$\frac{7}{25}$
5.	$\frac{3}{8}$	$\frac{6}{8}$	$\frac{7}{9}$	$\frac{1}{8}$	$\frac{5}{8}$
6.	$\frac{2}{5}$	$\frac{4}{10}$	$\frac{4}{15}$	$\frac{7}{30}$	$\frac{9}{50}$
7.	$\frac{9}{10}$	$\frac{1}{40}$	$\frac{7}{85}$	$\frac{45}{90}$	$\frac{65}{1000}$
8.	$2\frac{1}{2}$	$67\frac{1}{2}$	$15\frac{1}{4}$	$37\frac{1}{2}$	$16\frac{3}{4}$
9.	$1\frac{2}{4}$	$7\frac{1}{4}$	$\frac{15}{16}$	$8\frac{1}{2}$	$17\frac{1}{2}$
10.	$\frac{99}{100}$	$16\frac{2}{8}$	$66\frac{2}{3}$	$33\frac{1}{3}$	$6\frac{1}{4}$
11.	$\frac{27}{82}$	$\frac{5}{16}$	$\frac{92}{40}$	$\frac{49}{50}$	$\frac{79}{80}$
12.	$\frac{23}{25}$	$\frac{19}{20}$	$\frac{63}{80}$	$\frac{15}{16}$	$\frac{9}{11}$

81. Reducing Decimals to Common Fractions

The fraction $\frac{5}{10}=\frac{50}{100}=\frac{500}{1000}$, etc.; observe also that .5 = .50 = .500, etc. It does not change the value of a fraction to multiply or divide both terms by the same number. Annexing naughts to the right of a decimal does not change its value, as,

$$\frac{5}{10}\times\frac{10}{10}=\frac{50}{100};\text{ also } .5=.50$$

To reduce a decimal to a common fraction, express the decimal in the form of a common fraction and reduce to its lowest terms. If it is a mixed number, change the decimal to a common fraction and add it to the whole number; thus $3.5=3\frac{1}{2}$.

Change to the form of a common fraction:

	(a)	(b)	(c)	(d)	(e)
1.	.1	.3	.7	.25	.075
2.	.06	2.16	3.75	18.2	.325
3.	$.16\frac{2}{3}$	$.06\frac{1}{4}$	$.33\frac{1}{3}$	$.66\frac{2}{3}$	7.5
4.	$.17\frac{1}{2}$	.625	$.12\frac{1}{2}$	$.67\frac{1}{2}$	$.02\frac{1}{2}$

	(a)	(b)	(c)	(d)	(e)
5.	4.45	12.50	$.82\frac{1}{2}$	$.04\frac{1}{2}$	40.67
6.	7.975	18.642	$.18\frac{3}{4}$	$1.7\frac{1}{2}$	$2.5\frac{1}{4}$
7.	.005	.010	2.50	7.45	6.040
8.	.85	.965	3.875	3.56	2.225
9.	.34	.88	.56	.100	.33
10.	.11	.12	44.44	100.01	.00001
11.	3.28	9.8	.96	.98	.95
12.	.375	.80	.99	.75	.55
13.	.125	.625	.372	.89	.35
14.	9.5	8.15	2.72	$3.33\frac{1}{3}$	19.25

82. Addition of Decimals

1. Add: .25, 3.07, $.62\frac{1}{2}$, 5.2049, 225.009

.25
3 .07
.625
5 .2049
225 .009
234 .1589

Change the complex decimals to simple form, as the $.62\frac{1}{2}$ to .625. Arrange the numbers in a column so that the decimal points are in a vertical line. Add as in the addition of whole numbers, placing the decimal point in the sum in line with the decimal points in the addends. Placing like units under each other in effect reduces the decimals to common denominator.

2. $2.56+.956+7.25+98.3+2.52.$

3. $.27+84.65+.425+9.38+.6208.$

4. .45 yd.+2.5 yd.+3.75 yd.+85 yd.+.375 yd.

5. $.15\frac{1}{2}+7\frac{1}{8}+.06\frac{3}{4}+34\frac{7}{8}+.22\frac{1}{2}$

6. 39.37 in.+.19 in.+2.54 in.+.19 in.+30.48 in.

7. $1.45\frac{3}{4}+16\frac{1}{2}+26.37\frac{1}{2}+58\frac{1}{4}+7.9.$

8. $\$37.50+\$5.51+\$426.78+\$4.17\frac{1}{2}+\$4056.98\frac{1}{2}.$

9. $7.654+98.74+.0016+4006.009.$

10. $120.302+9.00834+.007+5.2+47.328.$

11. $100.001+2.7+8\frac{1}{4}+3.3333+12.4+42.589.$

12. $.10\frac{1}{2}+.287+.004+45.02+325.2+89.87.$

13. A man traveled on Monday 235.876 miles, on Tuesday 217.57 mi., on Wednesday 199.8 mi., on Thursday 211.817 mi. How far did he travel in the four days?

14. The three sides of a triangular field are 35.87 rd., 42.175 rd., and 31.916 rd., respectively. What length of fence will enclose it?

15. A farmer cultivated 1.75 A. in potatoes, .125 A. in beans, 25.756 A. in corn, 10.375 A. in cotton. How many acres did he cultivate in all?

16. James can plow .05 of a field in a day, B can plow .06 in a day and C can plow .07 in a day. What part of the field can they all plow in a day?

17. A had $\$\frac{3}{4}$, B had $\$\frac{3}{5}$, and C had \$1.19. How much money had they all?

18. Find the sum of one hundred twenty-five thousandths, seven tenths, nine hundredths, and forty-five ten thousandths.

19. Three members of a boys' corn club grew respectively 87.75 bu., 101.125 bu., and 99.824 bu. How many bushels did they all grow? What was the average number of bushels grown to the acre if each boy planted one acre?

20. Mr. Brown has 219.75 A. in his farm; Mr. Simpson has 21.19 A. more in his farm than has Mr. Brown. How many acres in both farms?

21. James sold 49.7 bu. of corn and had 73.29 bu. left. How many bushels had he before selling?

22. The subtrahend was 9.204, and the remainder was 7.619. What was the minuend?

23. A cotton picker earned \$2.62 on Monday, \$3.11 on Tuesday, \$1.12 on Wednesday, and \$2.83 on Thursday. How much did he earn for the four days?

24. If a man on an automobile tour runs 205.4 miles the first day, 183.26 miles the second day, and 215 miles the third day, how far does he travel in the three days?

83. Subtraction of Decimals

1. From 6.4 take 3.25.

6.40
3.25
3.15

Arrange the numbers so that tenths are under tenths, hundredths under hundredths, etc., and so that the decimal points are in a vertical line.

In case there are mixed decimals, reduce them to simple form as in addition of decimals.

When the number of decimal places in the minuend and the subtrahend are not the same, add naughts until they are the same and then proceed as in simple subtraction, placing the decimal point in the remainder in line with the decimal points in the minuend and subtrahend.

	a	b
2.	.85 − .67	$93\frac{1}{2} - .75$
3.	.927 − .45	487.25 − 97.875
4.	$.87\frac{1}{2} - .018$	98.15 − 43.256
5.	9.00 − 4.67	112.357 − 65.84
6.	37.92 − 14.86	2864 − 9.56
7.	$2.8 - .62\frac{1}{2}$	$538\frac{1}{2} - 417\frac{1}{4}$
8.	.0625 − .025	$\$87\frac{1}{2} - \$65\frac{3}{4}$
9.	.01935 − .00095	92.86 − 89.375
10.	1258.86 − 275.9241	$238.75 - 214\frac{1}{4}$
11.	7.6437 − 2.54	$3.3\frac{1}{8} - 1.6\frac{5}{6}$
12.	219.15 ft. − 7.25 ft.	$29\frac{7}{8} - 15\frac{5}{8}$
13.	\$37.50 − \$37.15	14.65 − 9.9
14.	5280 ft. − 379.265 ft.	750 − 4.897
15.	218.75 lbs. − 196 lbs.	3.891 − 1.102
16.	32.5 mi. − 27.67 mi.	$562\frac{1}{2} - 37\frac{1}{4}$
17.	.98253 − .90079	$845 - 98\frac{3}{4}$
18.	8.00132 − 7.35298	5.61 − 3.8
19.	9.8 − 1.97658	7.19 − 4.005
20.	25.005 − 2.5005	$.963 - .89\frac{3}{4}$

21. A fruit dealer bought 1 bu. of apples. He sold .85 of them. What part of a bushel had he left?

22. B owns .42 of 1 lot; A owns the remainder. What part of the lot does A own?

23. Ellen spent .25 of her money for a book, .125 for ribbon, .18 for stationery, .11 for candy. What part of her money had she left?

24. The son received $\frac{2}{5}$ of the father's estate, the daughter received .27, and the mother the remainder. What part of the estate did the mother receive?

25. Find the difference between $\$\frac{7}{8}$ and \$0.625.

26. The distance to a certain place is 10 miles. How much further has one to travel after he has gone 6.75 miles?

27. A car of coal contained 37.85 tons. After 23.67 tons had been unloaded, how much coal was left in the car?

28 Henry grew 87.37 bu. of corn on an acre. After selling 48.9 bu., how many bushels had he left?

29. Find the sum of $\frac{1}{2}$, 1.25, $\frac{3}{5}$, .625, and $\frac{3}{4}$, and subtract it from 4.896.

84. Multiplication of Decimals

1. $\frac{5}{10}\times\frac{5}{10}=\frac{25}{100}$

$.5\times.5=.25.$

2. $1.25\times10=12.50.$

3. $\frac{25}{100}\times\frac{21}{100}=\frac{525}{10000}$. $.25\times.21=.0525.$

Note that there are as many decimal places in the product as there are decimal places in the multiplicand and the multiplier.

From No. 2 we learn that moving the decimal point one place to the right is the same as multiplying by 10. To move it two places to the right is the same as multiplying by 100, three places to the right by 1000, and so on. Hence, to multiply when the multiplier is a power of ten, move the decimal point as many places to the right as there are naughts in the multiplier.

Direction for multiplying decimals:

Multiply as in multiplying whole numbers and point off as many decimal places in the product as there are decimal places in the multiplicand and multiplier.

Exercise:

1.	$.18\times.22$	16.	$.5\times.5$
2.	$.14\times36$	17.	$10\times.01$
3.	$.05\times.03$	18.	$10\times.001$
4.	2.5×2.87	19.	4.8665×15
5.	$45\times.16$	20.	$30\frac{1}{4}\times1.25$
6.	$6.7\times.85$	21.	$.048\times7$
7.	$.097\times.23$	22.	$.7854\times1000$
8.	375×1.005	23.	$.0048\times.006$
9.	1.19×30	24.	38.75×75
10.	29.45×88.5	25.	9.832×800
11.	3.45×100	26.	$.395\times.97$
12.	62.5×60	27.	7389×65.439
13.	$.1233\times468$	28.	39.37×7.92
14.	2.17×62.83	29.	4.44×2.22
15.	3.1416×9	30.	6.35×43.25

31. How much was paid for 48.75 acres of land at \$58.50 an acre?

32. If a train travels at the rate of 27.5 miles an hour, how far will it travel in 12 hours?

33. An acre of ground yields 33.7 bushels of wheat. What will the wheat bring in the market at \$1.97 per bushel?

34. The value of a franc in U. S. Money is \$0.193. What is the value of 1500 francs?

35. There are 39.37 inches in a meter. How many inches are there in 145 meters?

36. A cubic foot of water weighs 62.5 pounds. What will 1000 cubic feet weigh?

37. It costs \$2.47 per ton to crush lime for liming soil. If it requires 4 tons for one acre, what will it cost to lime 5 acres?

38. If gasoline costs \$0.27 per gallon, what will it cost to fill a 15 gallon tank?

39. An automobile travels at the rate of 17.3 miles per hour for 7 hours. How far has it traveled?

40. A submarine made an average speed of 11.8 knots per hour for 21 hours. How many knots did it register?

41. A dealer bought 350 tons of coal. After he sold .54 of it, how much had he left?

42. A farmer bought a horse for $250. He sold him at a gain of .18 of the cost. How much did he gain?

43. James had 24 marbles in his pocket. He lost through a hole .25 of them. How many did he lose?

44. What is the value of 50.5 bu. of corn at $1.375 a bushel?

45. In a certain state the average rainfall was 3.57 inches per month. What was the rainfall for a year?

46. James Howard bought a $1000 Liberty bond. The government pays him .045 of it in interest each year. How much does the government pay him a year?

47. James spent $80 and William spent .85 as much. How much did William spend?

48. Multiply one thousand by one thousandth.

85. Division of Decimals

1. Divide 5.2525 by 1.25.

Think of the operation in the form of a fraction. Since it does not alter the value of a fraction to multiply both terms by the same number, we multiply the divisor and dividend each by 100. The divisor thus becomes 125 and the dividend 525.25. Thus:

$$\frac{5.2525 \times 100}{1.25 \times 100} = \frac{525.25}{125}$$

Then,

$$\begin{array}{r} 4.202 \\ 125\overline{)525.250} \\ 500 \\ \hline 252 \\ 250 \\ \hline 250 \\ 250 \\ \hline \end{array}$$

You will note that after the last division there was a remainder of 25. A naught was added to the dividend and the division was made complete. After adding the naught, the dividend becomes thousandths; therefore, the quotient becomes thousandths.

Direction: Move the decimal point to the right in both divisor and dividend as many places as there are decimal places in the divisor.

2. $.25 \div 3.$

$$\begin{array}{r} .083+ \\ \hline 3).25\,0 \\ 24 \\ \hline 1\,0 \\ 9 \\ \hline 1 \end{array}$$

Make the division as in whole numbers. Point off as many places in the quotient as there are places in the dividend after the division is completed.

Or, write the point in the quotient directly over the point in the dividend.

In such cases, the quotient should be carried to a specified number of places, or to a sufficient number of places to show that the decimal continues to repeat. The continuation may be represented by a + as .083+. Or, the remainder may be added to the quotient in the form of a fraction, $.083\frac{1}{3}$.

3.	$96.16 \div 8$	**15.**	$41.85 \div 9.3$
4.	$60.84 \div 12$	**16.**	$8.00 \div 16$
5.	$.054 \div 6$	**17.**	$38.088 \div 211$
6.	$.09 \div 15$	**18.**	$3.1416 \div 4$
7.	$48.27 \div .03$	**19.**	$.5236 \div 13.09$
8.	$2.75 \div 35$	**20.**	$.6666 \div 74$
9.	$.216 \div 2.7$	**21.**	$3.215 \div 115$
10.	$15.2 \div .008$	**22.**	$325.92 \div 35$
11.	$15.2 \div 800$	**23.**	$3.8042 \div 25$
12.	$1.64 \div 820$	**24.**	$23.22 \div 27$
13.	$375 \div 2.5$	**25.**	$4.576 \div 65$
14.	$45.2 \div .02$		

NOTE: If accuracy to a stated number of decimal places is desired in any of the operations in decimals, it is necessary to carry the fraction to one more place than the number stated and add 1 to the last figure in the number required if the next figure is 5 or more; otherwise drop the next figure.

Thus, in Ex. 2 above, if accuracy to two decimals were required, the answer would be .08. For $.25 \div 6$, the answer correct to three decimal places would be .042, for

$$\begin{array}{r} .0416 \\ \hline 6).2500. \end{array}$$

26. If 18 dozen eggs cost \$9.00, what will 1 dozen cost?

27. If 25 pounds of sugar cost \$5.50, what is the cost per pound?

28. A man sold his hogs at \$0.13 per pound which netted him \$188.89. What was the weight of the hogs?

29. Breakfast bacon is \$0.50 per pound. How many pounds can be bought for \$4.50?

30. There are $30\frac{1}{4}$ square yards in a square rod. How many square rods in 907.5 square yards?

31. How many bushels of wheat at \$1.10 per bushel can be bought for \$55?

32. It is 621 miles from Nashville to New Orleans. How many hours will it take a passenger train to make the trip at the average rate of 27.2 miles per hour?

33. The first non-stop flight across the Atlantic Ocean made by an aeroplane was made from St. John's, Newfoundland, to Clifden, Ireland, a distance of 1960 miles, in 16.2 hours. What was the average rate of speed per hour?

34. The distance from New York to San Francisco is 3191 miles. How many hours will it take to make the trip in an aeroplane at an average rate of 110.5 miles per hour?

35. A boy belonging to a corn club produces 85 bushels of corn on an acre, which he sells for \$92.00. What was the price per bushel?

36. A bicycle wheel is 8.75 ft. around. How many revolutions will it make in going a mile (5280 ft.)?

37. A field containing 3.79 A. of land was subdivided into town lots containing .21 A. How many lots?

38. How many boxes holding 3.25 bu. each will it take to hold 65 bu.?

39. At \$0.18 a quart, how many gallons of cherries can be bought for \$7.20?

40. If a boy takes 2.67 ft. at a step, how many steps will he take in traveling 867.75 ft.?

41. If 8.7 yd. of cloth cost \$20.01, what will 9.3 yd. cost?

86. REVIEW PROBLEMS

1. How long will it take you to do a piece of work if you can do .25 of it in an hour?

2. James said he lacked 1.87 gal. of having 3 gallons of berries. How many gallons had he?

3. A man bought a farm for $18500. He sold it at a profit of .15 of the cost. (a) What was his profit? (b) For how much did he sell the farm?

4. A boy bought a pig for $20. He sold it for $30. (a) How much was his profit? (b) What part of the cost was his profit? (c) What part of the selling price was his profit?

5. A farmer grew 750 bu. of wheat. He sold $.33\frac{1}{3}$ of it at $1.25 a bushel. How much did he receive for what he sold?

6. J. E. Nichols grew 113 bu. of corn on an acre. If he sold .45 of it at $1.75 a bushel, how much did he receive?

7. How far would an aviator travel in 7.5 hours if he travels an average of 107.35 mi. an hour?

8. If brick are selling at $18.50 a thousand, what will 15.8 thousand cost?

9. A boy sold 1.3 bu. of corn for $2.08. What would be the value at the same rate of 3.7 bushels?

10. From Chicago to New Orleans by rail is 1078 miles. At the rate of 39.5 miles an hour, how long would it take to make the run?

11. A merchant bought potatoes at 60¢ a peck, and sold them at 75¢ a peck. (a) What was his gain. (b) His gain was what part of the cost? (c) What decimal part?

12. Multiply $(.4+.7)$ by $(2.9-.17)$.

13. Mr. Brown sold .27 of his 200 A. farm; Mr. Sanford .22 of his 180 A. farm. Which sold the more land? How much more?

14. William could do .27 of a piece of work in a day, and Thomas could do .42 of it in a day. What part of the work can they both do in a day?

87. REVIEW QUESTIONS

1. What is a decimal?

2. What is the difference between a decimal and a common fraction?

3. How can you tell the name of the denominator of a decimal?

4. What is the effect of placing naughts at the right of a decimal? Illustrate.

5. What is the effect of prefixing naughts to a decimal? Illustrate.

6. What is a pure decimal? a repeating decimal? Illustrate each.

7. What common fractions can be changed to pure decimals? Illustrate.

8. What is a complex decimal? Illustrate with $\frac{3}{11}$.

9. What common fractions can be changed only to complex or repeating decimals? Illustrate with $\frac{2}{7}$.

10. What is a mixed decimal?

11. In what two ways may mixed decimals be read? Illustrate with 6.25.

12. How are common fractions reduced to decimals?

13. How are decimals reduced to common fractions?

14. In addition and subtraction of decimals, why do you write tenths under tenths, hundredths under hundredths, etc.?

15. How many decimal places do you point off in the product? Why?

16. How many decimal places do you point off in the quotient? Why?

17. If you had .25 of a number how would you find .37 of the same number? Make and solve a problem to illustrate.

18. If you had the sum of three numbers and two of them given, how would you find the third number?

19. If you buy an article and sell it for more than cost, what do you call the excess above cost?

20. If you buy an article and sell it for less than cost, what would you call excess of cost above selling price?

21. Read: 786.50009; $.00\frac{1}{8}$; $.002\frac{1}{7}$; 8.7.

22. Write one thousandth; one millionth.

23. Why do you perform the fundamental operations in decimals just as you do in whole numbers?

24. If you know the cost of .87 of an acre, how would you find the cost of 3.25 A.?

25. What is the meaning of the word decimal?

26. Write in words: 40.1086001, .11001, 565.0.

27. If a merchant sells goods at 1.25 of cost, how much does he gain?

28. If a merchant sells goods at .85 of cost, how much does he lose?

29. If you had the product of three decimals and two of them given, how would you find the third? Illustrate with an example.

30. Write five common fractions that cannot be reduced to pure decimals?

31. If Sarah bought dress goods at \$0.85 a yard, and paid \$7.65 for the goods, how would you find the number of yards she purchased?

32. Read the following mixed decimals in two ways: 6.5, 7.85, 78.5, 10.1, 1.010.

33. What is the difference in result obtained by multiplying a number by .75 or dividing the same number by $1\frac{1}{3}$. Illustrate with examples.

88. PERCENTAGE

James has 100 peaches in a basket. He gave 6 peaches out of the hundred to Thomas. Thomas has then $\frac{6}{100}$ of James's peaches. James gave Lucy 7 peaches out of the hundred. Lucy has $\frac{7}{100}$ of the 100 peaches. James gave Ellen 20 peaches out of the 100. Ellen has $\frac{?}{100}$ of the peaches.

Write the number of hundredths Ellen has.

James gave William 35 peaches out of the 100. William has $\frac{?}{100}$ of the peaches. What part of 100 peaches is 17 peaches? 50 peaches? 75 peaches? 29 peaches? 18 peaches?

In all of the fractions above, what is the denominator?

In naming fractions whose denominator is 100, we may use the word **per cent** to take the place of the denominator. The word per cent means *by the hundred.*

To illustrate: Instead of saying James gave Thomas 6 peaches out of his 100 peaches or James gave Thomas $\frac{6}{100}$ of his 100 peaches, we may say James gave Thomas 6 per cent of his 100 peaches.

$$17 \text{ per cent} = \frac{17}{100} = .17.$$
$$50 \text{ per cent} = \frac{50}{100} = .50.$$
$$75 \text{ per cent} = \frac{75}{100} = .75.$$
$$40 \text{ per cent} = \frac{40}{100} = .40.$$

This sign (%) is used instead of the words per cent.

$$60\% = \frac{60}{100} = .60.$$
$$47\% = \frac{47}{100} = .47.$$
$$29\% = \frac{29}{100} = .29.$$
$$18\% = \frac{18}{100} = .18.$$
$$90\% = \frac{90}{100} = .90.$$
$$100\% = \frac{100}{100} = 1.00.$$

How many hundredths in all of James's peaches? How many % in all of James's peaches? How many hundredths in all of any number? How many per cent in all of any number?

Express the following per cents as common fractions and reduce to lowest terms:

1.	10%	15%	20%	24%
2.	14%	25%	35%	38%
3.	21%	37%	28%	22%
4.	40%	45%	48%	50%
5.	60%	65%	70%	75%
6.	80%	84%	85%	90%
7.	$125\% = \frac{125}{100} = \frac{5}{4} = 1\frac{1}{4}$.			
8.	150%	175%	130%	140%

Express the following per cents in decimals:

Example: $5\frac{1}{8}\% = .05\frac{1}{8}$ $125\% = 1.25$.

1.	$6\frac{1}{4}\%$	8%	38%	26%
2.	$16\frac{2}{3}\%$	$62\frac{1}{2}\%$	65%	70%
3.	$66\frac{2}{3}\%$	$33\frac{1}{3}\%$	47%	5%
4.	$87\frac{1}{2}\%$	90%	96%	100%
5.	165%	134%	148%	196%
6.	$166\frac{2}{3}\%$	175%	$187\frac{1}{2}\%$	191%

Change the following fractions first to their equivalent decimals in hundredths and then to their equivalent per cents:

Examples: $\frac{3}{5} = .60 = 60\%$. $\frac{1}{7} = .14\frac{2}{7} = 14\frac{2}{7}\%$.

1.	$\frac{2}{5}$,	$\frac{1}{8}$,	$\frac{3}{4}$,	$\frac{1}{2}$,	$\frac{1}{8}$,	$\frac{7}{9}$,
2.	$\frac{5}{8}$,	$\frac{1}{12}$,	$\frac{2}{7}$,	$\frac{4}{5}$,	$\frac{5}{6}$,	$\frac{3}{10}$,
3.	$\frac{7}{8}$,	$\frac{19}{20}$,	$\frac{7}{12}$,	$\frac{26}{25}$,	$\frac{3}{7}$,	$\frac{7}{11}$,
4.	$\frac{49}{50}$,	$\frac{24}{25}$,	$\frac{17}{20}$,	$\frac{10}{11}$,	$\frac{8}{9}$,	$\frac{5}{18}$,
5.	$\frac{43}{50}$,	$\frac{5}{9}$,	$\frac{1}{15}$,	$\frac{13}{20}$,	$\frac{39}{40}$,	$\frac{47}{50}$.

Write the following decimals as per cents:

Examples: $.137 = \frac{137}{1000} = \frac{13.7}{100} = 13.7\%$.
$.1356 = \frac{1356}{10000} = \frac{13.56}{100} = 13.56\%$.

1.	.787	.12	.165	.5378
2.	.965	.14	.21	$.33\frac{1}{3}$
3.	.375	$.37\frac{1}{8}$	$.66\frac{2}{3}$	$.87\frac{1}{2}$

Equivalents that should be memorized:

$4\% = \frac{1}{25}$	$5\% = \frac{1}{20}$
$6\frac{1}{4}\% = \frac{1}{16}$	$8\frac{1}{3}\% = \frac{1}{12}$
$10\% = \frac{1}{10}$	$12\frac{1}{2}\% = \frac{1}{8}$
$16\frac{2}{3}\% = \frac{1}{6}$	$20\% = \frac{1}{5}$
$25\% = \frac{1}{4}$	$33\frac{1}{3}\% = \frac{1}{3}$
$37\frac{1}{2}\% = \frac{3}{8}$	$50\% = \frac{1}{2}$
$60\% = \frac{3}{5}$	$62\frac{1}{2}\% = \frac{5}{8}$
$66\frac{2}{3}\% = \frac{2}{3}$	$75\% = \frac{3}{4}$
$80\% = \frac{4}{5}$	$83\frac{1}{3}\% = \frac{5}{6}$
$87\frac{1}{2}\% = \frac{7}{8}$	$90\% = \frac{9}{10}$

Oral Problems

How to find a given per cent of a number, as, $66\frac{2}{3}\%$ of \$24; 5% of \$80.

Solution: $66\frac{2}{3}\% = \frac{2}{3}$. $\frac{1}{3}$ of \$24 = \$8. Therefore $\frac{2}{3}$ of \$24 = 2 × \$8 = \$16, Ans.

$5\% = \frac{1}{20}$. $\frac{1}{20}$ of \$80 = \$4, Ans.

1. $12\frac{1}{2}\%$ of \$32 = __
2. 10% of \$100 = __
3. $8\frac{1}{3}\%$ of \$84 = __
4. $16\frac{2}{3}\%$ of \$48 = __
5. 20% of \$60 = __
6. 25% of 36 mi. = __
7. $37\frac{1}{2}\%$ of \$24 = __
8. 40% of 40 bu. = __
9. 50% of \$100 = __
10. 60% of 35 yd. = __
11. $66\frac{2}{3}\%$ of 36 sheep = __
12. 75% of 48 hr. = __
13. 80% of \$45 = __
14. $87\frac{1}{2}\%$ of 56 A. = __
15. 90% of \$80 = __
16. 100% of \$40 = __
17. $83\frac{1}{3}\%$ of \$144 = __
18. $12\frac{1}{2}\%$ of 96 = __
19. $66\frac{2}{3}\%$ of 66 yd. = __
20. 75% of \$100 = __
21. 80% of 80 A. = __
22. $87\frac{1}{2}\%$ of 84 = __
23. $62\frac{1}{2}\%$ of \$40 = __
24. $6\frac{1}{4}\%$ of 32 bu = __
25. 50% of $\frac{3}{8}$ = __
26. $33\frac{1}{3}\%$ of $\frac{6}{7}$ = __
27. 25% of $\frac{8}{11}$ = __
28. 20% of $\frac{20}{21}$ = __
29. $87\frac{1}{2}\%$ of 72 apples = __
30. $62\frac{1}{2}\%$ of 40 sheep = __
31. $37\frac{1}{2}\%$ of 32 = __
32. $16\frac{2}{3}\%$ of $\frac{12}{13}$ = __
33. $6\frac{1}{4}\%$ of \$96 = __
34. $8\frac{1}{3}\%$ of 108 apples = __

89. Written Problems

1. Find 35% of \$687.

Solution: 35% = .35

$$\begin{array}{r} \$687 \\ .35 \\ \hline 3435 \\ 2061 \\ \hline \$240.45 \end{array}$$

Suggestion: Write the per cent in the form of a decimal and multiply as in multiplication of decimals.

2. Find 25% of \$650.
3. Find 7% of \$786.
4. Find 20% of 850 A.
5. Find 32% of 860.
6. Find $12\frac{1}{2}$% of 360.
7. Find 30% of \$12.67.
8. Find 19% of 7856 bu.
9. Find 58% of \$92.85.
10. Find 16% of 500 men.
11. Find 8% of \$86.70.
12. Find 17% of 9.26.
13. Find 25% of \$6.57.

We have given in these problems a whole or mixed number for the multiplicand, the *per cent* expressed decimally in hundredths for the multiplier, to find the product.

There are three numbers involved in every multiplication: the multiplicand, the multiplier, and the product. Likewise in percentage there are three numbers involved having different names but performing the same office as the corresponding numbers in multiplication.

In **percentage,** the multiplicand is called the **base.** The multiplier is called the per cent or **rate.** The product is called the **percentage.** The problems above are problems in **percentage.**

14. Find 65% of \$765.

Solution:

$$\begin{array}{r} \$765 \text{ Base} \\ .65 \text{ Rate} \\ \hline 3825 \\ 4590 \\ \hline \$497.25 \text{ Percentage.} \end{array}$$

The base multiplied by the rate equals the percentage.

NOTE: In your solutions, name the base, rate, and percentage.

15. Find $16\frac{2}{3}\%$ of \$24. **16.** Find 25% of 480 sheep.

17. Find 18% of \$75.50. **18.** Find $66\frac{2}{3}\%$ of 144 bu.

19. Find 65% of \$87.86. **20.** Find $87\frac{1}{2}\%$ of \$368.

21. Find 10% of \$187. **22.** Find $37\frac{1}{2}\%$ of \$792.

23. A father is 60 years old. The age of his son is 60% of the father's age. How old is the son?

24. In a spelling lesson of 20 words, Lucy spelled 90% of them. How many words did she spell?

25. A box that will hold 384 bushels is $62\frac{1}{2}\%$ full. How many bushels does it contain?

26. A book dealer bought a gross of pencils. He sold $33\frac{1}{3}\%$ of them the first day of school. How many dozen pencils did he sell?

27. Charles had 25 apples. He gave to his playmates 80% of them. How many apples did he give away?

28. A man has a farm of 360 A. 35% of his farm is pasture land. How many acres are pasture land?

29. Find 125% of 640 A.

Solution: $125\% = 1.25$

640 A. Base
1.25 Rate
3200
1280
640
800.00 A. Percentage.

30. Find $137\frac{1}{2}\%$ of \$368. **31.** Find 825% of 800.

32. Find 6.7% of 750. **33.** Find $87\frac{1}{2}\%$ of 464 bushels.

34. Find $\frac{1}{8}\%$ of \$900.

Solution: $\frac{1}{8}\% = .00\frac{1}{8}$. \$900 × $.00\frac{1}{8}$ = \$3.00

35. Find $\frac{1}{4}\%$ of \$800. **36.** Find $\frac{3}{5}\%$ of \$750.

37. Find $\frac{7}{9}\%$ of 540. **38.** Find $2\frac{1}{8}\%$ of \$720.

39. Find 9% of \$146. **40.** Find $\frac{2}{7}\%$ of \$21.

41. William earned during vacation \$45. He spent 20% of his earnings for books. How much did he spend for books?

The **base** is the number of which some **per cent** is taken.

The **rate** is the **per cent** taken.

The **percentage** is the result obtained by taking a certain **per cent** of the base.

90. Finding the Per Cent One Number Is of Another

When the product and the multiplicand are given, how do you find the multiplier?

1. What % of 15 is 5?

5 is $\frac{1}{3}$ of 15. $\frac{1}{3}=33\frac{1}{3}\%$.

Oral and Written Problems

What % of:

2.	6 is 3?	**3.**	6 is 4?
4.	15 is 10?	**5.**	21 is 14?
6.	20 is 16?	**7.**	40 is 35?
8.	30 is 27?	**9.**	12 is 9?
10.	15 is 12?	**11.**	300 is 150?
12.	16 is 4?	**13.**	120 is 108?

Solution:

.90 or 90% Rate

Base 120)108.00 Percentage

108 00

The percentage divided by the base equals the rate.

What % of:

14.	750 is 300?	**15.**	828 is 207?
16.	1275 is 425?	**17.**	1680 is 504?
18.	180 is 70?	**19.**	654 is 109?

20. A man earns \$900 a year and spends \$750. What % of his earnings does he spend?

21. In a spelling examination of 50 words, a pupil spelled 48 words correctly. What % of the words did he spell?

22. There are 12,297 school teachers in Tennessee. Of this number 9,360 are women. What % of the teachers are women?

23. A dairyman has 80 cows. 60 of them are Jerseys. What % of his herd are Jerseys?

24. If I buy a horse for $150 and sell it at a gain of $45, what % do I gain?

25. A library contains 2500 volumes, of which 625 are works on history. What % of the volumes are works on history?

26. In a school of 175 students, 90 are girls. What % of the students are girls?

27. A ball player was at the bat 60 times and made 18 hits. What % was his batting?

28. A ball team played 120 games and won 75. What % did they win?

29. A dairyman sold 30 of his 45 cows. What % of his cows did he sell?

30. In a test made by a class in agriculture, it was found that 200 of the 240 grains being tested would produce plants. What % of the seed was good?

31. A certain farmer plowed a part of his corn land in the autumn and the remainder in the spring of the year. On the land plowed in the autumn, he grew 40 bu. of corn to the acre. On that plowed in the spring, he grew 8 bu. less to the acre. What was his % of loss on the land plowed in the spring?

32. In a school year of 180 days, a boy attended school 150 days. What was his % of attendance?

33. There are 300 pages in a certain book and the first chapter in the book contains 90 pages. What % of the book is in the first chapter?

34. A farmer owned 360 acres of land. He gave to his son 120 A., to his daughter 80 A. What per cent of the farm did the son receive? The daughter?

91. To Find the Base When the Percentage and Rate Are Given

How do you find the multiplicand when the product and multiplier are given?

Product ÷ multiplier = multiplicand.

1. If $\frac{3}{4}$ of a number is 36, what is the number?

If 75% (rate) is 36 (percentage), what is the number (base)?

Solution: (a) 75% of the number = $\frac{3}{4}$ of the number.
$\frac{3}{4}$ of the number = 36,
$\frac{1}{4}$ of the number = 12, Why?
$\frac{4}{4}$ of the number = 48, Why?

Or, (b) $36 \div \frac{3}{4} = 36 \times \frac{4}{3} = 48$

Oral Problems

Find the number of which:

2.	\$15 is 75%	**3.**	\$30 is $66\frac{2}{3}$%
4.	\$24 is $12\frac{1}{2}$%	**5.**	\$36 is 60%
6.	\$20 is 80%	**7.**	\$5 is 10%
8.	24 A. is 30%	**9.**	72 A. is $16\frac{2}{3}$%
10.	20 is 5%	**11.**	36 is 50%
12.	42 is $87\frac{1}{2}$%	**13.**	63 is 90%
14.	20 is 20%	**15.**	18 is $37\frac{1}{2}$%
16.	96 is 75%	**17.**	80 is $62\frac{1}{2}$%
18.	90 is $66\frac{2}{3}$%	**19.**	78 is $33\frac{1}{3}$%
20.	25 is $41\frac{2}{3}$%	**21.**	35 is $83\frac{1}{3}$%
22.	30 is $8\frac{1}{3}$%	**23.**	48 is $66\frac{2}{3}$%

Written Problems

Find the number of which:

24. 18% is 84.

Solution: 18% = .18
$84 \div .18 = 466\frac{2}{3}$

Percentage ÷ rate = base.

25.	$33\frac{1}{3}$% is 750	**26.**	42% is 8442
27.	25% is 927	**28.**	67% is 3274.29

29. 23% is $1809.49 **30.** $8\frac{1}{3}\%$ is 144 ft.

31. 17% is $492.32 **32.** 25% is 969

33. 39% is $3387.93 **34.** 20% is 750 ft.

35. A man spent $30 for a suit of clothes. This was 25% of his month's salary. What was his month's salary?

36. The average daily attendance of pupils in a certain school was 90% of the enrollment. If the number in attendance was 108, what was the enrollment?

37. John has $25 in his savings bank, which is $33\frac{1}{3}\%$ of the amount James has. How much money has James?

92. INTEREST

A farmer hired a team 1 day for $5.00. How much would the team cost him for 5 days?

A man paid $25 for the use of a house 1 month. What will his house rent cost for 6 months?

Money paid for the use of a team is called **hire.**

Money paid for the use of a house is called **rent.**

James Brown borrowed from Henry Lane $100 to use for 1 year. Brown paid Lane $6 at the close of the year for the **use** of the $100.

Money paid for the use of money is called **interest.**

The **principal** is the money borrowed.

The **amount** is the sum of the principal and interest.

Thus, $100 principal + $6 interest = $106, amount.

The interest is usually calculated at a certain rate per cent of the principal for one year.

1. What is the interest on $250 for 2 years at 6%?

Solution:

$250 Principal
.06 Rate
$15.00 Interest for one year.
2
$30.00 Interest for two years.

NOTE: Multiply the principal by the rate and that product by the time.

Find the interest on:

2. \$750 for 3 years at 6%.
3. \$1000 for $2\frac{1}{2}$ years at 5%.
4. \$650 for $3\frac{1}{3}$ years at 6%.
5. \$732 for $5\frac{1}{8}$ years at 6%.
6. \$9200 for $9\frac{1}{8}$ years at 9%.
7. \$2360 for $7\frac{1}{4}$ years at 6%.
8. \$7895 for $2\frac{1}{2}$ years at 6%.
9. \$8756 for $\frac{6}{8}$ years at 5%.
10. \$10200 for $4\frac{1}{2}$ years at 4%.
11. \$7963 for $5\frac{1}{4}$ years at 8%.
12. \$7850 for 2 years at $4\frac{1}{2}$%.
13. \$760 for $5\frac{1}{2}$ years at $5\frac{1}{2}$%.

What part of a year is 1 month? 2 months? 3 months? 4 months? 6 months? 8 months? 9 months? 10 months?

To find the interest for months, take such part of the interest for 1 year as the number of months is part of 1 year.

14. Find the amount of \$760 for 3 years 5 months at 6%.

Solution:

\$760 Principal
.06 Rate
\$45.60 Interest for 1 year.
\$45.60×3=\$136.80 Interest for 3 years.
$\frac{5}{12}$ of \$45.60= 19.00 Interest for 5 months.
\$155.80 Interest for 3 yr. 5 mo.
\$760+\$155.80=\$915.80, Amount.

Find the amount of:

15. \$360 for 3 yr. 7 mo. at 6%.
16. \$980 for 5 yr. 4 mo. at 8%.
17. \$760 for 4 yr. 5 mo. at 6%.
18. \$696 for 6 yr. 6 mo. at 6%.
19. \$560 for 4 yr. 4 mo. at 8%.
20. \$796 for 1 yr. 9 mo. at 4%.
21. \$120 for 2 yr. 11 mo. at 7%.
22. \$896 for 3 yr. 7 mo. at 5%.
23. \$365 for 2 yr. 6 mo. at 5%.

24. \$784 for 7 yr. 5 mo. at 7%.
25. \$586 for 6 yr. 11 mo. at 4%.
26. \$750 for 3 yr. 3 mo. at 12%.

93. Interest for Days

30 days is counted an interest month.

What part of a month is 1 day? 5 days? 6 days? 10 days? 12 days? 15 days? 18 days? 20 days? 24 days? 25 days?

To find the interest for days: Take such part of the interest for 1 month as the number of days is part of a month.

1. Find the interest on \$360 for 3 yr. 5 mo. 18 da. at 6%.

Solution:

\$360 Principal
.06
\$21.60 Interest for 1 yr.
\$21.60 × 3 = \$64.80 Interest for 3 yrs.
\$21.60 × $\frac{5}{12}$ = 9.00 Interest for 5 mo.
\$21.60 ÷ 12 × $\frac{3}{5}$ = 1.08 Interest for 18 da.
\$74.88 Int. for 3 yr. 5 mo. 18 da.

Find the interest on:

2. \$750 for 4 yr. 9 mo. 19 da. at 8%.
3. \$960 for 5 yr. 11mo. 17 da. at 5%.
4. \$636 for 2 yr. 7 mo. 11 da. at 6%.
5. \$936 for 5 yr. 5 mo. 13 da. at 10%.
6. \$696 for 7 yr. 7 da. at 5%.
7. \$326 for 3 yr. 3 mo. 2 da. at 6%.
8. \$536 for 9 yr. 10 mo. 10 da. at 8%.
9. \$540 for 7 yr. 11 mo. 29 da. at 9%.
10. \$13.80 for 5 yr. 7 mo. 27 da. at 8%.
11. \$2650 for 3 yr. 5 mo. 17 da. at 6%.
12. \$7860 for 7 yr. 7 mo. 11 da. at 5%.
13. \$675 for 2 yr. 5 mo. 12 da. at 6%.
14. \$1200 for 1 yr. 1 mo. 6 da. at 10%.
15. \$860 for 2 yr. 6 mo. 10 da. at 5%.

94. Promissory Notes

When money is borrowed, it is customary for the person who borrows to give to the one who lends a written statement acknowledging the debt and promising to repay the money at some certain time.

This statement is called a **promissory note** and is usually written in the following form:

Nashville, Tennessee, Jan. 1, 1923.

One year after date, I promise to pay Henry Lane, or order, One Hundred Dollars ($100) for value received, with interest at 6%.

JAMES BROWN.

The person signing the note is called the **maker.**

The person to whom the money is to be paid is called the **payee.**

The sum of money loaned is called the **face** of the note.

Who is the maker of the above note? The payee? What is the face of the note? What words in the note acknowledge the obligation to pay?

The day on which a note becomes due is called the **date of maturity.**

Find the amount of the following note at date of maturity:

1. Jackson, Miss., June 1, 1923.

Two years after date, I promise to pay John Smith, or order, Two Thousand Six Hundred Twenty-five Dollars ($2625) for value received, with interest at 6%.

JOHN HARPER.

Find the amount of a note for:

2. $168 due in 1 yr. 9 mo. 18 da. at 5%.

3. $396 due in 3 yr. 7 mo. 29 da. at 6%.

4. $480 due in 2 yr. 11 mo. 7 da. at 5%.

5. $756 due in 3 yr. 7 mo. 14 da. at 8%.

6. Write a promissory note, and find the amount of the note at date of maturity.

PART II

95. DENOMINATE NUMBERS

Before anything can be measured, there must be some unit of measurement provided. You could compare or measure the length of your desk with your pencil and find that the desk is four, five, or six times the length of the pencil. While the relative lengths would be clear to you, no one else would understand the comparison unless he knew the length of your particular pencil, as pencils are of different lengths.

But if you should take a ruler and measure your desk and find it to be 30 inches in length, every one would understand the length of your desk, for the *inch* is a recognized unit of measure established by law for ascertaining lengths.

A unit measure established by law or custom is called a **standard measure.**

A concrete number expressed in units of standard measure is called a **denominate number**, as \$5, 6 yd., 10 lb., 3 bu.

Each class of measurements has a legal standard unit from which other larger or smaller units may be derived. Thus, the **yard** is the legal standard unit for measuring lengths. But the smaller units, the **foot** and the **inch**, and the larger units, the **rod** and the **mile**, are derived from the yard. Often it is convenient to use more than one unit or denomination in measuring quantities. Thus, the length of the flag pole may be 18 ft. and 6 in.

A **compound denominate number** is composed of two or more denominations used to express one quantity.

Sometimes the smaller unit is expressed as a fractional part of the next larger unit, as $3\frac{1}{2}$ bu. instead of using the compound denominate number 3 bu. 2 pk.

96. Measures of Length

Table

12 in. = 1 ft.

3 ft. = 1 yd.

$5\frac{1}{2}$ yd. or $16\frac{1}{2}$ ft. = 1 rd. or pole.

320 rd. = 1 mi.

1 mi. = 320 rd. = 5280 ft. = 63360 in.

NOTE: Feet and inches are sometimes indicated thus: 3′ 6″ which is read *three feet and six inches*.

How many inches in 3 ft. 6 in.?

1 ft. = 12 in. and 3 ft. = 3 × 12 in. = 36 in. 36 in. + 6 in. = 42 in.

Hence, 3 ft. 6 in. = 42 in.

Reduce 30 in. to feet.

Since 12 in. = 1 ft., 30 in. will equal as many times 1 ft. as 12 is contained in 30.

$$\begin{array}{r} 2 \\ \hline 12)30 \\ 24 \\ \hline 6 \end{array}$$

12 is contained in 30, 2 times with 6 for remainder.

Hence, 30 in. = 2 ft. 6 in., or $2\frac{1}{2}$ ft.

Changing the form of compound numbers without changing their value is called *reduction*.

Reducing from a larger to a smaller unit is called *reduction descending*, as feet reduced to inches.

Reducing from a smaller to a larger unit is called *reduction ascending*, as inches reduced to feet.

1. Reduce 5 mi. 180 rd. 10 ft. to feet.

5
320
1600
180
1780
$16\frac{1}{2}$
890
10680
1780
29370
10
29380

Since there are 320 rod in 1 mi., in 5 mi. there are 5×320 rd. $= 1600$ rd. 1600 rd. $+ 180 = 1780$ rd.

As there are $16\frac{1}{2}$ ft. in 1 rd., in 1780 rd. there are $1780 \times 16\frac{1}{2}$ ft. $= 29370$ ft. 29370 ft. $+ 10$ ft. $= 29380$ ft.

2. Reduce 4 yd. 2 ft. 6 in. to inches.
3. Reduce 120 yd. 1 ft. 10 in. to inches.
4. Reduce $7\frac{3}{4}$ rd. to feet.
5. Reduce $\frac{1}{2}$ mi. to rods.
6. Reduce 5 mi. 180 rd. 10 ft. 6 in. to inches.
7. Reduce 8 mi. 140 rd. 3 yd. 7 in. to inches.
8. Reduce 43268 ft. to higher denominations.

(a) $16\frac{1}{2})43268 =$

(b)
2622
33)86536
66
205
198
73
66
76
66
$\frac{10}{2} = 5$

(c)
8
320)2622
2560
62

2622 rd. $= 8$ mi. and 62 rd.

Hence, 43268 ft. $= 8$ mi. 62 rd. 5 ft.

Multiplying divisor and dividend by the same number does not alter the quotient. The remainder must be divided by the same number by which the divisor and dividend are multiplied in order to correct the remainder: $\frac{10}{2} = 5$ ft.

9. Reduce 487936 inches to higher denominations.

10. Reduce 37834 yd. to higher denominations.

11. Reduce 22867 ft. to higher denominations.

12. Reduce 987436 in. to higher denominations.

13. Reduce 84672 ft. to higher denominations.

14. Reduce 637845 in. to higher denominations.

15. Reduce $\frac{7}{9}$ of a mile to lower denominations.

$\frac{7}{9}$ of a mile $= \frac{7}{9}$ of $\frac{3\,2\,8\,0}{1}$ rd. $= \frac{2\,2\,4\,0}{9}$ rd. $= 248\frac{8}{9}$ rd.

$\frac{8}{9}$ rd. $= \frac{8}{9}$ of $16\frac{1}{2}$ ft. $= \frac{\overset{4}{\cancel{8}}}{\underset{3}{\cancel{9}}}$ of $\frac{\overset{11}{\cancel{33}}}{\underset{1}{\cancel{2}}}$ ft. $= \frac{44}{3}$ ft. $= 14\frac{2}{3}$ ft.

$\frac{2}{3}$ ft. $= \frac{2}{3}$ of 12 inches $= 8$ inches.

Therefore, $\frac{7}{9}$ mi. $= 248$ rd. 14 ft. 8 in.

16. Reduce $\frac{2}{3}$ mi. to whole numbers of lower denominations.

17. Reduce $\frac{1}{2}$ rd. to whole numbers of lower denominations.

18. Reduce $\frac{7}{1\,1}$ mi. to whole numbers of lower denominations.

19. What fraction of a mile is 120 rd.?

Solution: 320 rd. $= 1$ mi.
1 rod is $\frac{1}{3\,2\,0}$ of mi.
120 rods is $\frac{1}{3\,2\,0}$ mi. $= \frac{1}{8\,\frac{2}{2}}$ mi. $= \frac{8}{8}$ mi.

20. What fraction of a mile is 180 rd. and 10 ft.?

21. What fraction of a yard is 2 ft. and 9 in.?

22. What fraction of a rod is 10 ft. and 9 in.?

23. What fraction of a mile is 180 rd. 10 ft. and 8 in.?

24. Change .75 rd. to units of lower denominations.

16.5
.75
825
1155
12.375 = number of ft.
12
4.500 = number of inches in .375 ft.

Since 1 rd. = 16.5 ft., .75 of a rd. = .75 of 16.5 ft. = 12.375 ft. 1 ft. = 12 in. .375 ft. = .375 of 12 in. = 4.5 in. Hence .75 rd. = 12 ft. 4.5 in.

97. Other Measures of Length

The following table was formerly used by surveyors, but is not in general use now. It is given here for reference only.

7.92 in. = 1 link (l.)

100 l. = 1 chain

4 rods or 66 ft. = 1 chain (ch.)

80 ch. = 1 mile

98. Table for Measuring Depths of Sea

6 feet = 1 fathom

608 feet = 1 cable length

10 cable lengths = 1 nautical mile (6080 ft.)

Written:

1. At a certain point in the sea the depth was found to be 6 cable lengths. How many feet deep was the sea?

2. How many feet in 20 fathoms?

3. There are 5280 ft. in a statute mile and 6080 ft. in a nautical mile. What part of a nautical mile is a statute mile?

4. How many more feet are there in 5 nautical miles than in 5 statute miles?

5. How many more feet are there in 9 cable lengths than in a statute mile?

Note: Other units of length: The **line** (you see on ruler) = the twelfth part of an inch; the **geographic mile** or **knot** (used in measuring distances at sea) = 1.152 statute miles. 1 **league** = 3 geographic miles; the **hand** (for measuring horses) = 4 in.; **span** = 9 inches; **cubit** = 18 inches.

99. SURFACE MEASURE

A **square** is a figure having four equal sides and four right angles.

The standard unit of square measure is the *square yard*, the other denominations being derived from it.

In practice, the size of the unit of measure used depends on the size of the surface measured.

We would measure the area of a leaf in your tablet in sq. in.; the area of your blackboard in sq. ft.; the area of the walls of your school room in sq. yd.; the area of your state in sq. mi. Each of these units is a square derived from its corresponding unit of length. The acre is a unit of area not corresponding to any measure of length.

Table

144 sq. in.	= 1 sq. ft.
9 sq. ft.	= 1 sq. yd.
$30\frac{1}{4}$ sq. yd.	= 1 sq. rd.
160 sq. rd.	= 1 A.
640 A.	= 1 sq. mi.

1 A. = 160 sq. rd. = 4840 sq. yd. = 43560 sq. ft.

You may take any unit of a compound denominate number and express its equivalent in terms of a higher or a lower unit. Thus, 800 sq. rd. may be expressed in acres by dividing 800 sq. rd. by 160, the number of sq. rd. in one acre; or 800 sq. rd. may be expressed in sq. yd. by multiplying 800 sq. rd. by $30\frac{1}{4}$ sq. yd., the number of sq. yd. in one sq. rd.

100. Exercise.

1. Reduce 12 A. 120 sq. rd. 6 sq. ft. to square feet.

Computation

12
160
720
12
1920
120
2040
$30\frac{1}{4}$
510
61200
61710
9
555390
6
555396

Process

1 A. = 160 sq. rd.
12 A. = 1920 sq. rd.
1920 sq. rd. + 120 sq. rd. = 2040 sq. rd.
1 sq. rd. = $30\frac{1}{4}$ sq. yd.
2040 sq. yd. = 2040 × $30\frac{1}{4}$ sq. yd. = 61710 sq. yd.
1 sq. yd. = 9 sq. ft.
61710 sq. yd. = 61710 × 9 sq. ft. = 555390 sq. ft.
555390 sq. ft. + 6 sq. ft. = 555396 sq. ft.

2. Reduce 5 A. 119 sq. rd. 7 sq. ft. to square feet.

3. Reduce 7 A. 3 sq. ft. to square feet.

4. Reduce 145 sq. rd. 5 sq. ft. 7 sq. in. to square inches.

5. Reduce 5 sq. mi. 240 A. 100 sq. rd. to square rods.

6. Reduce 20 A. to square yards.

7. How many square feet in 25 A.?

8. How many square inches in 125 sq. rd.?

9. Reduce 2 sq. mi. 140 sq. rd. to square feet.

10. What part of an acre is 140 sq. rd.?

11. What part of a square mile is 320 A.?

12. What part of a square rod is 25 sq. yd.?

13. Reduce 19259 A. to sq. mi.

14. Reduce 436624 sq. ft. to higher denominations.

(a) 48513—7
9)436624

(a) Number of sq. yd. = 436624 sq. ft. ÷ 9 sq. ft. = 48513—7 sq. ft. rem.

(b) $30\frac{1}{4}$)48513

(b) Multiply dividend and divisor each by 4.

1603
(c) 121)194052
121
730
726
452
363
$\frac{9}{4} = 22\frac{1}{4}$

(c) Number of sq. rd. = 48513 sq. yd. ÷ $30\frac{1}{4}$ sq. yd. = 1603—$22\frac{1}{4}$ sq. yd. rem.

(d) 10
160)1603
160
3

(d) Number of A. = 1603 sq. rd. ÷ 160 sq. rd. = 10—3 sq. rd. rem.

436624 sq. ft. = 10 A. 3 sq. rd. $22\frac{1}{4}$ sq. yd. 7 sq. ft.

The fraction of a square yard reduced to lower denominations:

$\frac{1}{4} \times \frac{9}{1}$ sq. ft. $= \frac{9}{4}$ sq. ft. $= 2\frac{1}{4}$ sq. ft.; 7 sq. ft. $+ 2\frac{1}{4}$ sq. ft. $= 9\frac{1}{4}$ sq. ft. = 1 sq. yd. $+ \frac{1}{4}$ sq. ft.; $\frac{1}{4}$ sq. ft. $= \frac{1}{4} \times 144$ sq. in. = 36 sq. in.

By adding like units, we have the simplified result: 436624 sq. ft. = 10 A. 3 sq. rd. 23 sq. yd. 36 sq. in.

15. Reduce 495840 sq. ft. to higher denominations.

16. Reduce 48676 sq. yd. to higher denominations.

17. Reduce 8267 sq. rd. to higher denominations.

18. Reduce 784936 sq. in. to higher denominations.

19. Reduce 864532 sq. rd. to higher denominations.

20. In the West, land is laid off into townships six miles square. Each township contains 36 sq. mi. How many acres in a township? Each township is divided into 36 equal sections. How many acres in a section?

101. SURFACE MEASUREMENTS

A surface has *length* and *breadth*; as your blackboard, schoolroom floor, tablet leaf, etc. Such surface is bounded by lines. When lines meet they form angles. Lines that lie in the same direction are *parallel*. Parallel lines can never meet.

Lines and Angles

Lines that lie in the same direction, as AB and CD, are parallel. Lines that are not parallel will meet if sufficiently produced. The difference in direction of lines that meet at a point is called an *angle*.

The size of an angle depends upon the difference in direction of the two lines that meet. When a straight line as AB is met by another straight line as CD, so as to make the two adjoining angles equal, each angle is a *right angle*. The lines AB and CD are then *perpendicular* to each other. What kind of an angle do lines make in forming a square corner?

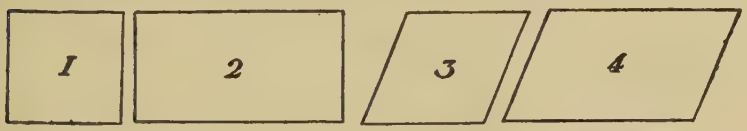
An angle less than a right angle is called an *acute* angle.

An angle larger than a right angle is called an *obtuse* angle.

The name of a figure describing a surface is determined by its number of sides and the size of its angles.

102. Quadrilaterals

A **quadrilateral** is a figure bounded by four straight lines.



Square Rectangle Rhombus Rhomboid

1. A **square** is a quadrilateral whose sides are all equal and whose angles are all right angles.

2. A **rectangle** is a quadrilateral whose opposite sides are equal and parallel and whose angles are all right angles.

What is the difference between a square and a rectangle?

3. A **rhombus** is a quadrilateral whose sides are all equal but whose angles are not right angles.

What is the difference between a square and a rhombus?

4. A **rhomboid** is a quadrilateral whose opposite sides are equal and parallel but whose angles are not right angles.

What is the difference between a rhomboid and a rectangle?

What is the difference between a rhombus and a rhomboid?

Trapezoid Trapezium

5. A **trapezoid** is a quadrilateral with only two of its sides parallel.

6. A **trapezium** is a quadrilateral which has no two of its sides parallel. **Parallelogram** is a term applied to quadrilaterals having opposite sides parallel.

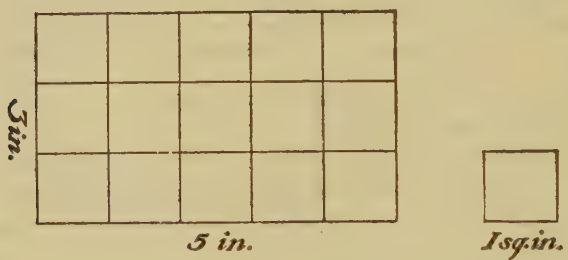
What quadrilaterals above are not parallelograms?

The side on which a figure appears to stand is called its *base*.

103. Areas of Rectangles

If we wish to find the area of a rectangle, we have only to ascertain the number of times it contains its unit of measure.

1. Find the area of a rectangle 5 inches in length and 3 inches in breadth.



Take the 1 sq. in. and see how many times it can be placed on the surface of the rectangle 5 inches long. We find the unit of measure can be placed on the length 5 times and on the breadth 3 times, thus making 3 rows of square inches with 5 square inches in a row. Hence, the area $=3\times5$ sq. in. $=15$ sq. in.

Or, area $=3\times5\times1$ sq. in. $=15$ sq. in.

2. Find the area of a rectangle 7 ft. 6 in. long and 6 ft. wide.

NOTE: Length and width must be expressed in like units.

Then, $7\frac{1}{2}\times6\times1$ sq. ft. $=45$ sq. ft.

The area of a rectangle is found by multiplying the product of its two dimensions by its unit of measure.

Thus, $A=l\times w\times u$, where A stands for area; l, for length; w, for width; and u, for the unit of measure.

Find the areas of rectangles having the following dimensions.

3.	24 ft. 6 in. by 20 ft.	**4.**	30 yd. by 27 yd. 2 ft.
5.	12 ft. 6 in. by 6 ft. 4 in.	**6.**	40 rd. by 30 rd.
7.	3 ft. by 40 in.	**8.**	40 rd. by 16 ft.
9.	16 ft. 8 in. by 12 ft.	**10.**	$40\frac{1}{2}$ rd. by 33 ft.
11.	$80\frac{1}{2}$ rd. by 60 rd.	**12.**	$16\frac{2}{3}$ ft. by 30 in.

13. Find the area of a rectangular grass plot 80 ft. long by $37\frac{1}{2}$ ft. wide.

14. How many acres in a square field each side of which is 80 rd.?

15. Find the area of a rectangular field 75 rd. long and $25\frac{1}{3}$ rd. wide.

16. What is the area of a hallway 160 ft. long and $12\frac{1}{2}$ ft. wide?

17. What is the area of the floor of a school room $23\frac{1}{2}$ ft. by 28 ft.?

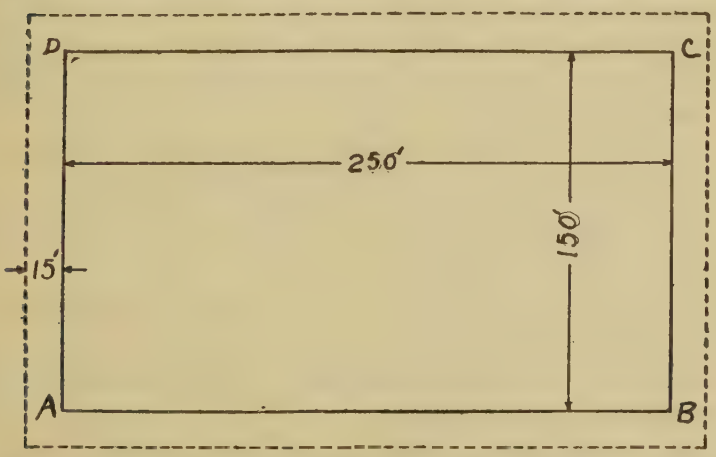
18. What is the area of a door $7\frac{1}{2}$ ft. by 4 ft.?

19. If a rectangle 5 ft. in width has an area of 80 sq. ft., what is the length?

20. If a rectangle 40 rd. long contains 1200 sq. rd., what is its width?

21. A rectangular field containing 9 A. is 60 rd. long; what is its width?

22. Find the area of a square each side of which is $5\frac{1}{2}$ yd.



23. ABCD is a rectangular plot of ground 250 ft. by 150

ft. On all sides of it is a road 15 ft. wide. Find the area of the road.

You may solve the above problem in either of two ways. Think out the two ways and state which you think is better.

24. On all sides of a rectangular park 750 ft. by 650 ft. is a street 30 ft. wide. Find the area of the street.

25. A city block 500 ft. square is surrounded by a sidewalk 6 ft. wide. Find the cost of constructing the sidewalk at 25¢ a square foot.

26. Find the cost of constructing a street 600 yd. long by 40 ft. wide at $1.50 a square foot.

27. Find the length of a rectangle whose area is 640 sq. rd. and whose width is 20 rd.

28. At 25¢ a square foot, the cost of constructing a sidewalk was $90. If the width of the sidewalk is $4\frac{1}{2}$ ft., what is its length?

29. Find the cost at $2.25 a square yard of constructing a concrete floor 25 ft. by 30 ft.

30. At 25¢ a square foot, find the cost of constructing a sidewalk 6 ft. wide around a square flower garden, each side of which is 200 ft.

31. A rug 9 ft. by 10 ft. has a painted border on the floor surrounding it $2\frac{1}{2}$ ft. wide. Find the area of the border.

32. Measure the top of your desk and find its area in square inches; in square feet.

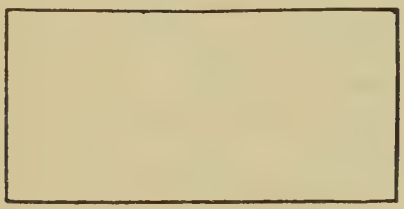
33. Find the area of one of the window-panes in your school-room.

34. Find the area of the floor of your school-room in square feet; in square yards.

35. How could you find the area of the ceiling of your school-room?

104. Area of Parallelograms

Any parallelogram has the same area as a rectangle whose base and altitude are equal to that of the given parallelogram. The *altitude* is the shortest distance between the base and the opposite side. It is perpendicular to, and forms right angles with, the base.



1. Cut from paper a parallelogram with base 3 in. and altitude 2 in. (2) Cut the parallelogram along the line of its altitude. (3) Place the left side of the parallelogram on the right side and your figure makes a rectangle.

Hence, the area of the parallelogram is $2 \times 3 \times 1$ sq. in. 6 sq. in.

Formula: $A = b \times h \times u$, where A stands for area; b for base; h for height or altitude; and u for unit of measure.

Find the area of parallelograms of these dimensions:

	Base	*Altitude*		*Base*	*Altitude*
1.	7 ft.	$6\frac{1}{2}$ ft.	**2.**	30 rd.	$20\frac{1}{2}$ rd.
3.	$6\frac{1}{2}$ mi.	6 mi.	**4.**	$7\frac{1}{2}$ in.	$4\frac{1}{2}$ in.
5.	$17\frac{1}{2}$ ft.	$3\frac{1}{2}$ yd.	**6.**	$40\frac{1}{2}$ rd.	25 ft.
7.	6 yd.	$8\frac{1}{2}$ ft.	**8.**	45 rd.	$27\frac{1}{2}$ ft.
9.	$3\frac{1}{4}$ in.	$3\frac{1}{3}$ ft.	**10.**	$16\frac{1}{2}$ ft.	$5\frac{1}{3}$ yd.

11. A flower plot in the shape of a rhomboid had a base of 16 ft. and an altitude of $7\frac{1}{2}$ ft. Find its area.

12. Find the area of a rhomboid whose base is 85 rd. and whose altitude is $37\frac{1}{2}$ rd.

13. Find the area of a rhombus whose base is 80 rd. and whose altitude is $50\frac{1}{2}$ rods.

14. If the area of a parallelogram is 320 sq. ft. and its base is 20 ft., find its altitude.

15. If the area of a rhomboid is 800 sq. rd. and its altitude is 20 rd., what is its base?

16. If the area of a field in the shape of a parallelogram is 12 A. and its length is 48 rd., find the width of the field.

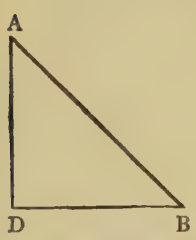
17. How many cards 4 in. by 6 in. can be cut from a cardboard 24 in. by 30 in.?

18. How many cards 5 in. by 7 in. can be cut from a cardboard 25 in. by 28 in.?

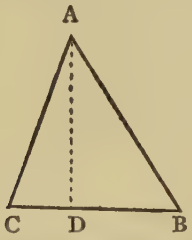
19. How many cards $4\frac{1}{2}$ in. by $6\frac{1}{2}$ in. can be cut from a cardboard $22\frac{1}{2}$ in. by 26 in.?

105. Triangles

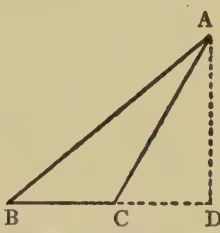
A **triangle** is a surface bounded by three straight lines.



Right Angled Triangle



Acute Angled Triangle



Obtuse Angled Triangle

The *base* of a triangle is the side on which it appears to rest.

The *vertex* of a triangle is the angle opposite the base, as the angle A.

The *altitude* is the perpendicular distance from the vertex to the base, or the base produced, as AD.

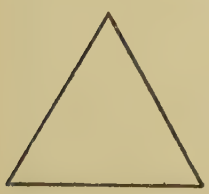
A triangle that has one right angle is called a **right angled triangle.**

A triangle whose angles are all acute is an **acute angled triangle.**

A triangle that has one obtuse angle is an **obtuse angled triangle.**

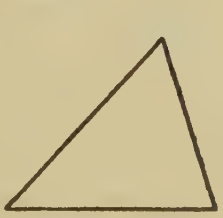
How many acute angles has a right angled triangle? An obtuse angled triangle?

The above triangles are named from their angles.



Equilateral Triangle

Isosceles Triangle



Scalene Triangle

An **equilateral** triangle has all its sides equal.

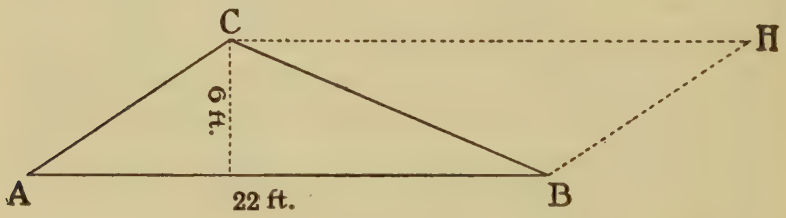
An **isosceles** triangle has two of its sides equal

A **scalene** triangle has no two of its sides equal.

These triangles are named from their sides.

106. Area of Triangles

1. Find the area of a triangle whose base is 22 ft. and whose altitude is 6 ft.



Draw CH parallel and equal to AB. Then draw BH. ABHC is a parallelogram whose base is 22 ft. and whose altitude is 6 ft. The triangle ACB is $\frac{1}{2}$ the parallelogram ABHC. The area of the parallelogram is equal to the product of its base by its altitude multiplied by its unit of measure. Hence the area of the triangle is equal to $\frac{1}{2}$ the product of the base by its altitude multiplied by the unit of measure. Area $= \frac{1}{2}$ (22×6×1 sq. ft.) = 66 sq. ft.

The base and altitude must be expressed in units of the same denomination.

2. Find the area of a triangle whose base is 8 ft. and whose altitude is 15 in.

Area $= \frac{1}{2}\times$ base (b) $\times$ altitude (a) $\times$ unit of measure (u), or Formula: $\mathbf{A} = \frac{1}{2}\times\mathbf{b}\times\mathbf{a}\times\mathbf{u}$.

Area $= \frac{1}{2}$ ($8\times1\frac{1}{4}\times1$ sq. ft.) = 5 sq. ft.

The unit of measure for surfaces is one square unit of the denomination in which the measurement is to be expressed.

Areas may, by the process of reduction, be changed to higher or lower units.

3. Find the area in acres of a triangular field whose base is 80 rd. and whose altitude is 80 rd.

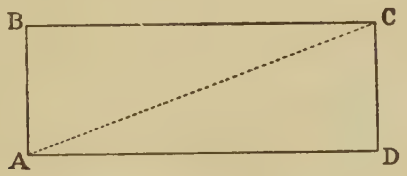
Area $= \frac{1}{2} \times 80 \times 80 \times 1$ sq. rd. $= 3200$ sq. rd.

3200 sq. rd. $\div 160$ sq. rd. $= 20$, number of acres.

Find the area of each of the following triangles:

	b (*base*)	*a* (*altitude*)		*b*	*a*
4.	12 ft. 6 in.	10 ft.	**5.**	$40\frac{1}{2}$ rd.	32 rd.
6.	6 yd. 2 ft.	3 yd.	**7.**	40 ft. 9 in.	20 ft.
8.	82 ft. 6 in.	3 yd.	**9.**	50 ft. 4 in.	32 ft. 6 in.
10.	68 rd. 5 ft.	33 ft.	**11.**	45 ft. 9 in.	16 ft. 3 in.
12.	80 rd. 5 ft.	49 ft. 6 in.	**13.**	$17\frac{1}{2}$ yd.	$37\frac{1}{2}$ ft.

14. Find the area of a right triangle whose base is 50 ft. and whose altitude is 25 ft.



What kind of a figure is ABCD? Why?

The line AC joining the opposite corners is called a *diagonal.*

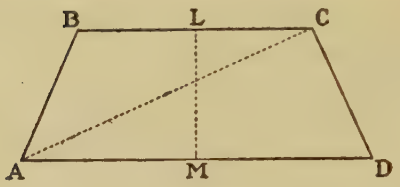
The diagonal divides the rectangle into what kind of triangles? What part of the rectangle is the right angled triangle?

15. Find the area of a right triangular grass plot whose base is 80 ft. and altitude 16 ft. 6 in.

16. Find the difference in area between two right triangles, one of which has a base of 20 ft. and an altitude of 12 ft. 6 in.; the other has a base of 25 ft. and an altitude of 10 ft. 6 in.

17. What is the area of a triangle whose base is 6 ft. 6 in. and whose altitude is 5 ft.?

18. What is the difference in area between two triangles one of which has a base of 8 ft. and altitude of 10 ft., and the other a base of 4 ft. and altitude of 5 ft.? What is the ratio of the two areas?



Trapezoid

107. Trapezoids

A quadrilateral with only two of its sides parallel is called a **trapezoid.** The parallel sides are called the bases of the trapezoid.

The perpendicular distance between the bases is called the altitude, as LM.

The line joining the opposite vertices is called the diagonal, as AC.

The diagonal divides the trapezoid into two triangles. The base of one triangle is 12 ft., the altitude 5 ft. The base of the other triangle is 8 ft., the altitude 5 ft.

The sum of the areas of the two triangles is the area of the trapezoid.

Area triangle ACD $= \frac{1}{2}$ (12×5×1 sq. ft.) = 30 sq. ft.
Area triangle ABC $= \frac{1}{2}$ (8×5×1 sq. ft.) = 20 sq. ft.
Area trapezoid = ---------- 50 sq. ft.

As all the factors are the same except 8 and 12, the upper and lower bases of the trapezoid, we may find the area thus: $\frac{1}{2}$ (8+12)×5×1 sq. ft. = 50 sq. ft.

The area of a trapezoid may be found by multiplying one half the sum of the parallel sides by the altitude and that product by the unit of measure.

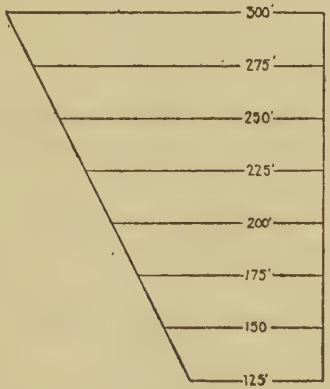
Formula: $\mathbf{A = \frac{1}{2} \times (B + b) \times a \times u}$, where **B** stands for lower base and **b** for upper base.

1. Find the area of a trapezoid whose parallel sides are 16 ft. and 24 ft. and altitude 15 ft.

2. Find the area of a lot in the shape of a trapezoid whose parallel sides are 100 ft. and 120 ft., and whose altitude is 60 ft.

3. Find the area of a field in the shape of a trapezoid having parallel sides of 120 rods and 16 rods and an altitude of 100 rods.

4. A man subdivided a tract of land in the following shape into lots 50 ft. in width. Find the area of each lot.



5. How many acres in a farm in the shape of a trapezoid whose parallel sides are 80 rd. and 120 rd. and whose altitude is 60 rd.?

6. (a) What is the difference in area between two trapezoids one of which has parallel sides of $8\frac{1}{2}$ ft. and $17\frac{1}{2}$ ft. and altitude of 6 ft., and the other having parallel sides of 17 ft., and 35 ft. and an altitude of 12 ft? (b) What is the ratio of the areas?

7. What is the altitude of a trapezoid whose area is 700 sq. ft. and whose parallel bases are 60 ft. and 80 ft.?

8. Find the area of a plot of ground in the form of a trapezoid whose parallel sides are $15\frac{1}{2}$ rd. and $7\frac{1}{2}$ rd., and whose altitude is $5\frac{1}{2}$ rd.

108. THE CIRCLE

A **circle** is a plane figure bounded by a curved line, every point of which is equally distant from a point within called the center.

The curved line bounding a circle is called the *circumference.* The distance from the center to the circumference is called the *radius.*

A line passing through the center of a circle and terminating in the circumference is called the *diameter.* The diameter is twice the length of the radius.

May a circle have more than one radius? More than one diameter? Illustrate your answer.

Measure the circumference of a silver dollar; the diameter.

Divide the length of the circumference by the length of the diameter.

Measure the circumference of a can lid, water pail, the top of a glass tumbler; then measure the diameter of each.

Divide the length of each circumference by the length of its diameter. You have found the circumference to be about $3\frac{1}{7}$ times the length of the diameter.

To be more exact, the circumference is 3.1416 times the length of the diameter. The Greek letter π may be used to represent 3.1416; diameter is represented by d; radius by r; and circumference by c.

If the diameter is given, how is the radius found? ($d=2r$.)

If the circumference is given, how is the diameter found?

Formula: $c=\pi\times d$; or $c=2\times\pi\times r$. Why?

109. Problems

Wherever decimals are involved in these problems, find results correct to three decimal places.

1. The diameter of a circle is 3 ft. Find its circumference.

2. The circumference of a tree is 18 ft. Find its diameter.

3. The diameter of a tumbler is $3\frac{1}{4}$ inches. Find its circumference.

4. What is the circumference of an 8 inch stove pipe?

5. The circumference of a wheel is 18 ft. 6 in. Find its radius.

6. The circumference of a circular grass plot is 31.416 ft. Find its diameter.

7. The circumference of a circular pond is 765 ft. 6 in. Find its diameter.

8. The radius of a circle is 5 ft. 4 in. Find its circumference.

9. Find the diameter of a circular race track whose circumference is 1 mile.

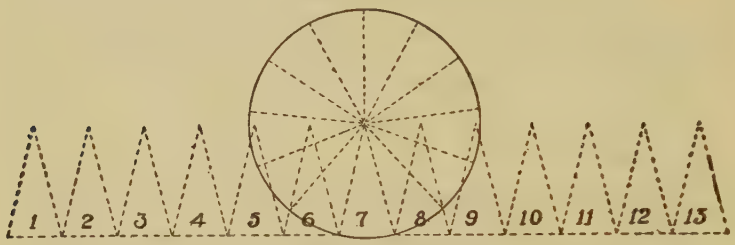
10. With a radius of 6 inches describe a circle. Find its circumference.

11. With a radius of 10 inches describe a circle, and from the same center with a radius of 7 inches describe a circle. What is the difference in the circumferences of the two circles mentioned above?

12. From the same center, with radii 3 in., 6 in., and 9 in., describe circles. What is the difference between the circumference of the circle whose radius is 9 in. and the sum of the circumferences of the two circles whose radii are 3 in. and 6 in.?

110. Measurement of Circles

The area of a circle is determined by the number of times it contains its unit of measure.



Imagine the circle cut into small sectors which we may call triangles, as illustrated in the figure. The altitude of each triangle is the radius of the circle, and its base is a part of the circumference. The area of each triangle is half the product of its base by its altitude by the unit of measure. The sum of the areas of the triangles is the area of the circle.

$A_1 = \frac{1}{2} \times b_1 \times r \times u$
$A_2 = \frac{1}{2} \times b_2 \times r \times u$
$A_3 = \frac{1}{2} \times b_3 \times r \times u$, etc.

A of circle $= \frac{1}{2} \times (b_1 + b_2 + b_3 + \text{etc.}) \times r \times u$; but $b_1 + b_2 + b_3 + \text{etc.} = c$.

Hence, we find the area of a circle equals one-half the product of the circumference by the radius by the unit of measure.

I. Formula: $A = \frac{1}{2} \times c \times r \times u$.

The circumference of a circle is $2 \times \pi \times r$, that is $c = 2 \times \pi \times r$.

Hence $A = \frac{1}{2} \times 2 \times \pi \times r \times r = \pi \times r^2$, that is $A = \pi \times r^2$.

NOTE: $r \times r = r^2$, just as $3 \times 3 = 3^2$.

We may find the area of a circle by multiplying the square of the radius (r^2) by π (3.1416). Hence, Formula I becomes Formula II, and is expressed thus: $A = \pi \times r^2$.

Find the area of a circle:

1. Whose radius is 6 ft.
2. Whose diameter is 10 ft.
3. Whose circumference is 21.9912 ft.
4. Whose radius is 5 ft.

5. Whose diameter is 7 rd.

6. Whose radius is 3 ft. 6 in.

7. Whose circumference is 219.912 ft.

8. Whose circumference is 502.656 ft.

9. What is the area of a circular grass plot whose radius is 12 ft.?

10. Over how many square feet of level ground can a pony graze if tethered with a rope 25 ft. in length and free to graze in all directions?

11. What is the area of a circular swimming pool 200 ft. in diameter?

12. What is the area of the base of a circular silo 12 ft. in diameter?

13. What is the area of a circular clock face $5\frac{1}{2}$ inches in diameter?

14. What is the difference in area between a square whose side is 4 ft., and a circle whose diameter is 4 ft.?

15. What is the area of a circular island whose diameter is 30 rd.?

16. Explain the formula: $c = \pi \times 2 \times r$; $A = \pi \times r^2$.

17. Which has a greater area, a circle whose diameter is 10 ft. or a square whose side is 10 ft.? How much greater?

18. Compare the area of a circle whose diameter is 10 ft. with the area of a square whose diagonal is 10 ft. (Area of square $= \frac{1}{2}$ the product of its diagonals.)

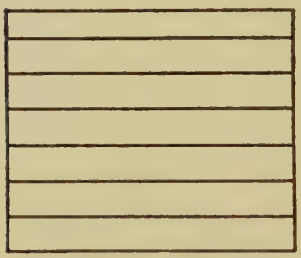
19. What is the diameter of a tree which is 22 ft. around? What is the area of its cross section?

20. A certain city is the center of a section with a 50-mile radius. Find the area of the section.

111. CARPETING

Carpeting is made in strips most frequently either $\frac{3}{4}$ of a yard or 1 yd. in width. The dealer will not split the width of a strip. Hence, if a fraction of the width of a strip is required, the whole strip must be bought. The carpet usually runs the length of the room.

1. How many yards of carpeting $\frac{3}{4}$ yd. wide are required for a room 18 ft. long, 15 ft. wide, if the carpet runs lengthwise?

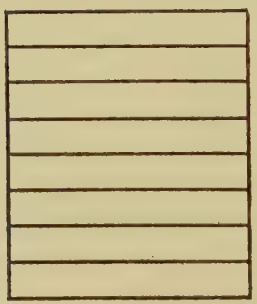


Divide the width of the room by the width of one strip of carpet to find the number of strips.

15 ft. = 5 yd., width of room.

5 yd. $\div \frac{3}{4}$ yd. $= 6\frac{2}{3}$, or 7, number of strips.

The room is 18 ft. long. The length of each strip must be 18 ft. = 6 yd. Multiply the length of each strip by the number of strips. 7×6 yd. = 42 yd.



If the strips run crosswise, divide the length of the room by the width of a strip. 18 ft. = 6 yd., length of room. 6 yd. $\div \frac{3}{4}$ yd. = 8, number of strips. 15 ft. = 5 yd., the length of 1 strip. Multiply the length of one strip by the number of strips.

8×5 yd. = 40 yd.

It will be seen in this problem that it takes 2 yards more to carpet the room if the strips run lengthwise. How do you account for this?

2. A floor 18 ft. by 16 ft. is covered with a carpet $\frac{3}{4}$ yd. wide, running lengthwise.

(1) How many strips are there?

(2) How many yards long is each strip?

(3) How many yards of carpet altogether?

(4) What would be the cost of the carpet at $1.35 a yard?

3. What will it cost to carpet a hallway 261 ft. long by 15 ft. wide with carpet 1 yd. wide running lengthwise at $1.20 a yard?

4. How many yards of carpet 30 in. wide will be required for a floor 21 ft. long and 15 ft. wide, (a) if the strips run lengthwise? (b) If the strips run crosswise?

Find the number of yards of carpet $\frac{3}{4}$ yd. wide and running lengthwise required to cover a room:

5. 15 ft. by 19 ft. **6.** 21 ft. by 24 ft.

7. 25 ft. by 30 ft. **8.** 12 ft. 6 in. by 14 ft.

9. 14 ft. by 6 ft. 6 in. **10.** 20 ft. by 24 ft.

11. How many yards of carpet 1 yd. wide will it take for a floor 13 ft. 6 in. wide and 15 ft. 6 in. long, the carpet running lengthwise? What will the carpet cost at $1.75 a yard?

12. How many square yards of linoleum will it take to cover a room 15 ft. by 18 ft.?

13. What will it cost to cover a hallway 12 ft. by 65 ft. with linoleum 48 in. wide, at $1.75 a linear yard, the linoleum to be laid lengthwise of the hall?

14. How many yards of carpeting 28 in. wide will be needed for a floor $12\frac{1}{2}$ ft. by 15 ft., strips running lengthwise?

15. Find the cost of carpeting a room 16 ft. by 18 ft. with carpet 1 yd. wide, strips running lengthwise, at $1.80 a yard.

16. Find the cost of carpeting the room described in problem 15 if the strips run crosswise.

112. PAPERING

Wall paper is sold by the single roll or double roll. The single roll is 8 yd. long, the double roll 16 yd. long, and the standard width is 18 in. Hence, there are 4 sq. yd. or 36 sq. ft. in a single roll and 8 sq. yd. or 72 sq. ft. in a double roll. Dealers do not sell a part of a roll. Hence, when a fraction of a roll is needed, the whole roll must be bought.

How many single rolls of paper are required to paper a room 15 ft. by 16 ft. and 9 ft. from the top of the base board to the ceiling, making no allowance for doors and windows?

15×16×1 sq. ft. = 240 sq. ft., area of ceiling.
15 ft. + 15 ft. + 16 ft. + 16 ft. = 62 ft., perimeter of room.
62×9×1 sq. ft. = 558 sq. ft., area of walls.
240 sq. ft. + 558 sq. ft. = 798 sq. ft., whole surface to be papered.
36 sq. ft. of paper = 1 roll.
798 sq. ft. ÷ 36 sq. ft. = $22\frac{1}{8}$ or 23, number of rolls.

The perimeter of the room multiplied by the height by the unit of measure equals the area of the walls. Why?

The sum of the areas of the walls and ceiling divided by the area of a roll equals the number of rolls required.

1. How many single rolls of paper will be required to paper a room 15 ft. by 12 ft. and $8\frac{1}{2}$ ft. from base board to ceiling?

2. How many double rolls of paper will be required to paper a hallway 12 ft. by 80 ft. and $13\frac{1}{2}$ ft. from floor to ceiling?

3. Find the cost of paper for a room 18 ft. by 20 ft. and 9 ft. to the ceiling at 35¢ a single roll.

4. Find the cost of a border for a room 16 ft. by 18 ft. at 27¢ a yard.

5. Find the cost of papering the ceiling of a hallway 16 ft. by 48 ft. at 85¢ a single roll.

6. Find the cost of papering a bed-room 12 ft. by 14 ft. and 8 ft. from the base board to the border at 37¢ a roll, the border costing 17¢ a yard.

7. Find the cost of paper for the ceiling of a room 15 ft. by 16 ft. at 90¢ a single roll.

NOTE: Local customs, as to allowances for openings and matching and methods of computing the number of rolls required, vary. Teacher and students should consult local paper dealers and hangers.

113. PLASTERING

Plastering is estimated by the square yard.

It is customary in most places to deduct one half the area of the openings from the total area of the surface plastered.

1. At 36¢ a square yard, how much will it cost to plaster a room 16 ft. long, 15 ft. wide, and 10 ft. high, allowing for 2 doors 7 ft. 6 in. by 4 ft. and two windows 6 ft. 6 in. by 4 ft.

$16 \times 15 \times 1$ sq. ft. $= 240$ sq. ft., area of ceiling.
Perimeter of room $= 15$ ft. $+ 15$ ft. $+ 16$ ft. $+ 16$ ft. $= 62$ ft.
$62 \times 10 \times 1$ sq. ft. $= 620$ sq. ft., area of walls.
240 sq. ft. $+ 620$ sq. ft. $= 860$ sq. ft., total area of walls and ceiling.
Area of 1 door $= 7\frac{1}{2} \times 4 \times 1$ sq. ft. $= 30$ sq. ft.
Area of 2 doors $= 2 \times 30$ sq. ft. $= 60$ sq. ft.
Area of 1 window $= 6\frac{1}{2} \times 4 \times 1$ sq. ft. $= 26$ sq. ft.
Area of 2 windows $= 2 \times 26$ sq. ft. $= 52$ sq. ft.
60 sq. ft. $+ 52$ sq. ft. $= 112$ sq. ft., area of doors and windows.
$\frac{1}{2}$ area of openings $= \frac{1}{2}$ of 112 sq. ft. $= 56$ sq. ft.
860 sq. ft. $- 56$ sq. ft. $= 804$ sq. ft.
804 sq. ft. $\div 9$ sq. ft. $= 89\frac{1}{3}$, number of sq. yd.
Cost $= \$0.36 \times 89\frac{1}{3} = \$32.16$.

2. How much will it cost at 28¢ a square yard to plaster a room 20 ft. long, 18 ft. wide, 10 ft. high, allowing for 2 doors 7 ft. 3 in. by 4 ft. and 4 windows 6 ft. 3 in. by 4 ft.?

3. How much will it cost to plaster the ceiling of a hallway 65 ft. long and 15 ft. wide at 27¢ a square yard?

4. How much will it cost at 32¢ a square yard to plaster a school room 30 ft. long, 25 ft. wide, and $13\frac{1}{2}$ ft. high, allowing for 2 doors 7 ft. 6 in. by 5 ft. and for 5 windows 8 ft. by 4 ft.?

5. Find the cost at 27¢ a square yard of plastering a bedroom 14 ft. long, 13 ft. wide, and 8 ft. high, allowing for 1 door 7 ft. 3 in. by 4 ft. and 2 windows 6 ft. 6 in. by 3 ft.

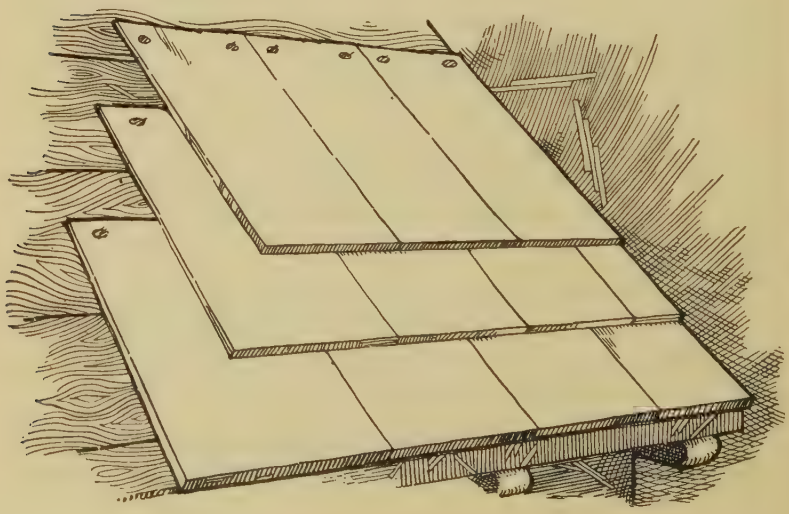
6. Find the cost of putting on a black-board $3\frac{1}{2}$ ft. by 24 ft. at 15¢ a square foot.

7. Find the cost at 24¢ a square yard of plastering a closet 8 ft. long, 6 ft. wide, 7 ft. high, allowing for 1 door 6 ft. by 3 ft.

8. How much will it cost to plaster the ceiling of a hallway 250 ft. long and 16 ft. wide at 35¢ a square yard?

114. ROOFING AND FLOORING

The unit of measure for roofing and flooring is the **square** which is 10 ft. by 10 ft., or 100 sq. ft.



In roofing with shingles or slate, each course is partly overlapped so that only a part of the shingle (usually 4 or $4\frac{1}{2}$ in.) is exposed to the weather. Shingles are usually made 16 in. long and 4 in. wide, and are put up in bundles of 250 shingles each.

In all problems in this text-book, the weather exposure is $4\frac{1}{2}$ in. unless otherwise stated.

1. If $4\frac{1}{2}$ in. of each shingle are exposed to the weather and the shingles are 4 in. wide, how many square inches of roof will each shingle cover?

2. How many square feet will a bundle cover?

3. If a bundle covers $31\frac{1}{4}$ sq. ft., how many square feet will 4 bundles (or 1000 shingles) cover?

4. How many bundles will it take to cover a chicken house whose roof is 10 ft. by $12\frac{1}{2}$ ft.?

5. How many bundles of shingles will be required to cover a garage whose roof is 18 ft. in width on each side and 35 ft. in length?

6. What will it cost to put a tar paper roof on a barn each side of which roof is 35 ft. wide and 70 ft. in length, at $7.50 a square?

7. If 1000 shingles are needed to the square, what will be the cost of shingles at $5.50 per M to cover two sides of a roof, each side of which is 24 ft. by 50 ft.?

8. What will it cost to put a flat composition roof on a building 30 ft. by 80 ft. at $25 a square?

9. How much will it cost to floor a school room 25 ft. by 30 ft. at $15 a square, adding $\frac{1}{5}$ for waste?

10. How much will it cost to lay a hardwood floor in a room 15 ft. by 20 ft. at $40 per square, adding $\frac{1}{3}$ for waste?

11. How much will it cost at $17.50 per square to lay a floor in a hall 12 ft. by 80 ft., adding $\frac{1}{4}$ for waste?

115. SOLID MEASURE

A **solid** is a figure or object having length, width and height.

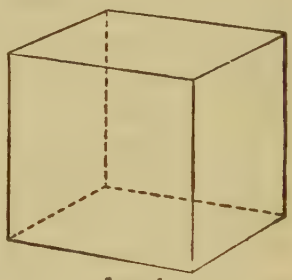
A solid having six equal square faces is called a **cube**.

A cube each edge of which is an inch is called a **cubic inch.**

Figure I represents 1 cu. in. A cubic foot is a cube each edge of which is 1 ft. In actual size each dimension of 1 cu. ft. is 12 times as much as each like dimension in 1 cu. in.

A cubic yard is a cube each edge of which is 1 yd.

The number of cubic units in a solid is called its **volume.**



1 cu. in.

FIG. I

Cube the number 12, or find 12^3.

Cube the number 3, or find 3^3.

Table

1728 cu. in. = 1 cu. ft.
27 cu. ft. = 1 cu. yd.
128 cu. ft. = 1 cord.
1 cu. yd. = 27 cu. ft. = 46,656 cu. in.

Cubic measure is used for measuring the volumes of solids.

A **rectangular solid** is a solid each of whose faces is a rectangle.

The volume of a rectangular solid is found by multiplying the length by the width by the height and that product by the unit of measure.

1. How many cubic feet in a block of ice 3 ft. by 2 ft. by $1\frac{1}{2}$ ft.?

Solution: $3 \times 2 \times 1\frac{1}{2} \times 1$ cu. ft. $= 9$ cu. ft.

2. How many cubic feet in a box 6 ft. long, 4 ft. wide, and $2\frac{1}{2}$ ft. deep?

3. How many cubic feet in a cellar 8 ft. by 12 ft. by 6 ft.?

4. (a) How many cubic feet in a school room 30 ft. long, 24 ft. wide, and $12\frac{1}{2}$ ft. from floor to ceiling? (b) If each student needs 200 cu. ft. of air space, how many students would the room accommodate?

5. What is the volume of two cubes whose edges are 5 in. and one-half foot, respectively?

6. How many cubic feet of dirt was removed from a ditch 4 ft. wide, 6 ft. deep, and 300 yd. long?

7. How many small boxes 1 ft. by 2 ft. by 2 ft. can be placed in a large box 5 ft. by 8 ft. by 4 ft.?

8. The dimensions of a common brick are $8'' \times 4'' \times 2''$. What is its volume?

9. (a) The volume of a common brick is what part of 1 cu. ft.? (b) How many common bricks can be placed in the volume of 1 cu. ft.?

10. After allowance for mortar, it is estimated in practice that 22 common bricks will make 1 cu. ft. How many bricks will be required to build a wall having a volume of 2500 cu. ft.?

11. How many bricks will it take to build a wall 200 ft. long, 18 in. wide, and 16 ft. high?

12. Find the volume of a wall 12 ft. by 8 ft. by 18 in.

13. How many common bricks can be laid in a wagon bed 12 ft. by $3\frac{1}{3}$ ft. by 8 in.?

14. How many cubic yards of stone will be required to cover a road 5280 ft. long, 24 ft. wide, 8 in. thick?

15. How many cubic feet of concrete in a floor 24 ft. by 30 ft. and 6 inches thick?

16. How many cubic feet in a concrete walk 400 ft. long, 5 ft. wide, 4 in. thick?

17. How many cubic yards of stone in a wall 300 ft. by 3 ft. by 4 ft.?

18. What will it cost to build a stone wall 200 yd. long, 1 yd. thick, and $1\frac{1}{3}$ yd. high, at $6.75 a cubic yard?

19. How many cubic yards in a cellar 12 ft. x 8 ft. x $5\frac{2}{3}$ ft.?

20. How many cubic inches in a crate $18\frac{1}{2}$ in. x 20 in. x $24\frac{1}{2}$ in.?

21. How many cubical blocks 2 in. x 2 in. x 2 in. can be put in a box 2 ft. x 3 ft. x 4 in.?

A CORD OF WOOD

22. A cord of wood is a stack of wood 8 ft. long, 4 ft. wide, and 4 ft. high. How many cubic feet does it contain?

23. How many cords in a stack of wood 24 ft. long, 4 ft. wide, and 12 ft. high?

24. How many cords in a stack of wood 32 ft. long, 4 ft. wide, and 10 ft. high?

25. What is the value of a stack of wood 132 ft. long, 8 ft. wide, and 8 ft. high, at $4.50 a cord?

26. Find the cost of a stack of wood 40 ft. long, $5\frac{1}{2}$ ft. high, and 4 ft. wide, at $3.75 a cord.

116. Reduction

1. Reduce 3 cu. yd. 19 cu. ft. to cubic feet.

2. Reduce 17 cu. yd. 24 cu. ft. 1350 cu. in. to cubic inches.

3. Reduce 65 cu. ft. 295 cu. in. to cubic inches.

4. 1 cu. yd. = ? cu. ft. = ? cu. in.

5. Reduce 983847 cu. in. to higher denominations.

6. Reduce 27635 cu. ft. to cubic yards.

7. Reduce 10 cu. yd. to cubic inches.

8. What is the ratio of 18 cu. ft. to 1 cu. yd.?

9. What is the ratio of 144 cu. in. to 1 cu. ft.?

10. What is the ratio of 1600 cu. in. to 9 cu. ft.?

11. Reduce .4 cu. yd. to units of lower denomination.

.4	Since there are 27 cu. ft. in 1 cu. yd., in .4 cu. yd. there are .4 of 27 cu. ft. = 10.8 cu. ft. Since there are 1728 cu. in. in 1 cu. ft., in .8 cu. ft. there are .8 of 1728 cu. in. = 1382.4 cu. in. Hence, .4 cu. yd. = 10 cu. ft. 1382.4 cu. in.
27	
10.8	
1728	
1382.4	

Reduce to lower denominations:

12. .125 cu. yd. **13.** .49 cu. ft. **14.** .75 cu. yd.

15. .$33\frac{1}{3}$ cu. yd. **16.** .45 cu. ft. **17.** 2.3 cu. yd.

117. CAPACITY MEASURE

When we speak of the capacity of a box, wagon bed, cistern, silo or any container, we mean the quantity it will hold of some unit of measure based upon volume.

Dry Measure

Dry measure is used for measuring grain, fruit, vegetables, etc. The standard unit of dry measure is the *bushel*, whose volume is 2150.42 cu. in.

How much would a box 13 in. by 13. in. by 12 in. lack of holding a bushel?

Table

2 pints (pt.) = 1 quart (qt.)
8 qt. = 1 peck (pk.)
4 pk. = 1 bushel (bu.)
1 bu. = 4 pk. = 32 qt. = 64 pt.

Exercise

1. What part of 1 pk. is 6 qt.?
2. What part of 1 bu. is 3 pk.?
3. How many pecks in $2\frac{1}{2}$ bu.?
4. Reduce 3 bu. 3 pk. 7 qt. 1 pt. to pints.
5. Reduce 3 pk. 6 qt. 1 pt. to pints.
6. Reduce 7 qt. 1 pt. to pints.
7. Reduce 19 bu. 1 pk. 2 qt. to quarts.
8. Reduce 7 bu. 1 pt. to pints.
9. Reduce 7 bu. 1 pk. to quarts.
10. Reduce 128 pt. to bushels.

11. Reduce 232 qt. to higher denominations.

12. Reduce 721 pt. to higher denominations.

13. How many cubic inches in $\frac{1}{2}$ bu.? $\frac{3}{4}$ bu.? $\frac{2}{3}$ bu.?

14. How many bushels in a box whose volume is 25805.04 cu. in.?

15. How many bushels in a box 9 ft. by 4 ft. by 3 ft. 6 in.?

16. A box having a volume of 240 cu. ft. is 8 ft. long and 6 ft. wide; what is its depth?

Hint: If you have the product of three factors and two of them given, how is the third found?

17. What is the length of a box if it has a volume of 84 cu. in., and its width is 4 in. and depth 3 in.?

18. How many bushels will a bin 6 ft. by 5 ft. by 3 ft. hold?

19. How many bushels will a bin 8 ft. by $3\frac{1}{2}$ ft. by 2 ft. hold?

20. How many bushels of wheat can be put in a granary whose dimensions are 25 ft. by 20 ft. by 12 ft.?

21. How deep would a box 14 in. by 15 in. have to be to hold 1 bu.?

22. How deep would a box 20 in. square have to be to hold $3\frac{1}{2}$ bu.?

23. What part of a bushel would a box 8 in. by $10\frac{1}{2}$ in. by $12\frac{1}{2}$ in. hold?

Name all the articles you can that are bought and sold by the bushel.

118. Liquid Measure

Liquid measure is used for measuring practically all kinds of liquids.

The standard unit for liquid measure is the *gallon*, which contains 231 cu. in.

Name articles that are bought and sold by the pint, quart, or gallon.

Table

4 gills (gi.) = 1 pt.
2 pt. = 1 qt.
4 qt. = 1 gal.
1 gal. = 4 qt. = 8 pt. = 32 gi.

Sometimes larger units of measure than the gallon are used.

$31\frac{1}{2}$ gallons = 1 barrel (bbl.)
2 bbl. or 63 gal. = 1 hogshead (hhd.)

1. What part of a gallon is 2 gi.? 2 pt.? 3 qt.?

2. Reduce 5 gal. 3 qt. 1 pt. 3 gi. to gills.

3. Reduce 2 qt. 1 gi. to gills.

4. Reduce 10 gal. 1 qt. to pints.

5. Reduce 5 gal. 1 gi. to gills.

6. Reduce 65 gi. to higher denominations.

7. Reduce 95 pt. to higher denominations.

8. Reduce 18 qt. to higher denominations.

9. If there are 231 cu. in. in a gallon, how many cubic inches in a liquid quart?

10. If there are 32 qt. in a bushel, how many cubic inches in a dry quart?

11. How much larger is a dry quart than a liquid quart?

12. There are 8 dry gallons in a bushel. How many cubic inches in a dry gallon?

NOTE: While the measures of pints and quarts are used in both dry measure and liquid measure, they are not the same in capacity. The dry units of measure are larger than the liquid units of measure having the same name.

Dry gallon = 268.8 cu. in.
Liquid gallon = 231 cu. in.
Dry quart = 67.2 cu. in.
Liquid quart = $57\frac{3}{4}$ cu. in.

13. How many gallons in a tank that contains 9240 cu. in.?

14. How many gallons in a barrel that contains 9752 cu. in.?

15. How many gallons in a swimming pool 100 ft. by 50 ft. by $4\frac{1}{2}$ ft. in depth?

NOTE: Count $7\frac{1}{2}$ gal. to 1 cu. ft., as this is nearly exact.

16. There are 728436 cu. ft. in a reservoir. How many barrels of water will it contain?

17. How many gallons of gasoline will a tank hold that has a capacity of 4389 cu. in.?

18. An oil well produced 200 bbl. of oil a day. How much will it produce in 18 days at that rate?

19. How many quart bottles will it take to hold $7\frac{1}{2}$ gal. of milk?

20. How many barrels of water will a cistern having a capacity of 754 cu. ft. hold?

21. How many quart cans can be filled from 200 gal. of blackberries, if the berries lose 35% of their volume in cooking?

22. What is the cost of 4 bu. 3 pk. of potatoes at $1.40 a bushel?

23. A dairyman sold 20 gal. 3 qt. of milk at 16¢ a quart. How much did he receive?

24. A tank holding 60 gal. of gasoline lost by leakage 25% of its contents. How many gallons were left in the tank?

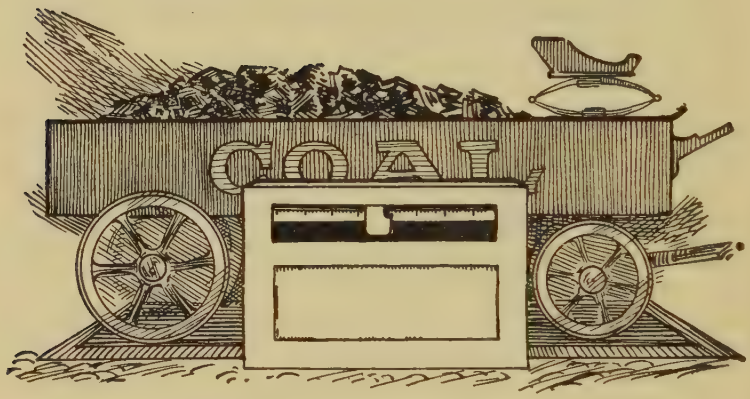
25. Find the difference in volume between 8 dry gallons (a bushel) and 8 liquid gallons.

26. If a merchant buys chestnuts by the dry quart and sells them by the liquid quart, how many cubic inches does he gain. What part of a dry quart is the gain? of a liquid quart?

27. How many cubic inches does a gallon of berries contain? a gallon of milk? a gallon of onion sets? a gallon of vinegar?

28. Which is heavier, a gallon of milk or a gallon of cream? Give a reason for your answer.

119. MEASURES OF WEIGHT



Avoirdupois weight is used in weighing such articles as coal, cattle, hogs, groceries, grain, etc.

The standard unit of avoirdupois weight is the *pound*, which contains 7000 grains.

Table

16 ounces (oz.) = 1 pound (lb.)
100 lb. = 1 hundred weight (cwt.)
2000 lb. = 1 Ton (T.)
1 oz. av. = $437\frac{1}{2}$ grains.
1 T. = 20 cwt. or 2000 lb. = 32000 oz.

The *long ton*, which is seldom used now in business practice, contains 2240 lb.

120. Troy Weight

Troy weight, which is used in weighing gold, silver, and jewels, is too limited in its use to demand extensive treatment. The following table is given for reference:

24 grains	= 1 penny-weight (pwt.)
20 pwt.	= 1 ounce (oz.)
12 oz.	= 1 lb.
1 lb. = 12 oz. = 240 pwt. = 5760 grains.	
1 oz. troy = 480 gr.	

121. Apothecaries' Weight

The **apothecaries' weight,** which is used only in filling medical prescriptions, is of practical use to druggists only.

The table is given here for reference:

20 grains	= 1 scruple (sc. or ℈)
3	= 1 dram (dr. or ʒ)
8 ʒ	= 1 ounce (oz. or ℥)
12 ounces	= 1 lb. or ℔.

These problems involve avoirdupois weight.

1. What part of 1 lb. is 4 oz.? 8 oz.? 12 oz.?
2. What will 5 lb. 8 oz. of steak cost at 28¢ a pound?
3. What is the value of 3 T. 1500 lb. of hay at $24 a ton?
4. Reduce 2 T. 700 lb. 10 oz. to ounces.
5. Reduce 69 lb. 15 oz. to ounces.
6. Reduce 5 cwt. 79 lb. to pounds.
7. Reduce 19 cwt. 11 oz. to ounces.

8. Reduce 25 T. 1800 lb. to pounds.

9. Reduce 40000 oz. to higher denominations.

10. Reduce 7580 lb. to higher denominations.

11. Reduce 80000 cwt. to tons.

12. Reduce 90000 oz. to higher denominations.

13. Reduce .85 lb. to ounces.

$$\begin{array}{r} 16 \\ .85 \\ \hline 80 \\ 128 \\ \hline 13.60 \end{array}$$

Since there are 16 oz. in 1 lb., in .85 lb. there is .85 of 16 oz. = 13.6 oz.

14. Reduce .85 cwt. to lower denominations.

15. Reduce .75 T. to pounds.

16. If .18 T. of hay cost $3.78, what will .29 T. cost?

17. What will 375 lb. of pork cost at $18 per cwt.?

18. When sugar is selling at 16¢ a pound, what will be the cost of .25 lb.?

19. When coal is selling at $6.50 a ton, what will be the cost of 1600 lb.?

20. There are 80 lb. in a bushel of coal. How many bushels in a car containing 30 T.?

21. Which is heavier, a pound of feathers or a pound of gold? How much?

122. Capacity and Weight Equivalents

Weights are now used in business transactions very largely instead of *dry measure*. On this account, states have fixed by law or custom the number of pounds that shall be accepted for the measured bushel. Most states accept the following standards of weight per bushel:

barley, 48 lb.	buckwheat, 52 lb.
corn shelled, 56 lb.	oats, 32 lb.
potatoes, 60 lb.	rye, 56 lb.
clover seed, 60 lb.	timothy seed, 45 lb.
wheat, 60 lb.	corn in ear, 70 lb.

Exercise

1. What is the value of 2400 lb. of wheat at $1.25 a bushel?

2. How many bushels in a ton of oats?

3. What part of a bushel of corn is 14 lb.?

4. What is 42 lb. of corn worth at $1.20 a bushel?

5. What is 40 lb. of clover seed worth at $12 a bushel?

6. How many bushels in 2 T. of corn?

7. A farmer carried 3200 lb. of potatoes to market, which he sold for $1.50 a bushel. How much did he receive for them?

8. Coyle Black grew 4144 lb. of shelled corn on an acre of ground. How much was the corn worth at $1.15 a bushel?

9. Newton Calthrop grew enough corn when shelled to fill a bin with a capacity of 191387.38 cu. in. What was his corn worth at $1.05 a bushel?

10. What is 16 lb. of barley worth at the rate of $1.50 a bushel?

11. What are 50 lb. of potatoes worth at $1.80 a bushel?

12. What are 190 lb. of timothy seed worth at $7.50 a bushel?

13. A farmer exchanged 3000 lb. of wheat for 3360 lb. of rye. If wheat is worth $1.25 a bushel and rye is worth 95¢ a bushel, did he gain or lose by the transaction? How much?

14. A farmer had 80 bu. of wheat in a box; 25% of it was destroyed by weevil. How many bushels had he left?

15. Ellen went to market and bought 6 lb. of potatoes at the rate of 60¢ a peck. How much did they cost?

16. Jenkins sold 200 bu. potatoes at 3¢ a pound. How much did he receive for them?

123. MEASURES OF TIME

Table

60 seconds (sec.)	= 1 minute (min.)
60 min.	= 1 hour (hr.)
24 hr.	= 1 day (da.)
7 da.	= 1 week (wk.)
365 da.	= 1 common year (yr.)
366 da.	= 1 leap year
100 yr.	= 1 century

The standard unit of time is the *day*. The day is divided into 24 equal parts, each of which is called an hour. The day extends from midnight to midnight. For convenience in counting time, the day is divided into two equal divisions. The first division of 12 hr. extends from midnight to 12 o'clock (noon). It is 12 o'clock (noon) when the sun is directly over the meridian of a place. Time from midnight to 12 o'clock (noon) is designated as A.M., (ante meridian), meaning before noon. Time from noon to midnight is called P.M., (post meridian), which means after noon.

The solar year contains 365 da. 5 hr. 48 min. 46 sec. There are 365 days in a common year, and 366 days in a leap year. The difference in length between the common year and the solar year is 5 hr. 48 min. 46 sec. This difference makes almost a day's time in 4 years. Every year represented by a number that is divisible by 4 is a leap year, as 1896, 1916, 1920, except the year with which a century ends, in which case it must be divisible by 400, as 1600, 2000.

Name the leap years in the following: 1828; 1918; 1904; 1899; 1900; 2200; 2400;

The year is divided into 12 months.

The following lines will help you remember the number of days in each.

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone—
To it we twenty-eight assign,
Till leap year gives it twenty-nine."

Oral Exercise

1. How many days in 7 wk.? In 50 wk.?

2. What part of a day is 6 hr.? 8 hr.? 12 hr.? 18 hr.? 21 hr.? How many hours in 2 da.? 4 da.? 5 da.? 10 da.? 20 da.?

3. How many minutes in 2 hr.? 3 hr.? 5 hr.? 6 hr.? 10 hr.? 12 hr.?

4. What part of an hour is 5 min.? 10 min.? 15 min.? 30 min.? 45 min.?

5. How many seconds in 4 min.? 7 min.? 8 min.? 11 min.? 15 min.?

6. What part of a minute is 2 sec.? 3 sec.? 4 sec.? 6 sec.? 12 sec.? 40 sec.?

7. What part of the month of June is 3 da.? 5 da.? 6 da.? 10 da.? 12 da.? 15 da.?

8. What part of a week is 3 da.? 5 da.? 4 da.? 7 da.?

9. What part of a century is 5 yr.? 10 yr.? 20 yr.? 25 yr.? 50 yr.? 75 yr.? 90 yr.?

10. What part of the month of February, 1919, is 2 da.? 4 da.? 7 da.? 14 da.? 21 da.?

Written Exercise

1. Reduce 4 wk. 6 da. 15 hr. to hours.

2. Reduce 5 hr. 48 min. 46 sec. to seconds.

3. Reduce 10 da. 20 min. to seconds.

4. Reduce 6 yr. 300 da. 14 hr. to hours.

5. Reduce 6 da. 7 hr. 20 min. to seconds.

6. Reduce 9 da. 8 hr. to minutes.

7. Reduce to higher denominations 63361 sec.

8. Reduce to higher denominations 84371 min.

124. BOARD MEASURE

The unit of measure for lumber is the *board foot*, which is the amount of lumber contained in a board 1 foot square and 1 inch or less thick. How many times is a board foot contained in a board 1 ft. wide, $2\frac{1}{2}$ ft. long and 1 in. thick?

In board measure, anything less than one inch in thickness is counted as a full inch.

Thickness of more than 1 inch is counted at actual thickness.

A board $\frac{1}{2}$ in. thick, 12 in. wide, and 15 ft. long contains 15 ft. board measure.

1. How many board feet in a board 20 ft. long, 6 in. wide, and 1 in. thick?

Solution: 6 in. $=\frac{1}{2}$ ft. $20\times\frac{1}{2}\times1\times1$ board foot $=10$ board feet.

From the above, it will be seen that the number of board feet in a piece of lumber is found by *multiplying the length expressed in feet by the width in feet and this product by the thickness expressed in inches.*

2. How many board feet are there in a board 14 in. wide, 12 ft. long, and 2 in. thick?

Solution: 14 in. $=1\frac{4}{12}$ ft. $\frac{\cancel{14}}{\cancel{12}}\times\frac{\cancel{12}}{1}\times\frac{2}{1}\times1$ bd. ft. $=28$ bd. ft.

Find the number of board feet in a board:

3. 10 in. wide, 12 ft. long, $1\frac{2}{3}$ in. thick.

4. 9 in. wide, 16 ft. long, $\frac{7}{8}$ in. thick.

5. 7 in. wide, 18 ft. long, 1 in. thick.

6. 16 in. wide, 12 ft. long, $2\frac{1}{2}$ in. thick.

7. 15 in. wide, 10 ft. long, 1 in. thick.

8. 8 in. wide, 15 ft. long, $1\frac{1}{2}$ in. thick.

9. 9 in. wide, 12 ft. long, $\frac{1}{2}$ in. thick.

10. 11 in. wide, 12 ft. long, $1\frac{1}{4}$ in. thick.

11. 6 in. wide, 16 ft. long, $2\frac{1}{4}$ in. thick.

12. 5 in. wide, 15 ft. long, 1 in. thick.

13. 6 in. wide, 12 ft. long, 3 in. thick.

14. 4 in. wide, 16 ft. long, 3 in. thick.

15. How much lumber 1 inch thick and 6 inches wide will it take to build a fence 4 boards high and 80 ft. in length?

16. How many feet of lumber in a piece of timber 16 ft. long, 12 in. wide, and 8 in. thick?

17. How many feet of lumber 1 in. thick, 6 in. wide, and 16 ft. long will be required to build a fence 4 boards high around a rectangular field 32 rd. by 48 rd.?

18. How many feet of lumber in a piece of timber 18 in. $\times$ 18 in. $\times$ 16 ft.?

19. How many feet of lumber will it take to lay a floor in a room 16 ft. by 18 ft., allowing $\frac{1}{10}$ for wastage?

125. FOREIGN MONEY

English Money

When one goes to market in England, instead of paying for articles purchased with dollars and cents, he uses money expressed in pounds, shillings, and pence.

Table

4 farthings (far.) = 1 penny (d.)	£1 = $4.8665
12 d. = 1 shilling (s.)	1 s. = $.24332
20 s. = 1 pound (£)	1 d. = $.02027+.
£1 = 20 S. = 240 d.	

The symbol for pound, like the symbol for dollar, is written before the number; thus, £5.

Exercises, Oral and Written

1. How many shillings in £5? £7½? £25?
2. How many pence in 5s.? 6½ s.? 12½ s.?
3. How many farthings in 6¼ d.? 12½ d.? 15d.? 20d.?
4. Reduce £4 15s. 5d. to pence.
5. Reduce £30 19s. to shillings.
6. Change 2475 d. to higher denominations.

English money does not mean anything to us unless we know its value in terms of our own money.

The English farthing, like our mill, is not coined, but is written as the fraction of a penny.

7. Elizabeth bought 7 yd. of cloth at 1 s. per yd. Find the cost in U. S. money.

8. Mr. Smith bought a farm of 80 A. at £45 an acre. Find the cost of the farm in U. S. money.

9. Ellen bought 8 pencils at 2 d. each. Find the cost in our money.

10. A tourist wanted to convert $970 into English £'s. If the £ is worth $4.85, how many £'s did he receive?

126. French Money

The *franc* is the standard unit of French money.

100 centimes = 1 franc (fr.) 1 franc = $.193.

1. A soldier paid 4 francs for a dinner. How much was that in our money?

2. An American soldier paid 40 francs for a pair of shoes. How much did the shoes cost in U. S. money?

3. A lady bought a dress in Paris for 200 francs. How much U. S. money did it cost?

4. How many francs in 500 centimes? In 250 centimes?

5. If the French war debt is 100,000,000,000 francs, how many dollars is equivalent to the amount of her debt?

6. If the French soldier was paid 2 francs a day, how much was his pay in U. S. money for 30 da.?

7. A tourist wanted to exchange $300 for francs. How many francs should he receive?

8. If a horse sold in France for 700 francs, for how much did he sell in U. S. money?

127. MEASURES OF CIRCLES AND ANGLES

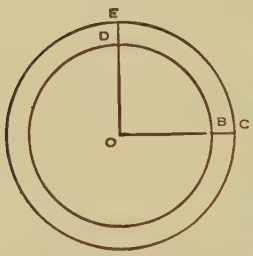
Table

60 seconds (″) = 1 minute (′)

60′ = 1 degree(°)

360° = 1 circumference

1° = 60′ = 3600″



Any part of the circumference of a circle is called an arc. What part of the circumference of the large circle is the arc CE? If there are 360° in the circumference of the circle, how many degrees are there in the arc CE? The small circle within the larger has the same center as the larger circle. What part of the circumference of the small circle is the arc BD? The angle COE is measured by the arc CE. The arc CE = 90°. Hence, the angle COE = 90°. In the same way, it may be shown that the angle BOD = 90°. Every circle, whether large or small, contains 360°. The size of the angle is determined by the difference in direction of two lines meeting at a point, and not by the length of the lines forming the sides of the angle.

An angle of 90° is a *right angle.*

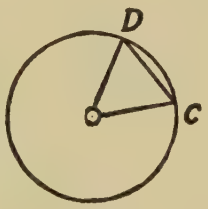
An angle less than 90° is an *acute angle.*

An angle greater than 90° is an *obtuse angle.*

What kind of an angle is each of the following:

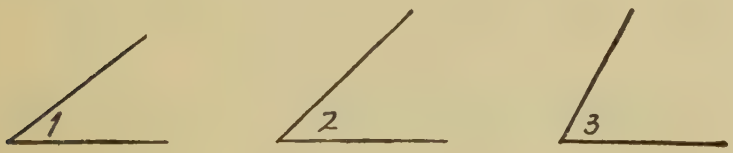
To measure the angle COD, describe a circle, thus:

The arc CD measures the angle O. For example, if you can lay off the arc CD on the circumference 6 times, the angle is $\frac{1}{6}$ of 360° = 60°. The angle is always such a part of 360° as the arc is part of the circumference.



NOTE: The line CD connecting the ends of the arc is called a **chord.**

Estimate in degrees the size of the following angles:



How would you test your estimate by measurement?

1. How many degrees in $\frac{1}{2}$ of a circumference? $\frac{3}{4}$? $\frac{3}{8}$? $\frac{7}{12}$? $\frac{1}{5}$? $\frac{1}{3}$? $\frac{5}{6}$? $\frac{11}{12}$?

2. How many degrees in .9 of a circumference? in .15?

3. What part of a circumference is 30°? 45°? 60°? 90°? 135°? 180°?

4. What kind of an angle is an angle of 65°? 95°? 40°? 145°? $37\frac{1}{2}°$ 10°? 100°?

5. Reduce 20° 15″ to seconds.

6. Change 3961″ to higher denominations.

7. What part of a circumference is 30′? 45′? 48′?

8. Change 40′ 59″ to seconds.

9. If the circumference of the earth is 25000 miles, how many miles in 1° of the circumference?

10. You have learned in your geography that it is 90° from the equator to the poles. What is the distance in miles?

11. Chicago is 4° farther north than San Francisco. How many miles farther north is Chicago than San Francisco?

12. The northern boundary of North Dakota is 15° farther north than the southern boundary of Kentucky. How many miles farther north is the northern boundary of North Dakota than the southern boundary of Kentucky?

13. Memphis is 5° due north of New Orleans. Find the distance between the two cities.

14. It is nearly 29° from the Canadian line to the most southern point of Texas. Find the distance in miles.

15. Reduce .87° to lower denominations.

16. How many right angles in 360°?

17. The sum of two angles is 90° If one angle contains 60°, how many degrees in the other angle?

18. What is the size of the angle made by the hands of a clock at 3 o'clock? at 4 o'clock? at 10 o'clock?

19. If a man should travel 10,000 miles along the equator, how many equatorial degrees would he cover?

20. If he should travel 20,000 miles along the equator, how many equatorial degrees would he cover?

128. ADDITION AND SUBTRACTION OF DENOMINATE NUMBERS

The fundamental operations of denominate numbers are performed just as with other numbers. Whenever the sum of the units of any denomination equals or exceeds one unit of the next higher, the number is reduced to the next higher denomination.

Written Exercise

1. Add:

	bu.	pk.	qt.
	6	3	7
	5	2	5
	9	1	3
	21	3	7

The sum of the qt. column ÷ 8 qt. = 1 pk. 7 qt. Place 7 qt. under qt. column and add 1 pk. to pk. column. The sum of the pk. column = 7 pk. = 1 bu. 3 pk. Write 3 pk. under pk. column and add 1 bu. to bu. column.

The sum is 21 bu. 3 pk. 7 qt.

2. Subtract 20 rd. 5 ft. 3 in. from 35 rd. 1 ft. 7 in.

rd.	ft.	in.
35	1	7
20	5	3
14	12	4
		6
14	12	10

7 in. − 3 in. = 4 in.

Write 4 under inches. You cannot subtract 5 ft. from 1 ft. Take 1 rd. from 35 rd. and change it to feet. 1 rd. = $16\frac{1}{2}$ ft. $16\frac{1}{2} + 1 = 17\frac{1}{2}$. $17\frac{1}{2} - 5 = 12\frac{1}{2}$ Write 12 under feet and change $\frac{1}{2}$ ft. to in. $\frac{1}{2}$ ft. = 6 inches, which must be added to inches. 20 rods from 34 rods leave 14 rods. The result is 14 rd. 12 ft. 10 in.

Find the difference between:

3.

bu.	pk.	qt.
20	2	7
12	3	5

4.

hr.	min.	sec.
22	35	49
17	36	42

Add:

5.

mi.	rd.	ft.	in.
80	110	5	11
90	85	10	9
67	25	15	6
85	125	00	1

6.

T.	lb.	oz.
19	1700	15
29	1400	11
35	10	10
27	5	8

7. From 2 mi. 116 rd. 5 ft. 7 in. subtract 1 mi. 96 rd. 6 ft. 9 in.

8. From 366 da. subtract 365 da. 5 hr. 48 min. 46 sec.

9. From 2 T. 500 lb. 9 oz. subtract 1 T. 700 lb. 12 oz.

10. From 27 da. 9 hr. 15 min. 21 sec. subtract 15 da. 7 hr. 16 min. 22 sec.

11. From 31° 6′ 24″ subtract 15° 9′ 19″.

12. From 31 gal. 2 qt. 1 pt. subtract 25 gal. 3 qt. 1 pt.

Find the sum of:

13. 7 A. 10 sq. rd. 9 sq. yd.; 15 A. 140 sq. rd. 25 sq. yd.; 17 A. 95 sq. rd. 8 sq. yd.; 20 A. 99 sq. rd. 19 sq. yd. 7 sq. ft.

14. 29 gal. 3 qt. 1 pt. 2 gi.; 17 gal. 1 qt. 1 pt. 3 gi.; 15 gal. 1 qt.; 25 gal. 1 gi.

15. 17 T. 1275 lb. 10 oz.; 15 T. 1000 lb. 12 oz.; 23 T. 875 lb. 6 oz.; 12 T. 195 lb.; 15 T. 13 oz.

16. 29 sq. yd. 6 sq. ft. 133 sq. in.; 11 sq. yd. 106 sq. in.; 6 sq. ft. 27 sq. in.; 5 sq. yd. 2 sq. ft. 117 sq. in.

17. 25 cu. yd. 17 cu. ft.; 16 cu. yd. 907 cu. in.; 49 cu. yd.; 24 cu. yd.; 31 cu. yd. 11 cu. ft., 875 cu. in.

18. 5 wk. 4 da. 15 hr. 49 min.; 23 wk. 5 da. 16 min.; 8 wk. 7 hr. 40 min.; 3 wk. 7 da. 18 hr. 50 min.

19. From 117 cu. yd. 17 cu. ft. 1428 cu. in. subtract 84 cu. yd. 21 cu. ft. 1360 cu. in.

129. FINDING TIME BETWEEN TWO DATES

Name the months of the year and give the number of days in each. What is the 1st month? 5th month? 9th month? How do you determine whether a given year is a leap year? How many days has the month of February in a leap year?

To find the difference in time between two dates, consider the number of the year as so many years, the number of the month as so many months, the number of the day as so many days, and subtract.

1. Robert E. Lee was born Jan. 19, 1807; Abraham Lincoln was born Feb. 12, 1809. Find the difference in time between dates of their birth.

yr.	mo.	da.
1809	2	12
1807	1	19
2	0	23

19 da. cannot be subtracted from 12 da. Take 1 mo. from 2 mo. and add it to number of da. 1 mo. =30 da. 30 da.+12=42 da. 42 da.−19 da.= 23 da. 1 mo.−1 mo. =0.

The difference in time=2 yr. 23 da.

NOTE: In finding the time between dates, 30 da. is counted as 1 mo.

Find the time from:

2. July 5, 1917 to Jan. 1, 1920.

3. May 10, 1867 to Mar. 13, 1920.

4. Sep. 25, 1916 to Nov. 20, 1918.

5. Apr. 29, 1896 to June 17, 1907.

6. Aug. 1, 1917 to Jan. 10, 1920.

7. A man gave his note Feb. 25, 1916. He paid the note Jan. 15, 1917. How long did the note run?

8. U. S. Grant was born April 27, 1822. He was inaugurated president Mar. 4, 1869. How old was he at the date of his inauguration?

9. Find from your history the dates of birth and death of the following well-known Americans, and find the exact age of each at death: George Washington, Thomas Jefferson, Alexander Hamilton, Daniel Webster, and Henry Clay.

10. Find the exact number of days from Jan. 10 to Mar. 15, 1920.

130. Multiplication and Division

1. Multiply 2 sq. rd. 9 sq. yd. 6 sq. ft. 132 sq. in. by 6.

sq. rd.	sq. yd.	sq. ft.	sq. in.
2	9	6	132
			6
13	$27\frac{3}{4}$	5	72
		6	108
13	28	3	36

6×132 sq. in. $=792$ sq. in. $=5$ sq. ft 72 sq. in. Write 72 sq. in. and carry 5 sq. ft. 6×6 sq. ft. $=36$ sq. ft. 36 sq. ft. $+5$ sq. ft. $=41$ sq. ft. $=4$ sq. yd. 5 sq. ft. Write 5 sq. ft. and carry 4 sq. yd. 6×9 sq. yd. $=54$ sq. yd. 54 sq. yd. $+4$ sq. yd. $=58$ sq. yd. $=1$ sq. rd. $27\frac{3}{4}$ sq. yd. Write $27\frac{3}{4}$ sq. yd. and carry 1 sq. rd. 6×2 sq. rd. $=12$ sq. rd. 12 sq. rd. $+1$ sq. rd. $=13$ sq. rd. The fraction $\frac{3}{4}$ sq. yd. may be reduced to whole numbers of lower denominations. $\frac{3}{4}$ sq. yd. $=\frac{3}{4}$ of 9 sq. ft. $=6\frac{3}{4}$ sq. ft. $\frac{3}{4}$ sq. ft. $=\frac{3}{4}\times144$ sq. in. $=108$ sq. in. Therefore, $\frac{3}{4}$ sq. yd. $=6$ sq. ft. 108 sq. in. Adding this result, we have for the amount, 13 sq. rd. 28 sq. yd. 3 sq. ft. 36 sq. in.

Multiply:

2. 6 mi. 180 rd. 9 ft. by 8.
3. 14 A. 119 sq. rd. 25 sq. yd. by 9.
4. 29 bu. 7 qt. 1 pt. by 7.
5. 5 T. 320 lb. 15 oz. by 6.
6. 24 da. 19 hr. 41 sec. by 10.
7. 21 gal. 3 qt. 1 pt. by 12.
8. 27 cu. yd. 17 cu. ft. 1200 cu. in. by 7.
9. 6 wk. 6 da. 19 hr. 45 min. by 5.
10. $65°\ 19'\ 40''$ by 12.
11. £5 12 s. 9 d. by 8.
12. $31\frac{1}{2}$ gal. 1 pt. by 9.
13. 10 bu. 7 qt. 1 pt. by 9.
14. 2 mi. 120 rd. 10 ft. by 9.
15. Divide 39 rd. 10 ft. 9 in. by 9.

	rd.	ft.	in.
9)	39	10	9
	4	6	$8\frac{1}{3}$

39 rd. $\div9=4$ rd. with 3 rd. for remainder, which must be reduced to ft. 1 rd. $=16\frac{1}{2}$ ft., 3 rd. $=49\frac{1}{2}$ ft. $49\frac{1}{2}$ ft. $+10$ ft. $=59\frac{1}{2}$ ft. $59\frac{1}{2}$ ft. $\div9=6$ ft. with a remainder of $5\frac{1}{2}$ ft. 1 ft. $=12$ in.; $5\frac{1}{2}$ ft. $=5\frac{1}{2}\times12$ in. $=66$ in. 66 in. $+9$ in. $=75$ in. 75 in. $\div9=8\frac{1}{3}$ in.

Divide:

16. 32 bu. 3 pk. 1 pt. by 7.

17. 20 T. 1600 lb. 11 oz. by 8.

18. 20 mi. 185 rd. 8 ft. by 9.

19. 17 wk. 5 da. 14 hr. 40 min. by 6.

20. 48 lb. 19 oz. by 13.

21. 61 gal. 1 pt. by 7.

22. 84° 6′ 56″ by 8.

23. £19 10 s. 8 d. by 8.

24. 22 bu. 7 qt. 1 pt. by 9.

25. How many times is 231 cu. in. contained in 1 cu. ft.? 582 cu. in.?

131. Division of One Compound Number by Another

1. How many vessels, each holding 1 gal. 1 qt., will it take to hold 6 gal. 1 qt. of milk?

1 gal. 1 qt. = 5 qt.

6 gal. 1 qt. = 25 qt. 25 qt. ÷ 5 qt. = 5, number of vessels.

To divide one compound number by another, reduce each to the same denomination and divide as in simple numbers

2. How many sacks, each holding 1 bu. 3 pk., will it take to hold 12 bu. 1 pk. of grain?

3. How many boards 2 ft. 3 in. long can be cut from a board 11 ft. 3 in. long?

4. How many vessels holding 6 gal. 1 qt. each can be filled from a vessel containing 50 gal. of gasoline?

5. If a farmer puts 1 bu. 3 pk. of corn in a sack, how many sacks will it take to hold 15 bu. 3 pk.?

6. If a man travels 5 mi. 180 rd. in one hour, how long will it take him to travel 50 mi. 20 rd.?

7. A farmer owned a tract of land containing 361 A. 110 sq. rd. which he divided into smaller tracts, each containing 40 A. 30 sq. rd. How many small tracts were there?

8. How many cans holding 1 qt. 1 pt. each will be required to hold 7 gal. 3 qt. 1 pt. of cherries?

9. How long will it take a farmer to plow 11 A. 40 sq. rd. of land, if he can plow each day 2 A. 40 sq. rd.?

10. James travels 2 ft. 5 in. each step. How many steps will he take in traveling 241 ft. 8 in.?

132.

Expressing One Quantity as a Fraction of Another Quantity

1. 2 ft. 3 in. is what part of a yard?

2 ft. 3 in. = 27 in.
1 yd. = 36 in.
Hence 27 in. = $\frac{27}{36}$ yd. = $\frac{3}{4}$ = .75.

2. What decimal part of a day is 6 hr.?

Reduce:

3. 4224 ft. to the decimal of a mile.

4. 3 pk. 1 qt. to the decimal of a bushel.

5. 1 da. 18 hr. to the decimal of a week.

6. 8 ft. to the decimal of 1 rd.

7. 40 sq. rd. to the decimal of an acre.

8. 18 cu. ft. to the decimal of a cubic yard.

9. 16 ft. 6 in. to the decimal of 33 ft.

10. 80 sq. rd. to the decimal of an acre.

11. 800 lb. to the decimal of a ton.

12. What fractional part of a week is 2 da. 8 hr.?

13. What fractional part of a day is 5 hr. 48 min.?

14. What fractional part of a yard is 9 in.?

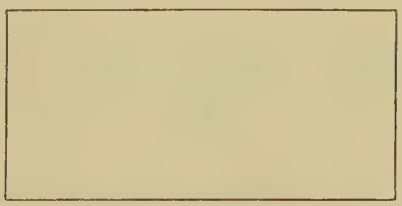
133. DRAWING TO A SCALE

It is many miles across your state. Notice the map of your state and observe its length. A comparison of the length of your map to the actual length of your state will determine the scale. If the length of your map is 6 in. and the length of your state is 360 miles, 1 inch on the map will represent 60 miles of the actual length of your state. We would say the map is drawn on a scale of 60 miles to the inch.

1. Suppose your school room is 24 ft. by 30 ft. Make a drawing of the floor on a scale of 6 ft. to 1 in. What would be the size of your map?

2. Make a drawing on a scale of 200 ft. to 1 in. of a square grass plot, each side of which is 400 ft.

3. Make a drawing on a scale of 50 ft. to 1 in. of a town lot 150′ by 300′.



4. Actually measure with your ruler the dimensions of the drawing of this rectangle. If each inch represents 300 ft., what are the actual dimensions of the rectangle?

5. Draw a map of your state on a scale of 50 mi. to 1 in. Locate its four largest cities, and measure the distance between them.

6. Look at the map of the United States and ascertain by use of the scale the distances between the following places: Charleston, S. C., to New York; New York to Atlanta; Boston to Louisville; Chicago to Memphis; Boston to San Francisco; Wilmington, N. C., to San Antonio; Lake Michigan to Gulf of

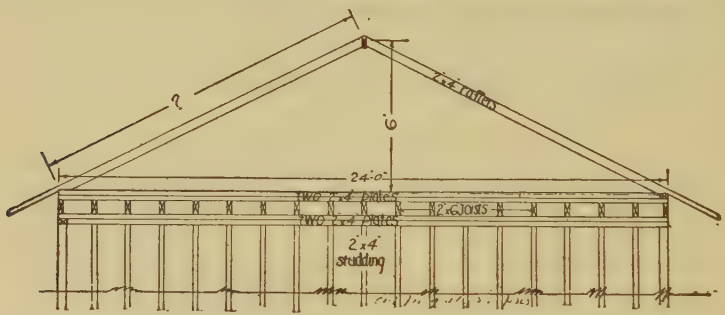
Mexico; Little Rock to Portland; Washington to Birmingham; Minneapolis to Key West; Richmond to Dallas.

7. Draw a map of your school ground on a scale of 1 in. to 20 ft., and locate your school building on your map.

8. Draw a map of your base ball diamond on a scale of 30 ft. to 1 in.

9. If the map of a certain state in the shape of a rectangle is 2 in. by 3 in., what is the area of the state if the map is drawn on a scale of 100 miles to 1 in?

10. What is the length of a line drawn to the scale of 150 mi. to 1 in., if it correctly represents 7500 mi. in distance?



11. Measure with your ruler the width of the roof (24′0″); the height of the ridge pole above the upper plates (6′).

12. From the results of your measurements, find the scale for the drawing. Hint: $24 \div 3$; $6 \div \frac{3}{4}$.

13. Measure the line indicating the slope of the roof (?) and find its length by use of the scale.

14. How much does the roof project below the plates?

15. The height of the roof divided by its width is the pitch of the roof; find the pitch of the roof, expressing it as a fraction in its lowest terms.

134. COMPUTING TIME

1. How far does the hour hand of a clock move while the minute hand moves around the face of the clock?

2. How many times as fast does the minute hand move as the hour hand?

3. What are the positions of the hour and minute hands at 3 o'clock? At 3:30 o'clock?

4. How many times does the second hand of a clock move around in 1 minute? in 5 minutes? in 60 minutes?

5. The second hand moves how many times as fast as the minute hand? as the hour hand?

6. A clock that strikes hours will strike how many times in a day?

7. If a watch loses 20 seconds an hour, how much time will it lose in a week?

8. In problem 7, when it is 9 A.M. by the watch at the close of the week, what is the actual time?

9. If a watch gains 15 seconds a day, and is set right June 1, how much ahead of time is it July 1?

10. Mr. Brown left the station at 11:50 A.M. and reached his destination in 3 hr. 40 min. What time was it by his watch when he reached his destination?

11. At the rate of 15 miles an hour, how long will it take to make a journey of 48 mi.? (Give answer in hours and minutes.)

12. James begins to study his lessons at 7:15 P. M. If he studies 2 hr. and 10 min. and then retires, at what time does he retire?

13. James started on a visit to his uncle's at 8:30. It

took him 5 hr. and 35 min. to make the trip. At what time did he reach his uncle's?

14. If a train runs a mile in $1\frac{5}{8}$ min., what is its rate of speed per hour?

15. Name all the leap years between 1891 and 1931.

16. Two watches were set together at 12 o'clock on Tuesday; one of them gained 10 sec. an hour and the other lost 15 sec. an hour. How far apart were the watches at 12 o'clock on the following Tuesday?

17. A tourist left one city in his car at 10 A.M. and reached the next city in 3 hr. and 40 min.

(a) If the cities were 66 mi. apart, at what rate did he travel per hour?

(b) What time was it when he reached the next city?

18. A farmer went to work at 15 min. before 6 A.M. and worked 5 hr. and 50 min. before he stopped to go to his dinner. At what time did he stop for dinner?

19. A man looked at his watch at 6:30 A.M. What time was it by his watch 10 hr. and 15 min. later?

20. Henry traveled at the rate of 24 mi. an hour. His speedometer registered 2175 mi. when he started on his trip; when he returned it registered 2235 mi. How long did he ride?

21. If a man can plant 8 A. of corn in 10 hr., how much can he plant in 6 hr.?

22. If James was 11 yr. and 3 mo. old June 1, 1919, how old will he be in 1 yr. and 4 mo. from that date?

23. If a father's age is 48 yr. and 7 mo., how old is his son, who is 25 yr. and 6 mo. younger than his father?

24. If Laura is 10 yr. 1 mo. and 19 da. old, and Emma is 2 yr. 3 mo. and 10 da. older, how old is Emma?

25. How many minutes in a week?

26. A train running 35 mi. an hour was 48 min. in going from one town to the next. If the train made 2 stops enroute of $1\frac{1}{2}$ min. each, how far apart were the towns?

135. AVERAGES

If a person were to inquire about the amount of goods a merchant sold, he would not consider a statement of sales on one particular day to be an accurate statement of the business carried on, as it might be far above or below the usual daily transactions, and therefore misleading. But if the total amount of sales were given for a definite period, say 5 mo. or 5 yr., and that total amount of sales divided by the number of months or years the merchant had been in business, the result would be more reliable, as the days of large sales and small sales would be averaged.

The average of a series of quantities is their sum divided by the number of the quantities.

1. A farmer had 5 oxen whose weights each were as follows: 925 lb., 1150 lb., 1400 lb., 825 lb., 1600 lb. Find the average.

925 lb.
1150
1400
825
1600
5)5900 lb.
1180 average weight.

Find the sum of the weight of the oxen and divide by the number of them.

2. A poultryman had 300 hens that laid an average of 10 eggs a month. How many dozen eggs did he receive in 5 mo.?

$300 \times 10 \times 5 = 15000$ and $15000 \div 12 = 1250$, no. of doz.

3. If flour sold at \$6.50 a barrel in 1912, \$9.50 in 1916, \$11.25 in 1917, and \$12.50 in 1920, what was the average cost of a barrel during that period?

4. The six children in a family are aged respectively 24 yr., 20 yr., 17 yr., 14 yr., 11 yr., and 9 yr. Find the average age.

5. It rained during a month on 6 different days as follows: .25 in., .375 in., 1.3 in., 1.132 in., .116 in., and .03 in. Find the average fall of rain for each day.

136. REVIEW

What is the standard unit for board measure?

Wood or timber that is cut or sawed into building material is called **lumber,** which is measured by the board foot.

Lumber is usually bought and sold for so much per thousand (M) feet.

1. How many board feet in a board 10 ft. long, 6 in. wide, and 1 in. thick?

2. How many board feet in a board 15 ft. long, 1 ft. wide, and $\frac{7}{8}$ in. thick? (For thickness less than 1 in. count 1 in.)

3. How many board feet in a board 18 ft. long, 12 in. wide, and $1\frac{1}{2}$ in. thick? (For thickness more than 1 in. count exact thickness.)

4. How many board feet in 15 boards 14 ft. long, 7 in. wide, and 1.2 in. thick?

NOTE: In answers to your problems, count to the nearest board foot.

5. How many board feet in 16 sticks of timber 16 ft. long, 4 in. wide, and 2 in. thick?

6. How many board feet in 40 joists 18 ft. long, 6 in. wide, and 2 in. thick?

7. Find the cost of 60 pieces of timber 16 ft. long, 5 in. wide, and 2 in. thick, at \$0.075 per board foot.

8. At \$90 a thousand, find the cost of 40 boards $16' \times 9'' \times \frac{3}{4}''$.

$40 \times 16 \times \frac{9}{12} \times 1 = 480$ board feet.
$\$90 \div 1000 = \$.09$, the cost of 1 board foot.
$480 \times \$.09 = \$43.20$.
Or, cost $= 40 \times 16 \times \frac{9}{12} \times \frac{1}{1000} \times \$90 = \$43.20$.

9. What is the cost of 60 boards $16' \times 1' \times 1\frac{1}{2}''$ at \$85 per M.?

NOTE: Remember that all boards having less than 1 in. in thickness are counted as 1 in. and that all thicknesses over 1 in. are counted at actual thickness.

10. (a) How many board feet will it take to lay a floor in a room 16′ x 18′ with boards 1 in. thick, if you allow 20% of the actual number of feet for waste in cutting and fitting the boards?

(b) What will it cost to lay the floor if lumber is worth $90 per M. and the labor costs $25?

Hint: Think of one board the length and width of the room.

11. How many board feet will be required to build a fence 300 ft. in length, with boards 16 ft. long, 6 in. wide, and 1 in. thick, the fence to be 5 boards high?

12. What is the cost of 60 joists 16′ x 3″ x 4″ at $65 per M.?

NOTE: When lumber is spoken of, the term feet is often used for board feet.

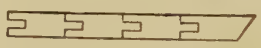
13. What is the cost of 835 ft. of lumber at $56 per M.?

14. How many board feet will be required to build a wall 18 ft. long and 10 ft. high with boards 18′ x 5″ x $\frac{7}{8}$″, allowing 1 in. for overlapping as shown in picture?

15. Find the number of feet of lumber required to build a box 6′ x 5′ x 4′, adding $\frac{1}{10}$ for waste.

Hint: How many board feet in the bottom of the box? in the sides? in the ends?

16. How many board feet will be required for the walls and ceiling of a room 15′ long x 16′ wide x 9′ high, deducting $\frac{1}{5}$ for doors and windows?



17. How many feet of lumber will be required to floor a room 24′ x 28′, adding $\frac{1}{4}$ for loss made by tongue and groove?

18. If a board is 3 in. wide and $\frac{1}{2}$ in. is lost by tongue and groove, what part of width of the board is the tongue and groove?

Hint: If 1 in. was taken up by tongue and groove, what part would be lost?

19. If a board is 4 in. wide and $\frac{3}{4}$ in. is lost by tongue and groove, what part of the width of the board is the tongue and groove?

20. (a) Find the number of board feet in a hardwood floor of a library 18′ by 24′, using boards 2 in. wide and adding $\frac{1}{4}$ for loss of tongue and groove.

(b) What would be the cost of the floor if the lumber cost $125 per M. and the labor and nails cost $50?

21. How many feet board measure in a piece of wood 16′ x 4″ x 4″?

137. MEASUREMENTS

The size of the unit of measure used depends upon the size of the quantity measured.

Name the unit of measure for:

1. Distance from New York to New Orleans.
2. Distance across your schoolroom.
3. The length of your tablet.
4. The area of your state.
5. The area of your farm.
6. The area of your schoolroom floor.
7. The area of a leaf in your tablet.
8. The weight of a diamond.
9. The weight of an ox.
10. The weight of a car of coal.
11. The size of a bottle of milk.
12. The contents of a barrel.
13. The size of a small solid.
14. The size of a cellar.
15. The time since the discovery of America.
16. The time since Christmas.
17. The time since last Sunday.
18. The time you recite.
19. The amount of the Victory Loan.
20. The cost of an orange.
21. A pile of wood.
22. The height of a horse.
23. The depth of the ocean.
24. A small pile of potatoes.
25. A wagon load of potatoes.

In practice where absolute exactness is not demanded, approximate measures are often used.

Suppose a farmer has his wagon box (or bed) filled with corn in the ear and there are no scales convenient to weigh it; he can obtain the approximate amount (a near approach to the exact amount) by taking the approximate number of cubic feet in a bushel .

1 ton coal = 42 cu. ft.
1 ton hay, well packed = 450 cu. ft.
1 bu. shelled corn = $1\frac{1}{4}$ cu. ft.
1 cu. ft. of water weighs $62\frac{1}{2}$ lb.
1 gal. of water = $8\frac{1}{3}$ lb.
1 cu. ft. of any liquid = $7\frac{1}{2}$ gal.
1 bu. apples, coal, potatoes, turnips, corn on ear, etc. = $1\frac{5}{9}$ cu. ft.

1. How many bushels of potatoes in a wagon box 12′4″ long, 3′6″ wide, and 20″ deep?

Solution: 12′4″ = 148″
3′6″ = 42″

$$\frac{148\times42\times20}{1728}\div\frac{14}{9}=\frac{\overset{37}{\cancel{148}}\times\overset{1}{\cancel{42}}\times\overset{5}{\cancel{20}}}{\underset{432}{\underset{108}{\underset{36}{\underset{4}{\cancel{1728}}}}}}\times\frac{\overset{1}{\cancel{9}}}{\underset{1}{\cancel{14}}}=\frac{185}{4}=46\tfrac{1}{4}\text{ bu.}$$

2. How many bushels of turnips will a box 6′ x 5′ x 4′ 6″ hold?

3. How many apples in a bin $3\frac{2}{3}$′ x 6′ x 5′4″?

4. How many bushels of shelled grain in a box 6′ x 3′6″ x 4′?

5. How many bushels of potatoes can be shipped in a car 36′ x 8′6″ x 8′?

6. How many gallons of water in a cistern 5 ft. in diameter and 5 ft. in depth?

7. How many tons of well packed hay are in a barn 50′ x 30′ x 12′?

138. TELEGRAPHIC MESSAGES

If one wishes to communicate to a person at a distance in the quickest time possible, he will use either the telephone or the telegraph.

The cost of sending messages varies with the distance they are sent.

Charges are usually based on a flat rate of so much for a message of not more than ten words and an additional charge of so much for each additional word. Charges to foreign countries are usually fixed at so much per word.

The words in the body of the telegram only are counted in determining the number of words; no charges are made for the name and address of the parties.

Montgomery, Ala., April 1, 1920.

John Doe & Co., Chicago, Ill.
Will ship tomorrow two hundred bales at price quoted.

HERBERT MASON.

Who sent the telegram?

To whom was it sent?

How many words in the telegram?

1. It costs 70¢ to send a telegram from Chattanooga to New York at the regular rate of 10 words. How much will it cost to send a telegram of 15 words if 10¢ a word is charged for each excess word?

2. What will it cost to send a telegram of 13 words from New Orleans to Toronto at 90¢, and 15¢ for each excess word, and 10¢ exchange?

3. Write a telegram, and find the cost of sending the message from your telegraph office to a party in San Francisco, at \$1.30, and 20¢ for each excess word.

NOTE: The message rate is based on 10 words as the maximum.

4. What will it cost to send a message of 15 words from Atlanta to Paris if the message from Atlanta to New York is 70¢ with 15¢ for each excess word, and the message from New York to Paris costs 31¢ for each word?

5. What will it cost to send a message of 15 words from Nashville to London, if the message from Nashville to New York is 70¢ and 5¢ for each excess word, and the message from New York to London costs 31¢ for each word?

6. What is the cost of sending a message of 9 words from Shreveport to Santiago, Chili, at 56¢ for each word?

7. What is the cost of sending a message of 19 words from Jackson, Miss. to Havana, Cuba, at 15¢ for each word?

8. What is the cost of sending a message of 15 words from Charleston, S. C., to Tokyo, Japan, at \$1.37 a word?

9. Find the cost of sending a message of 17 words from Louisville, Ky., to Buenos Aires, at 56¢ for each word.

10. Write a telegram to a person in London.

11. How much would it cost to talk over long distance telephone 5 minutes, if the rate for 3 min. is \$1.50, and 25¢ is added for each excess minute?

12. If the rate over long distance between two distant cities is \$2.25 for 3 min. and 25¢ additional for each excess minute, how much will it cost to talk 18 min.?

13. If it costs \$2.25 to talk in New York 3 min. to a person in San Francisco, and 30¢ for each additional minute, find the cost of talking 7 min.

139. POSTAL SERVICE

The handling of mail in the United States is an enterprise that probably affects more people than any other public service carried on by the government. To defray the cost for the transmission of mail, every one who utilizes the service pays for same by purchasing postage or by paying the government a fixed rate of so much per pound when mailing matter has been received at the post office.

Matter accepted in the mail is classified into the first, second, third, or fourth class.

The first class includes all matter wholly or partly in writing, such as letters and post cards. The cost of mailing a post card is 1¢ and letter postage is charged at 2¢ an ounce or fractional part of an ounce.

Second class matter includes newspapers and periodicals and when mailed by others than the publisher or a news agent the rate is 1 cent for 4 ounces.

Third class matter embraces circulars, newspapers and periodicals not admitted to the second class, nor embraced in the term book. The rate of postage on unsealed third class matter is 1 cent for each two ounces or fraction thereof, and the limit of the weight is 4 pounds.

Fourth class matter embraces that known as domestic parcel post mail and includes merchandise, farm and factory products, etc.

Limits upon packages to be sent parcel post are as follows:

(1) For local delivery or for delivery within the first **three zones** (see Table of Rates below), the package must not exceed 70 lb. in weight.

(2) For all other zones the weight limit is 50 lb.

(3) No package is accepted which has a greater total length and girth than 84 inches.

Rates of postage on parcel post packages are as follows:

(a) Parcels weighing 4 ounces or less, except books, seeds,

plants, etc., 1 cent for each ounce or fraction thereof, any distance.

(b) Parcels weighing 8 ounces or less containing books, seeds, cuttings, etc., 1 cent for each 2 ounces or fraction thereof, regardless of distance.

(c) Parcels weighing more than 8 ounces, containing books, seeds, plants, etc., and all other parcels of fourth class matter weighing over 4 ounces are chargeable according to distance or zone at the pound rates shown in the following table:

Fourth-Class (Domestic Parcel Post) Zone Rates

Weight in Pounds	1st		ZONES						
	Local	to 50 miles	2nd to 150 miles	3rd to 300 miles	4th to 600 miles	5th to 1000 miles	6th to 1400 miles	7th to 1800 miles	8th Over 1800 miles
1	$0.05	$0.05	$0.05	$0.06	$0.07	$0.08	$0.09	$0.11	$0.12
2	.06	.06	.06	.08	.11	.14	.17	.21	.24
3	.06	.07	.07	.10	.15	.20	.25	.31	.36
4	.07	.08	.08	.12	.19	.26	.33	.41	.48
5	.07	.09	.09	.14	.23	.32	.41	.51	.60
6	.08	.10	.10	.16	.27	.38	.49	.61	.72
7	.08	.11	.11	.18	.31	.44	.57	.71	.84
8	.09	.12	.12	.20	.35	.50	.65	.81	.96
9	.09	.13	.13	.22	.39	.56	.73	.91	1.08
10	.10	.14	.14	.24	.43	.62	.81	1.01	1.20
11	.10	.15	.15	.26	.47	.68	.89	1.11	1.32
12	.11	.16	.16	.28	.51	.74	.97	1.21	1.44
13	.11	.17	.17	.30	.55	.80	1.05	1.31	1.56
14	.12	.18	.18	.32	.59	.86	1.13	1.41	1.68
15	.12	.19	.19	.34	.63	.92	1.21	1.51	1.80
16	.13	.20	.20	.36	.67	.98	1.29	1.61	1.92
17	.13	.21	.21	.38	.71	1.04	1.37	1.71	2.04
18	.14	.22	.22	.40	.75	1.10	1.45	1.81	2.16
19	.14	.23	.23	.42	.79	1.16	1.53	1.91	2.28
20	.15	.24	.24	.44	.83	1.22	1.61	2.01	2.40

1. What is the cost of sending a 19 lb. package by parcel post from your office to a town located in the 4th zone?

2. What is the cost of sending a 15 lb. package from New Orleans to St. Louis?

NOTE: Look in your geography and ascertain the distance.

3. What is the cost of sending a package weighing 17 oz. as first class mail matter?

4. What is the difference in cost between sending 6 oz. as first class mail and as second class mail matter?

5. What is the cost of sending $5\frac{1}{4}$ lb. of candy from Atlanta to Charleston?

6. What is the cost of sending $7\frac{1}{2}$ lb. of pork sausage from your home to the third zone?

7. How many miles is it across the 4th zone along a diameter?

8. Find the cost of sending a package from your home to St. Louis.

9. What is the cost of sending 18 lb. to the 6th zone?

10. Name all the large cities in the local zone in which you live.

11. How much more will it cost to send to the 8th zone than to the 5th zone a 12-lb. package of books?

12. How much postage will John have to pay on 4 letters, 3 of which weigh less than 1 oz. each, and the other one $2\frac{1}{2}$ oz.?

13. John also mails a package of magazines which weighs 8 oz. Find the cost of postage.

14. It costs 10¢ in addition to the postage to register a package. Mary registers a package which weighs 6 oz. How much postage does she pay?

15. What will the postage be to the 7th zone on a package containing photographs and weighing 15 oz.?

16. Would a package of seeds weighing $6\frac{1}{2}$ lb. be sent by ordinary mail or parcel post? What would it cost to send it 350 mi.?

17. How would a turkey weighing 12 lb. be sent? How much would the charge be for 150 mi.?

18. Find the cost of sending a 3 lb. book by parcel post, 930 mi.

19. Mary received a 3-lb. box of nuts and candy from home. The parcel post charge was 15¢. Can you tell the distance exactly? Within what limits was the distance?

20. Find the cost of mailing proof sheets which weigh 4 lb. 3 oz. (1¢ for 2 oz.)

140. STEPS IN PROBLEM SOLVING

While much of your work in arithmetic consists of making computations in addition, subtraction, multiplication, and division with whole numbers, common and decimal fractions, and denominate numbers, a very important part of your work in arithmetic, both in school and out, will consist of problem solving. The intelligent solution of a problem involves four steps:

I. A very careful reading of the problem.

II. The selecting of the proper principles to use, and deciding how and in what order to use them.

III. The actual computation.

IV. Checking and interpreting results.

Illustration:

John earned $12.60 by working a week of 6 days. At that rate, how much will he earn if he works 10 da.?

I. Read carefully, and answer:

(1) How many days did John work?

(2) How much did he receive for the work?

(3) What does the phrase, *at that rate*, mean?

(4) What are you required to find in order to complete the solution?

II. Think of the principles necessary to use:

(1) How many steps are there to be taken?

(2) What is the first step?

(3) What principle, or operation, is required for the first step?

(4) What is the second step?

(5) What principle, or operation, is required for the second step?

III. Make your computation:

(1) Can you divide $12.60 by 6 without your pencil?

(2) Can you multiply $2.10 by 10 without your pencil?

NOTE: Always do as much of the computation as possible, without the use

of your pencil. You will find after a little practice that you will make fewer errors if you make the computation without your pencil.

IV. Checking:

(1) About how many times as much should John get for 10 days' work as for 6 days' work. Should he get three times as much? Twice as much? Make your estimate carefully.

(2) If the result of your computation is about what your estimate led you to expect, then check by going over your work again hurriedly, or if you are not absolutely certain, check by inverse operations, that is, multiplication by division and division by multiplication.

(3) When you interpret the results, note that you first divide \$12.60 into 6 equal parts, giving \$2.10; and that in multiplying, you are taking \$2.10 ten times, thus giving the result in dollars. It is important that you consider the divisor in the first step and the multiplier in the second step as abstract numbers, and that you interpret the results as dollars.

To acquire skill in solving problems, you should be able to take these four steps rapidly and perform the computation accurately. In the illustration given above, the complete process would be about as follows:

I. Read: *John earned \$12.60 by working a week of 6 days. At that rate, how much will he earn if he works 10 days?*

II. Selecting principles: Two steps: (1) division, (2) multiplication.

III. Computation: (1) $\$12.60 \div 6 = \$2.10$.

(2) $\$2.10 \times 10 = \$21$.

Or, $$\frac{\$12.60 \times 10}{6} = \$21.$$

IV. Check: 10 days is almost twice as many as 6 days, hence he would earn *at the same rate* almost twice \$12.60; or, checking by inverse operations, (1) $\$21 \div 10 = \$2.10$, (2) $\$2.10 \times 6 = \$12.60$. Interpret each result as labeled in III.

141. PRACTICAL PROBLEMS

Solve orally the problems numbered with odd numbers. Solve with pencil those numbered with even numbers.

1. James has 25¢ and $\frac{4}{5}$ of his money is $\frac{1}{2}$ of John's. How much money has John?

2. A steamship is valued at $75,000. $\frac{2}{1\,5}$ of the value of the ship is $\frac{2}{5}$ of the value of the cargo. What is the value of the cargo?

3. John earned $1.75 a day and spent each day $1.25 of his earnings. How much did he have at the end of 6 days?

4. A carpenter earned $7.75 a day and spent $5.65 each day. How much money had he at the end of 26 da.?

5. A boy misspelled 2 words out of 10. At that rate, out of 50 words, how many would he miss?

6. A stock trader made $54 on every 6 horses he bought. How much would he make on 72 horses?

7. Luther gave Henry $\frac{1}{3}$ of his marbles and William $\frac{1}{4}$. He had 10 marbles left. How many had he before giving any away?

8. A farmer gave his son $\frac{1}{3}$ of his land, his daughter $\frac{1}{5}$. He had 287 A. left. How many acres had he before giving any land away?

9. Thomas and Frank bought a base ball for 90¢. If Thomas paid 10¢ more than Frank, how much did each pay?

10. Divide $3500 between two men so that one will have $500 more than the other.

11. $\frac{3}{7}$ of \$21 is $\frac{3}{11}$ of how many dollars?

12. Mr. Dale had \$4200 in a bank. $\frac{5}{7}$ of his money is $\frac{4}{15}$ of Mr. Dodge's. How much money has Mr. Dodge?

13. After adding 1 to the product of 3 by a certain number, the result was 7. What was the number?

14. After adding 45 to the product of 25 by a certain number, the result was 745. What was the number?

15. If $\frac{2}{5}$ of a pole is under water and 12 ft. is above water, how long is the pole?

16. A man spent $\frac{1}{4}$ of his income for house rent, $\frac{1}{8}$ for groceries, $\frac{1}{4}$ for clothing, and had \$600 left. What was his income?

17. A little girl lost $\frac{1}{4}$ of her money and had 15¢ left. How much money had she before losing any?

18. A man lost $\frac{2}{5}$ of his stock of goods by fire, and had left \$5850 worth of goods. What was his stock worth before losing any?

19. The Superintendent of Schools receives \$3600 a year. The high school principal receives $\frac{2}{3}$ as much as the Superintendent, and the grade teacher $\frac{1}{2}$ as much as the high school principal. How much does the grade teacher receive?

20. In a certain city the scholastic population is 13500, and 9500 of the population are in school. What part of the population is in school?

21. Add $\frac{2}{7}$ of 14 to $\frac{3}{5}$ of 25.

22. If a train travels at the rate of $33\frac{1}{2}$ mi. an hour, how far will it travel in 1 hr. and 20 min.?

23. If $\frac{5}{7}$ of \$42 is $\frac{5}{9}$ of my money, how much money have I?

24. A farmer had 42 A. of land. He sold $\frac{1}{3}$ of it at \$250 an acre, $\frac{3}{7}$ of it at \$200 an acre, and the remainder of it for \$160 an acre. How much did he receive for his land?

142. REDUCTION TO THE UNIT

1. If 12 yd. of cloth cost \$36, what should be the cost of 9 yd.?

We have given the cost of 12 yd. to find the cost of 9 yd.

If 12 yd. cost \$36, 1 yd. will cost $\frac{1}{12}$ of \$36=\$3.

If 1 yd. costs \$3, the cost of 9 yd. is $9\times\$3=\$27$.

It will be seen in this problem that we first go from the cost of a number of units to the cost of one unit, and then from the cost of one unit to the required number of units.

2. If a boy earns \$18 in 6 da., how many dollars should he earn in 11 da.?

3. How many trucks would be needed to carry 60 T. of coal if 5 trucks carry 25 T.?

4. If 95 sacks of flour weigh 2280 lb., what should 147 sacks weigh?

5. If I give \$336 for 56 T. of coal, how much should I give for 72 T.?

6. What will be the cost of 108 bu. of wheat if 66 bu. cost \$138.60?

7. If $\frac{7}{8}$ of a yard of cloth cost 49¢, what will $1\frac{3}{4}$ of a yard cost?

We have given the cost of a fraction of a yard to find the cost of another fraction of a yard.

If $\frac{7}{8}$ of a yd. costs 49¢, $\frac{1}{8}$ of a yard will cost $\frac{1}{7}$ of 49¢=7¢. If $\frac{1}{8}$ of a yard costs 7¢, $\frac{8}{8}$ of a yard or one yard will cost $8\times7¢=56¢$. If a yard costs 56¢, then $1\frac{3}{4}$ of a yard will cost $1\frac{3}{4}$ of 56¢=52¢.

8. If Ellen paid 18¢ for $\frac{3}{4}$ of a yard of ribbon, what should she pay for $\frac{5}{6}$ of a yard?

9. If a tourist runs his car 6 mi. in $\frac{1}{4}$ of an hour, how far should he run it in $1\frac{1}{2}$ of an hour?

10. If $\frac{3}{7}$ of an acre produced 210 lb. of lint cotton, how much should $\frac{9}{10}$ of an acre produce?

11. If $1\frac{5}{6}$ of a number is 45, what is $1\frac{1}{2}$ of the number?

12. Ralph grew 24 bu. of wheat on $\frac{8}{9}$ of an acre; how much should he grow on $\frac{2}{3}$ of an acre?

13. If $\frac{1}{1}\frac{5}{7}$ of a number is 60, what is $\frac{3}{4}$ of it?

14. Minnie gave 40¢ for $\frac{1}{3}$ doz. oranges; how much should $3\frac{1}{2}$ doz. cost?

15. If $\frac{3}{4}$ of a number is 72, what is $\frac{7}{8}$ of it?

16. Find the cost of 209 copies of a book of which 178 copies cost $156.64.

17. If $\frac{2}{3}$ of a number is 44, what is $\frac{4}{1}\frac{}{1}$ of it?

18. If 25 men can do a piece of work in 6 da., in how many days could 15 men do it?

We have given the number of men it will take to do a piece of work in a given time to find how long it will take a different number of men to do the same work.

25×6 da. $= 150$ da. $\quad 150$ da. $\div 15 = 10$ da.

If 25 men do the work in 6 da., 1 man will do the work in 25×6 da. $= 150$ da. If 1 man can do the work in 150 da., 15 men can do the work in $\frac{1}{1}\frac{}{5}$ of 150 da. $= 10$ da.

19. By traveling 15 mi. an hour, I make a trip in 20 hr.; how long would it take me to make the trip if I traveled 18 mi. an hour?

20. If 35 men can do a piece of work in 30 da., how long will it take 50 men to do the work?

21. If A makes a trip in 12 hr. traveling in an automobile at the rate of 20 mi. an hour, how long would it take to make the trip if he traveled on the railroad train at the rate of $37\frac{1}{2}$ mi. an hour?

22. If a supply of provisions lasts 840 men for 57 da., for how many days would the same supply last 1330 men?

23. A watch loses 20 sec. in 2 hr.; how much time will it lose in a day?

24. A watch loses 1 sec. in 2 hr.; how much time will it lose in 1 wk.?

25. A watch loses 2 hr. 13 min. a week; how much does it lose in 36 hr.?

26. A motor car goes 90 mi. an hour.

(a) How far does it go in a minute?

(b) How many feet does it go in a second?

27. If 125 men can do a piece of work in 120 da., how many men could do it in 100 da.?

28. If a quantity of bread will last 45 men 10 da., how long will it last 150 men?

29. If 5 men can plow a field in 12 da., how long will it take 10 men to plow the field?

30. If \$1.25 will buy 5 lb. of steak, how much can be bought for \$3.00?

31. An army of 25000 men had enough rations to last them 4 da.; if 5000 of the men were transferred to another division, how long would the rations last the men remaining?

32. A son inherited \$3200 which was $\frac{4}{9}$ of the estate. What was the value of the estate?

33. If $\frac{4}{25}$ of my income is \$800, what is my income?

34. If $\frac{5}{13}$ of an estate is worth \$2500, what part of the estate is \$650?

Solution: $\frac{5}{13}$ of estate = \$2500.

$\frac{1}{13}$ of estate = $\frac{1}{5}$ of \$2500 = \$500.

$\frac{13}{13}$ of the estate = $13 \times \$500 = \$6500$.

If the estate is worth \$6500, what part of the estate is \$650?

$\$1 = \frac{1}{6500}$ of the estate.

$\$650 = \frac{650}{6500}$ of the estate $= \frac{1}{10}$ of the estate.

35. If $\frac{4}{9}$ of a farm is 120 A., what part of the farm is 180 A.?

36. I have read 240 pages, which is $\frac{4}{7}$ of the book. What fraction of the book shall I have read when I have read to the end of page 350?

37. A pays $\frac{1}{10}$ of his income for taxes and has \$1800 left. (a) What is his income? (b) What are his taxes?

38. How many yards of silk can be bought for \$12.50 when silk costs \$1.25 a yard?

39. After cutting off $\frac{6}{7}$ of a rope and $\frac{1}{2}$ the remainder, there were left 12 ft.; what was the length of the rope?

143. Problems

1. Anna can make a dress in 2 da.; it requires Mary 3 da. to make the same dress. In what time can they both make it working together?

If Anna can make the dress in 2 days, in 1 day she can make $\frac{1}{2}$ of the dress. If Mary can make the dress in 3 days, in 1 day she can make $\frac{1}{3}$ of the dress. They can both make $\frac{1}{2}+\frac{1}{3}=\frac{3}{6}+\frac{2}{6}=\frac{5}{6}$ of the dress in a day. There are $\frac{6}{6}$ in the whole dress and if they can both make $\frac{5}{6}$ of the dress in 1 day, it will take them as many days as $\frac{5}{6}$ is contained in $\frac{6}{6}$. $\frac{6}{6}\div\frac{5}{6}=1\frac{1}{5}$ days.

2. If Rufus can do a piece of work in 4 days and Henry in 5 da., in what time can both do it working together?

Process: $\frac{1}{4}+\frac{1}{5}=\frac{5}{20}+\frac{4}{20}=\frac{9}{20}$.
$\frac{20}{20}\div\frac{9}{20}=2\frac{2}{9}$. Explain the process.

3. A farmer can plow a field with a tractor in 6 da. and his neighbor can plow it with a team of horses in 10 da. In what time can they both working together plow it?

4. Lucile could sweep her room in 15 min.; Ellen could sweep the same room in 12 min. How long will it take both of them together to sweep the room?

5. James could do a certain piece of work in 5 da., Henry in 6 da., and John in 9 da. In what time could all three do it working together?

6. A could mow a field in 5 da., B in 8 da., and C in 10 da. In what time could they all mow it working together?

7. Edward could do a piece of work in $\frac{1}{5}$ of a day, and Morris in $\frac{1}{6}$ of a day. In what time could they both together do it?

If Edward can do the work in $\frac{1}{5}$ of a day, he can do the work 5 times in 1 day. If Morris can do the work in $\frac{1}{6}$ of a day he can do the work 6 times in 1 day. If Edward can do the work 5 times in 1 day and Morris 6 times in 1 day, they can both do the work 11 times in a day. If they can do the work 11 times in a day they can do the work 1 time in $\frac{1}{11}$ of a day.

8. James could do a piece of work in $\frac{1}{8}$ of a day and William could do the same work in $\frac{1}{9}$ of a day. In what time could they both together do the work?

9. Samuel could do a piece of work in $\frac{3}{5}$ of a day and Richard could do the same work in $\frac{3}{7}$ of a day. In what time could both together do it?

Hint: How many times can each do the work in 1 day?

10. If Bessie can do a piece of work in $\frac{2}{3}$ of an hour, and Sylvia can do the same work in $\frac{3}{4}$ of an hour, in what time can they both together do it?

11. If a man owns $\frac{5}{8}$ of a grocery store and sells $\frac{1}{2}$ of his share for $2500, at that rate what is the whole store worth?

12. There were 36 pupils in the 8th grade of a grammar school; $\frac{4}{9}$ of them were boys. How many girls in the grade?

13. In a recent examination, a boy spelled 24 words out of 25. What part of the words did he miss? What per cent of the words did he miss?

14. James sold his pig for $\frac{2}{5}$ more than it cost him. If he received $14 for the pig, what was its cost?

15. Ellen sold her story book for 60¢, which was $\frac{1}{5}$ less than cost. How much did it cost?

Hint: How many fifths of selling price equal the cost?

16. Ophelia sold her story book for 60¢, which was $\frac{1}{5}$ more than it cost. How much did it cost?

17. I owned $\frac{3}{4}$ of a piece of property and sold $\frac{1}{2}$ of my share for $6,000; at that rate what was the whole property worth?

18. (a) Find $\frac{2}{5}$ of $60. (b) Find 60% of $60.

19. Find $\frac{1}{4}$ of 36 ft. Find 60% of 36 ft.

20. Find $\frac{1}{2}$ of $\frac{5}{6}$ of 60 in.

21. If 12 men can do a piece of work in 6 da., how long will it take 9 men?

22. A man bought 3 dairy cows. The first cost $125, the second cost $\frac{4}{5}$ as much as the first, and the third cost $1\frac{3}{4}$ times as much as the second. What is the cost of the second and third?

144. THRIFT PROBLEMS

1. In a school of 320 children, $\frac{5}{8}$ of them have bought Thrift Stamps to the amount of $1000. What is the average amount of money each child who is saving has invested?

2. William Johnson invested $125 in bonds. The government pays him each year 4% of the amount invested. How much does the government pay him each year?

3. Mr. and Mrs. Riley plan to put in the savings bank $100 each during the year. On July 1 Mr. Riley had deposited $\frac{3}{5}$ of his amount and Mrs. Riley had deposited $55. Which had the more money in the bank? How much more?

4. Eleanor had $21.40 to invest in War Savings Stamps. At $4.28 each, how many stamps could she buy?

5. If 200 out of 250 students at the Hemlock School bought Thrift Stamps, what part of the school invested?

6. William sold his pigs for $270. He put 75% of the money in a savings bank. How much did he put in the bank?

7. A farmer sold his cotton crop for $2500. He paid his debts with 65% of it and invested the remainder in live stock. How much did he spend for live stock?

8. A newsboy earns $3.75 a week. He puts in a savings bank $\frac{1}{3}$ of it. How much at that rate will he save in 1 year?

9. A workman earned $42 a week. If he saves $6.50 a week, how much will he save in 1 year?

10. Mr. Brown by buying his flour in 100 lb. sacks gets it for $7 a hundred. Mr. Smith buys 25 lb. sacks at $2.00 each. If each one of them uses 300 lb. during the year, how much more does Mr. Smith pay for flour than Mr. Brown?

11. Mrs. Parker buys her potatoes in small quantities at 6¢ a lb. Mrs. Thrift buys hers at $1.80 a bushel. How much more does Mrs. Parker pay per bushel than Mrs. Thrift?

145. PERCENTAGE

We have already learned that a common fraction whose denominator is 10 or some power of 10, such as 100, 1000, 10000, etc., is also a decimal fraction and may always be written in the form of a common fraction or in the form of a decimal, as $\frac{6}{100}$, 0.06.

Write each of the following fractions in the form of a decimal: $\frac{9}{100}$, $\frac{29}{100}$, $\frac{75}{100}$, $\frac{125}{100}$. How many places are required for hundredths?

A common fraction with 100 as denominator, or a two-place decimal, is frequently expressed in a third form, called the *per cent* form. That is, another name for hundredths is *per cent.* So that instead of $\frac{6}{100}$ or 0.06, we may write 6 *per cent.* Per cent comes from a Latin phrase and means by the hundred or hundredths.

Instead of the words *per cent*, the symbol % is usually written. This symbol is thought by some to be made up of the one and two naughts required to write 100. The naughts are placed on each side of the 1, and one of them is tied to the top of the 1. When used with a number, it means that the number is the numerator of the fraction whose denominator is 100.

Note the following equivalent forms:

$\frac{8}{100} = 0.08 = 8\%.$

$\frac{1}{5} = \frac{20}{100} = .20 = 20\%.$

$125\% = \frac{125}{100} = 1.25.$

$\frac{1}{2}\% = .00\frac{1}{2}, = .005.$

$\frac{7\frac{1}{2}}{100} = .07\frac{1}{2} = .075 = 7\frac{1}{2}\%.$

Note that there are the same number of per cent (%) as there are of hundredths. If, then, common fractions or decimals are first expressed as so many hundredths, there will be just as many per cent as there are hundredths.

146. Expressing Common Fractions in Hundredths

I. When the denominator is a divisor or multiple of 100.

Illustration: $\frac{1}{5}=\frac{1\times 20}{5\times 20}=\frac{20}{100}$

Reduce these fractions to hundredths (common fractions):

1. $\frac{1}{4}$, **2.** $\frac{1}{2}$, **3.** $\frac{2}{5}$, **4.** $\frac{7}{10}$, **5.** $\frac{6}{200}$.

By what do you multiply the numerator and denominator of the fraction $\frac{1}{4}$ in order to reduce it to hundredths? Does it change the value of a fraction if you multiply the numerator and the denominator by the same number?

In multiplying $\frac{1}{4}$ by $\frac{25}{25}$, you are really multiplying the fraction by what? Does it change the value of a number if it be multiplied or divided by *one*? By what did you divide the numerator and denominator each in changing $\frac{6}{200}$ to hundredths? What form of *one* did you use in reducing each of the other fractions to hundredths?

Multiplying or dividing both terms of a fraction by the same number is called reduction, and does not change the value of the fraction.

II. When the denominator is not a divisor or multiple of 100.

Illustration: $\frac{1}{6}=6)\overline{1.00}$ (.16⅔)

Reduce these fractions to hundredths (decimals):

1. $\frac{1}{8}$, **2.** $\frac{7}{6}$, **3.** $\frac{4}{7}$, **4.** $\frac{8}{9}$, **5.** $\frac{2}{11}$.

How do you change a common fraction to a decimal? The line separating the numerator and denominator may be thought of as a sign of division between the numerator (dividend) and the denominator (divisor).

Which of the following fractions would you change to hundredths by reduction? Which would you change to hundredths by dividing the numerator by the denominator?

1. $\frac{4}{5}$, **2.** $\frac{7}{9}$, **3.** $\frac{1}{6}$, **4.** $\frac{3}{15}$, **5.** $\frac{759}{1000}$.

How do you determine which is the better way to change a common fraction to hundredths?

147. Oral Exercise

Change to common fractions in their simplest forms:

1.	5 %	**6.**	25 %
2.	10 %	**7.**	$33\frac{1}{3}$%
3.	$12\frac{1}{2}$%	**8.**	50 %
4.	$16\frac{2}{3}$%	**9.**	$66\frac{2}{3}$%
5.	20 %	**10.**	75 %

Change to per cent:

1.	$\frac{2}{5}$	**6.**	$\frac{7}{10}$
2.	$\frac{3}{10}$	**7.**	$\frac{1}{2}$
3.	$\frac{3}{8}$	**8.**	$\frac{1}{12}$
4.	$\frac{1}{6}$	**9.**	$\frac{1}{9}$
5.	$\frac{3}{20}$	**10.**	$\frac{3}{4}$

Change to per cent:

1.	$0.33\frac{1}{3}$	**6.**	$0.00\frac{1}{8}$
2.	0.80	**7.**	$0.12\frac{1}{2}$
3.	0.7	**8.**	0.125
4.	0.06	**9.**	0.375
5.	$0.01\frac{1}{2}$	**10.**	0.750

Write the following in two other forms, taking care to see that the per cent symbol (%) is used in one of the three forms:

Illustration: $\frac{2}{5}=\frac{40}{100}=40\%$.

1.	$\frac{1}{8}$	**6.**	$12\frac{1}{2}$%	**11.**	225%	**16.**	$\frac{3}{4}$	**21.**	1750%
2.	25%	**7.**	$\frac{2}{8}$	**12.**	$.00\frac{1}{8}$	**17.**	$\frac{3}{4}$%	**22.**	$\frac{1}{6}$
3.	$\frac{3}{5}$	**8.**	$0.87\frac{1}{2}$	**13.**	$3\frac{3}{4}$	**18.**	$\frac{7}{8}$	**23.**	125%
4.	$8\frac{1}{3}$%	**9.**	0.002	**14.**	$2\frac{1}{4}$	**19.**	1	**24.**	0.0125
5.	0.035	**10.**	$\frac{1}{2}$	**15.**	$\frac{1}{2}$%	**20.**	$\frac{9}{10}$	**25.**	$2\frac{1}{2}$

Memorize this table of equivalents:

1.	$50\% = \frac{1}{2}$	**6.**	$20\% = \frac{1}{5}$
2.	$66\frac{2}{3}\% = \frac{2}{3}$	**7.**	$16\frac{2}{3}\% = \frac{1}{6}$
3.	$33\frac{1}{3}\% = \frac{1}{3}$	**8.**	$12\frac{1}{2}\% = \frac{1}{8}$
4.	$25\% = \frac{1}{4}$	**9.**	$10\% = \frac{1}{10}$
5.	$75\% = \frac{3}{4}$	**10.**	$100\% = 1$

What is an easy way to find $33\frac{1}{3}\%$ of a number?

Answer orally:

1.	20% of 35	**6.**	$8\frac{1}{3}\%$ of 36
2.	$66\frac{2}{3}\%$ of 21	**7.**	$33\frac{1}{3}\%$ of 75
3.	25% of 48	**8.**	$12\frac{1}{2}\%$ of 64
4.	50% of 50	**9.**	10% of 120
5.	75% of 60	**10.**	$16\frac{2}{3}\%$ of 54

James raised 80 bushels of corn. He paid 10% of the crop to his father for the use of the land. How many bushels did his father get?

Solution: 10% of 80 bu. $= \frac{1}{10}$ of 80 bu. $= 8$ bu.

Note in this problem that there are three terms or numbers. 80 bu. represents the whole crop; 10%, the fractional part paid for the use of the land; and 8 bushels, the number of bushels paid for the use of the land. Which two terms are of like kind?

In percentage the number, or term, of which you take some per cent or fractional part is called the *base*. The term expressing the per cent to be taken is called the *rate*.

The term you get by taking the given number of per cent of the base is called the *percentage*. The sum of the base and percentage is called the *amount*.

Point out the *base*, *rate*, and *percentage* in the above problem. The *base* and *percentage* will always be of like kind.

148. Oral Problems

Name the base, rate, and percentage, and solve each of the following problems.

1. Henry bought a book for 75¢ which he sold at a gain of 20%. How much did he gain?

2. John invested $20 in a pig, which he afterwards sold at a gain of 40%. How much did he gain?

3. A tank that will hold 90 gal. is $66\frac{2}{3}\%$ full. How many gallons in the tank?

4. There are 20 business houses in a block. 25% of them are grocery stores. How many grocery stores in the block?

5. Ellen planted 150 tomato plants; $33\frac{1}{3}\%$ of them died. How many plants lived?

6. A hen had 18 little chicks; $16\frac{2}{3}\%$ of them died. How many of them died?

7. James had 15 marbles. He lost 20% of them through a hole in his pocket. How many marbles did he lose?

8. It rained on $33\frac{1}{3}\%$ of the days in November. On how many days did it rain?

9. A new book costs $1.60. A second hand book could be bought for 75% of the cost of a new book. Find the cost of a second hand book.

10. James bought a suit marked $25 at a bargain sale at a reduction of 40%. What did the suit cost James?

11. Mary solved 80% of the problems assigned. If 20 problems were assigned, how many did Mary solve?

12. Sam bought a bicycle marked $60 at a discount of $12\frac{1}{2}\%$. What did he pay for it?

149. To Find Percentage when Base and Rate are Given

1. Find $16\frac{2}{3}\%$ of \$360.00.

Solution: $\frac{1}{6}$ of \$360.00 = \$60.00 (for $16\frac{2}{3}\% = \frac{1}{6}$).

2. Find 41% of \$193.00.

Solution:

\$193
.41 (for 41% = .41)
193
772
\$79.13

If we let b stand for base, r for rate, and p for percentage, we may express the solution of the two problems above by this formula:

I. $\mathbf{p = b \times r}$

NOTE: r should always be written as a common fraction or as a decimal before multiplying or dividing.

Solution of Example 2 by formula:

$p = b \times r$, $b = \$193.00$, $r = 41\%$.

Hence $p = \$193.00 \times .41 = \$79.13$.

Solve the following by formula:

3. 23% of \$840.00
4. $12\frac{1}{2}\%$ of \$480.00
5. 75% of \$636.00
6. 17% of 421
7. 15% of \$950.00
8. 29% of \$70.00
9. 80% of 10
10. 100% of 63.

11. Miss Wheeler's salary is \$125.00 per month. She pays 20% of it for board. How many dollars does she pay for board?

12. A miner who was getting \$1.20 per ton for mining coal was allowed a 25% increase. How much did he then get?

13. How much is saved in buying a suit marked \$36 at a reduction of $33\frac{1}{3}\%$? What does the suit cost?

14. Last year coal cost \$5.60 per ton. It costs at present 10% more. Find the present cost of a ton.

15. Mr. Martin paid \$1650 for his automobile. It decreased 10% in value the first year. What was it then worth?

16. If 40% of a ton of coal is lump, how many pounds of lump coal are there? If the other part is slack, how many pounds of slack coal are there?

17. A merchant finds that his operating expenses are 12% of his sales. If his sales for the year amount to $28,000, what are his operating expenses?

18. If there are 63,700 people in a town and if 21% of them are school children, how many school children are there?

19. A grocery merchant wishes to sell butter at a profit of 30% of the cost. What must he get for butter if he pays 60¢ per pound for it?

20. Mr. Smith paid $9600.00 for a house. He spent $628.50 repairing it. He then sold it at a profit of $12\frac{1}{2}\%$ of the original cost. How much was his net profit?

Suggestion: Net profit is found by taking all the expenses out of the entire profit.

150. To Find Rate when Base and Percentage are Given

NOTE: Answer all the questions you can orally.

1. What part of 10 is 5?
What per cent of 10 is 5?

Solution: Since 5 is $\frac{1}{2}$ of 10, and since $\frac{1}{2} = 50\%$, then 5 is 50% of 10.

2. 30 is what part of 90?
30 is what per cent of 90?

Solution: Since 30 is $\frac{1}{3}$ of 90, and since $\frac{1}{3} = 33\frac{1}{3}\%$, then 30 is $33\frac{1}{3}\%$ of 90.

What per cent of:

3. 20 is 5?
4. 36 is 24?
5. 100 is 20?
6. 180 is 60?
7. 24 is 3?
8. 2 is 1?
9. 25 is $12\frac{1}{2}$?
10. 100 is $16\frac{2}{3}$?
11. 3 is what per cent of 12?
12. 16 is what per cent of 48?
13. 45 is what per cent of 60?
14. 36 is what per cent of 108?

15. 48 is what per cent of 96?

16. 18 is what per cent of 25?

Solution: 18 is $\frac{18}{25}$ of 25. $\frac{18}{25}=\frac{72}{100}$.
Hence, 18 is 72% of 25.

17. What per cent of \$60 is \$17?

Solution: $\frac{17}{60}=17\div60=0.28\frac{1}{3}=28\frac{1}{3}\%$.
Hence, \$17 is $28\frac{1}{3}\%$ of \$60.

In exercise 17, which term is the *base*? the *percentage*? the *rate*? Which two were given? Which one was required? What operation was performed with the two given terms in order to find the third term? If we represent the solution in a formula, we will have: II. $\mathbf{r=p\div b.}$

18. What per cent of a week is one day?

Solution: $r=1\div7=0.14\frac{2}{7}=14\frac{2}{7}\%$.

19. If you spend 5 hr. each day in school, what per cent of the 24 hr. are you in school?

20. What per cent of the entire year is your school in session?

151. Problems

NOTE: Determine which number or term represents the base (b), which the percentage (p), and then solve by formula.

1. A pattern maker allows 1 ft. out of every 8 ft. for shrinkage. What per cent does he allow?

2. The total amount of lumber needed for a job is 1600 sq. ft. Of this amount, 480 sq. ft. must be quarter-sawed oak. What per cent is quarter-sawed oak?

3. What per cent of a gallon is a pint?

4. If a mixture of 25 gallons of alcohol and water has one pint of alcohol in it, what per cent of the entire mixture is alcohol?

5. A cord of wood contains 128 cu. ft. What per cent of a cord is there in a pile of wood 6 ft. by 4 ft. by 4 ft.?

6. Two neighbors buy a ton of coal together. One of them gets 850 lb. of it and the other gets the remainder. What per cent of the coal bill should each pay?

7. A bushel of wheat weighs 60 lb. What per cent of a bushel is 36 lb.?

8. A bale of seed cotton weighs 1800 lb. What per cent of a bale does a man pick in a day if he picks 125 lb.?

9. If a man pays $4200 for a house and sells it at once for $5600, what per cent profit does he make on the cost?

10. A baseball club played 120 games and won 80 of them. What per cent did the club win? What per cent did it lose?

Find two solutions for the second question.

152. To Find the Base when the Percentage and Rate are Given

1. $24 is $\frac{1}{2}$ of my money. How much have I?

Express $\frac{1}{2}$ as per cent. 50% of my money = $24. Then 100%, or all, of my money = $48. Therefore, I have $48.

2. Which of the three terms in the above problem is *rate*? *percentage*? *base*?

3. Since $0.50 \times 48 = 24$, what do you get when you divide 24 by 0.50? This formula shows how to find the base when the *percentage* and *rate* are given: III. $b = p \div r$.

NOTE: Remember that r should be expressed as a common fraction or decimal before dividing.

4. James spends $25, or 20% of his salary, for board. Find his salary.

Solution: (1) $b = p \div r$
$b = \$25 \div \frac{1}{5}$ (since $20\% = \frac{1}{5}$)
$b = \$25 \times \frac{5}{1} = \$125$, Salary.
Or, (2) $b = p \div r$.
$b = \$25 \div .20 = \$125$, Salary.

Find the number if:

5. (a) $12\frac{1}{2}\%$ of it is 8. (b) $37\frac{1}{2}\%$ of it is 24.

6. 14% of it is 56.
7. $3\frac{1}{8}$% of it is 20.
8. 75% of it is 120.
9. 45% of it is 72.
10. 120% of it is 360.

11. John bought a suit at a bargain sale at a discount of 30%, thus saving \$7.50 of the marked price. Find the marked price.

12. Mary bought a book of fifty street-car tickets for 20% less than 50 cash fares cost, and saved 50¢. Find what Mary paid for the book of tickets.

13. William's spelling grade was 75%. If he spelled 15 words correctly, how many words were there in the lesson?

14. The teacher finds that $12\frac{1}{2}$% of the children enrolled are absent today. If six are absent, how many are enrolled? How many are present?

15. Mr. Williams was given a \$15.00 increase on the month, which was 16% of his original salary; what was his original salary? What is his present salary? Can you solve the second question two ways?

153. Problems for Review

1. Mr. Adams has an income of \$2500. He spends 30% of his income for food, 20% for clothing, 16% for rent, 7% for books and general improvement, 5% for fuel and lights, 12% for insurance and surveys, and the remainder for church and charity. Find the amount spent for each purpose.

2. If a 24 yd. bolt of nainsook costs \$10.80 and will cut 8 garments, what will be the cost of each garment? A garment of equal quality costs \$2.25 when bought ready-made. What per cent of the cost of the cloth is saved by making the garment at home?

3. Here is a suggested wardrobe for a high-school girl. Make it out in the form of a bill and find the total cost:

2 voile dresses at $3.65 each; 3 gingham dresses at $2.65 each; 1 serge dress at $8.85; 1 silk dress at $27.50; 2 tailored waists at $2.45 each; 1 blouse at $5.85; 1 blue serge suit at $45.00; 1 silk underskirt at $4.65; 2 cotton skirts, home-made, at $1.75 each; 1 summer hat at $5.30; 1 winter hat at $4.75; 1 wool cap at $1.50; 1 pair of winter shoes at $8.95; 1 pair Oxfords at $7.85; 1 half dozen pairs of lisle hose at 70¢ each; 2 pairs of silk hose at $2.25 each; 6 combination suits at $1.75 each; 1 doz. handkerchiefs at 15¢ each; 1 pair of gloves at $1.85 each; 1 bungalow apron at $1.35; 1 kimona at $1.95; 3 ties at $1.00 each; miscellaneous, $5.00.

4. What per cent of the entire bill is the silk dress? The blue serge suit?

5. What per cent of the entire bill are the two pairs of shoes? The two hats?

6. Here are some prices of foods in a certain town for the years indicated. Copy this table and fill in the blanks, showing increase and per cent of increase over the 1914 prices.

FOOD	Sold by	Price 1914	Price 1920	Increase	% of Increase
Bacon	Pound	$ 0.20	$ 0.35		
Bread	Loaf	.05	.10		
Butter	Pound	.30	.70		
Eggs	Doz.	.30	.65		
Flour	24-lb.	.80	$1.65		
Potatoes	Peck	.40	.75		
Sugar	Pound	.05	.20		

7. Which food has had the largest per cent increase? If a thing doubles in value, what is the increase in per cent?

8. James did not test his seed corn, and only 75% of it germinated. He grew 60 bushels of corn on an acre. At that rate, how much would he have grown had all the seed germinated?

9. Kenneth raised 12 chickens at a cost of \$1.30. He sold them for \$9.00. How much profit did he make? What per cent is the profit of the cost?

10. A measured bushel of wheat weighed 60 lb. in July and 54 lb. in January. What per cent was lost in drying?

Can you solve the following? If not, supply the needed information and solve them:

11. What per cent profit does a man make on selling a house for \$6000?

12. John sold his bicycle for $33\frac{1}{3}\%$ less than what he paid for it. How much did he get for it?

13. If William makes a profit of 25¢ on his newspapers, how much did he pay for the papers?

14. A baseball club won 108 games. What per cent of the total number did the club win?

15. Ruth misspells five words of her spelling lesson. What grade should she have?

Solve these problems as stated. Then change those involving fractions so as to make them problems in percentage and solve them.

16. A merchant sells sugar for 20¢ per pound. He makes a profit of $\frac{1}{10}$ of the selling price. How much does he pay for the sugar?

17. Henry solves 16 problems and is unable to solve $\frac{1}{5}$ of the problems assigned. How many were assigned?

18. A farmer has $\frac{2}{3}$ of his farm planted in cotton and the remainder in corn. What part is in corn? What part of the cotton land is the corn land? What part of the corn land is the cotton land?

19. The vote for two of the presidential candidates in Calhoun County, Mississippi, in 1916 showed: Wilson 1225; Hughes 45. Hughes' vote was what per cent of Wilson's? Wilson's vote was what per cent of Hughes'?

APPLICATIONS OF PERCENTAGE

154. Commission and Brokerage

1. Mr. Adams, desiring to sell his farm, secured the services of Mr. Hunt, who is a real estate dealer. Mr. Hunt sold the farm to Mr. Field for $8500. Would Mr. Adams expect to receive all of the $8500? Mr. Hunt makes a charge of 5% of the selling price, which is called **commission,** and this is taken out of the amount received for the farm. Find Mr. Hunt's commission on the transaction described above. How much does Mr. Adams receive for the farm after paying the commission?

2. Brown & Co. are engaged in the mercantile business. They turn over to Mr. Campbell for collection a number of unpaid bills. The total amount of the bills was $784.65, and Mr. Campbell collects all of this amount except $123.15. He keeps as his commission 10% of the amount collected. How much does Mr. Campbell get for his work? How much does the firm of Brown & Co. get?

3. Mr. Cates grows and sells fruits. He ships a car load of peaches to the firm of I. B. Merriman & Co., Cincinnati. The firm sells the shipment to the merchants of Cincinnati, receiving for the entire carload $840.75. I. B. Merriman & Co. deduct for their services 8% of the amount received, and send the remainder to Mr. Cates. How much do I. B. Merriman & Co. make on the transaction? How much does Mr. Cates get?

4. Mr. Sells wishes to invest in some corn. He wires Mr. Wilson, Atlanta, to buy 2000 bushels of corn for him. Mr. Wilson buys 2000 bushels at $1.12\frac{1}{2}$ per bushel. He makes a charge against Mr. Sells of $\frac{1}{8}¢$ per bushel for his services. How

much does Mr. Wilson make on the deal? How much money will Mr. Sells need to send to Mr. Wilson to cover the cost of the corn and Mr. Wilson's commission?

A person or firm who buys or sells for some one else is called **an agent, a commission merchant, or a broker.**

The per cent of the sales, collections, or purchases, charged for services, is called the rate of commission or the **commission rate.** The money paid for the services is called **commission,** or **brokerage.** The person or firm who authorizes the agent, etc., to transact business for him is called the **principal.** The amount received by the principal after all expenses are paid is called the **net proceeds.**

Fill in the blanks:

	Sales	Rate	Commission	Net Proceeds
5.	$1,050.	5%	------	--------
6.	375.	-----	$30.	--------
7.	1,000.	$\frac{1}{8}$%	------	--------
8.	2,400.	$6\frac{1}{4}$%	------	--------
9.	7,240.	------	724.	--------
10.	16,000.	------	------	$15,960
11.	324.64	$12\frac{1}{2}$%	------	--------
12.	--------	15%	69.	--------
13.	--------	------	50.	9,950
14.	20,000.	$12\frac{1}{2}$%	------	--------

15. A commission merchant sold a shipment of strawberries for $845.60. The commission charge was 10% of the sales, and the drayage in Cincinnati amounted to $10.50. Find the commission. Find the net proceeds due the shipper.

16. If my broker buys 40,000 lb. of cotton for me at 30.25¢ per lb. and charges me $\frac{3}{4}$% commission, how much money will I need to send him?

17. Miss Carroll secured her position as teacher with the aid of a teachers' agency. She is to pay the agency 5% of her first year's salary. If she receives $120 per month for nine months, how much will she owe the agency?

18. A grocer shipped 480 crates of peaches to his commission merchant, who sold them at $2.25 per crate. The commission merchant deducts 15% commission, $36.40 freight, and $12.60 drayage. What was the commission? the net proceeds?

19. A real estate agent bought a farm for Mr. Smith for $12,500. Within the year he sold it for Mr. Smith at an advance of 10%. The agent charged 3% commission on each deal. How much did his commission amount to? How much did Mr. Smith make on the two farm deals?

20. A broker proposes to Mr. Allen to sell his corn crop of 12,000 bushels at a commission of $\frac{1}{8}¢$ per bushel or $\frac{1}{10}\%$ on the sales amount. If the corn market is $1.50 per bushel, which offer is better for Mr. Allen? How much better?

21. A commission merchant bought for John Smith 320 A. of land at $75 an acre. How much was his commission at 5%?

22. A man sent $5025 to a commission merchant to buy cotton. After taking out his commission of 5%, how much did he invest in cotton? What was his commission?

Hint: $5025 includes the cost of the cotton and his commission. A broker gets commission only on money invested. Let 100%=cost of cotton, and 5% of cost=his commission. Then 105% of cost=$5025.

23. A farmer sent an agent $205 with which to buy a horse, after taking out $2\frac{1}{2}\%$ commission. What was the cost of the horse? What was the commission?

155. Discount

Sometimes goods are sold at less than their marked price, or original value, because they are out of season, damaged, or in order to effect quick sales.

Any reduction from an original price is called a **discount.**

The rate per cent taken of the original price in finding the discount is called the **rate of discount.**

After the discount is taken from the original price, the remainder is called the **net price.**

Commercial, or **trade, discounts** are deductions usually taken from retail prices in order to make them conform to wholesale prices.

Find the net prices of the articles marked and discounted as follows:

1. \$30.00 at 15% discount.
2. \$48.60 at $33\frac{1}{3}$% discount.
3. \$63.96 at $12\frac{1}{2}$% discount.
4. \$20.00 at 90% discount.
5. \$50.00 at 50% discount.
6. \$90.00 at $87\frac{1}{2}$% discount.

Fill in the blanks:

	List Price	*Rate of Discount*	*Net Price*
7.	\$840.00	25%	-------
8.	125.00	------	\$100.0
9.	------	20%	80.00
10.	63.00	$33\frac{1}{3}$%	-------
11.	48.00	------	36.00
12.	144.00	------	72.00
13.	17.50	10%	-------
14.	25.00	8%	-------
15.	------	$16\frac{2}{3}$%	15.00
16.	32.00	$12\frac{1}{2}$%	-------

17.	\$ 24.00	------	\$ 18.00
18.	0.90	------	0.60
19.	10.00	4%	--------
20.	16.00	------	14.00
21.	0.75	------	0.60

22. A drummer gets $8\frac{1}{3}\%$ commission on the sales he makes. If his sales amount to \$57,000 a year, what commission does he receive? If his traveling expenses are \$1600.00 for the year, what is his net income per month?

23. An agent bought 1000 bu. of wheat for Mr. Ring at \$1.20 per bushel. What was the total cost per bushel to Mr. Ring if the agent charged a commission of $1\frac{1}{2}\%$?

24. A broker in New York sold some railroad stock for Mr. Boyd, the face value of which was \$8000.00. The broker received \$7200.00 for the stock. His commission was $\frac{1}{8}\%$. Find the amount of his commission. How much did Mr. Boyd receive? (Commission was charged on face value.)

25. Mr. Crouch's broker in Memphis bought 72,000 lb. of cotton at $30\frac{1}{4}¢$ per pound. The commission charge was $\frac{3}{4}\%$, and the drayage and storage amounted to \$110.50. How much should Mr. Crouch send his broker to cover all the cost?

26. My agent bought a house for me for \$4800.00, and sold it at a profit of 10%. If I had to pay the agent 3% commission for each of the two deals, how much profit did I make?

27. A hardware merchant bought a bill of goods listed at \$720, but got a discount of $16\frac{2}{3}\%$. What was the cost to the merchant?

28. Find the cost of a piano listed at \$600, but bought at a discount of $12\frac{1}{2}\%$.

29. In a certain state, ministers are allowed a discount of $33\frac{1}{3}\%$ on all railway tickets. How much will a minister in that state pay for a ticket on a 200-mile trip if the full fare is $3¢$ a mile?

156. Profit and Loss

1. Mr. Horner bought a house for $6000.00 and sold it for $7200.00. What was his profit in dollars? In per cent of the cost? In per cent of the selling price?

2. If Mr. Horner paid out $300.00 for repairs on the house before selling it, what was his net profit in dollars? What per cent of the total cost did he make? of the selling price?

NOTE: The difference between the cost and the selling price is called **gross profit.** Usually such expenses as clerk hire, rent, taxes, insurance, etc., called **overhead charges,** are added to the cost in order to find the **total** or **gross cost.** The difference between the total cost and the selling price is called the **net profit.**

3. A merchant finds that overhead expenses, such as clerk hire, delivery, rent, freight, etc., amount to 5% of the cost of staples, such as coffee, sugar, etc. If this merchant pays 10¢ per lb. for sugar and sells it at 12¢ per lb., what per cent is his net profit of the total cost? What per cent is his net profit of the selling price?

4. If your merchant pays $1.80 per bushel for apples and sells them at the rate of $3.60 per bushel, and if loss by decay, expenses of selling, etc., amount to 20% of the selling price, how much profit is he making in dollars? What per cent is this of the net cost? of the total cost? of the selling price?

NOTE: A smaller per cent gross profit is usually made on staples, such as sugar, coffee, etc., than on vegetables, fruits, and other perishable products. Why?

5. Your merchant bought 48 qt. of strawberries on Saturday at $12\frac{1}{2}$¢ per quart. He sold 44 qt. at 15¢ per quart on Saturday and the remainder spoiled before Monday. What per cent profit on the cost would he have made had he sold all of

them at 15¢ per quart? What per cent is the net profit of the cost?

NOTE: The net profit here is the gross profit less the loss by decay.

6. For how much will the grocer have to sell string beans that cost him \$1.60 per bushel, counting 20¢ per bushel for waste and overhead expenses, to make a profit of 25% on the total cost?

7. In a certain town, the street car company sells 5¢ car tickets in quantities for 4¢ each. What per cent of discount is the company giving? What per cent of saving is the patron making on the cost?

8. In the same town, school children get the 5¢ tickets in quantities at $2\frac{1}{2}$¢ each. What per cent discount is the company giving? What per cent saving is a schoolboy making on the cost?

9. A grain dealer bought 6000 bu. of wheat in the fall at \$1.20 per bushel. When he weighed the wheat in the spring, he found that it had lost 2 lb. to the bushel in drying. At what price per bushel must he sell it to make a profit of 10% of the cost?

10. Mr. Mathes bought a city lot for \$600.00. He paid out \$10.00 for taxes and sold it for \$480.00. What per cent of the selling price did he lose?

11. A man pays \$240 each for two horses. He sells one at a profit of 25% and the other at a loss of 25% of the cost. Did he gain or lose, and how much?

12. A man sells two horses for \$240 each. On one he gains 25% and on the other he loses 25% of the cost. Did he gain or lose, and how much?

13. A man sells two horses for \$240 each. On one he gains 25% and on the other he loses 20% of the cost. Did he gain or lose, and how much?

157. Bills and Receipts

A **bill** shows the names of the parties concerned in the transaction, the date of the sale of each item, the price of each item, and the total amount. When terms of payment other than cash are allowed, the terms are also shown on the bill.

The person or firm selling the goods is called the **creditor,** and the person or firm buying the goods is called the **debtor.**

When the words "Received Payment," or "Paid," together with the date of payment and signature of the creditor, are written or stamped on the bill, it becomes a **receipt.**

Nashville, Tenn., May 1, 1920.

R. J. Brown & Co.
Huntsville, Ala.

Bought of H. G. Hill & Co.

Terms: 5/10, n/30

1920								
Apr.	21	120 bbl. flour	@	$9.25	$1,110			
	23	90 bu. potatoes	@	$1.80	162			
	25	210 lb. butter	@	63¢	132	30		
	28	1 bag sugar, 100 lb.	@	19½¢	19	50		
					$1,423	80		
		Cash 5%			71	19	$1,352	61
		Received payment May 8, 1920 H. G. Hill & Co.						

1. Who sold the goods? Who bought them? Where do these firms do business? 5/10, n/30, means that if the account is paid within 10 days from the date of the bill, May 1, 1920, a

discount of 5% will be allowed; if not paid within 10 days but within 30 days, the full amount of the bill will be charged; and that interest begins after 30 days. How much would have been due had payment been postponed until May 25, 1920?

Make out receipted bills for the following accounts:

2. On June 1, 1920, Crouch Book Co., Johnson City, Tenn., ordered from American Publishing Co., Nashville, Tenn.: 30 doz. ruled tablets @ 8¢ each, 60 doz. sheets graph paper @ $8\frac{1}{3}$¢ per doz., 6 doz. arithmetics @ 45¢ each. Terms: 5/10, n/30. Paid June 9, 1920.

3. On July 10, 1920, the Chattanooga Plow Co., Chattanooga, Tenn., sold to the Bayless Hardware Co., Athens, Ga.: 12 cultivators at $48.50 each, 10 small wagons at $83.50 each, 5 large wagons at $112.60 each. Terms: 10/30, n/60, 6%. Paid Aug. 30, 1920.

4. On March 15, 1920, Harvey Roberts purchased from the John Wanamaker store of New York City, the following articles: 4 men's suits @ $40 each, 6 ladies' hats @ $7.50 each, 2 dozen pairs of hose @ $4 a dozen, 6 rugs @ $25 each, 12 dozen pairs of gloves @ $15 a dozen. Make out the bill sent Harvey Roberts by the John Wanamaker store on April 1, 1920, and give him a discount of 5% for cash payment.

5. Make out a bill to Joe Morse, who owes Henry Lovell for 1 automobile @ $1550, 2 tires @ $57.25 each, 6 inner tubes @ $1.75 each, and a kit of tools @ $27.50. Allow 6% discount for cash payment.

6. Inquire prices of four different articles on sale at some place of business in your community, make out a bill for certain quantities of them as having been sold to one of your classmates, and, acting as clerk for the dealer, receipt the bill as paid within 10 days on the terms n/10. Be sure to fill in the dates, prices, etc., complete.

158. FOOD PROBLEMS

The table below suggests a proper food budget, with the various kinds of foods that should be included and the amount to be spent for each, for one month of 30 days:

SIZE OF FAMILIES

Food	Two		Three		Four		Five		Six		Seven	
	Amt.	% of budget	Amt.	% of budget	Amt.	% of budget	Amt.	% of budget	Amt.	% of budget	Amt.	% of budget
Cereals	$ 7 50		$10 00		$12 00		$14 00		$16 00		$18 00	
Milk	5 00		8 00		11 50		13 00		15 00		18 00	
Meat	8 50		13 00		18 00		23 00		28 00		32 00	
Vegetables	5 00		7 50		9 00		11 00		13 00		15 00	
Fruit	2 50		4 00		6 50		8 00		10 00		12 00	
Fats	4 00		6 00		8 00		10 00		12 00		14 00	
Sugar	1 25		2 00		2 75		3 50		4 25		5 00	
Luxuries	2 25		3 50		4 25		7 50		9 25		12 00	
Food Budget	$36 00		$54 00		$72 00		$90 00		108 00		126 00	

What are cereals? fats? luxuries? Consult the dictionary if you are uncertain.

1. How much is allowed per month per person for food? How much per day for each person?

2. Fill in the blanks, "% of budget," for a family of two, for each food item.

Hint: % of cereals = $7.50 ÷ $36.

3. The cost of food for a family of 3 is what per cent of the cost of food for a family of 7?

4. Fill in the blanks, "% of budget," for a family of 4.

5. (a) The cost of meat for a family of 3 is what per cent of the cost of meat for a family of 6? (b) 3 is what per cent of 6?

6. A truck farmer with a family of 7 produced his own milk, vegetables, and fruit. What per cent of his food budget did he save?

159. VALUE OF GOOD ROADS

1. If it costs $\frac{2}{3}¢$ per bushel per mile to haul corn over an unimproved road and $\frac{3}{10}¢$ per bushel per mile over an improved road, how much more will it cost a farmer to haul his crop of 800 bushels over an unimproved road to a market 12 miles from the farm?

2. A paved road increased land values in a certain community 30%. How much will a farm of 75 acres be worth if it had a value of $60 per acre before the road was improved?

3. A horse can draw a load 75% greater in weight over a paved road than an unimproved road. If he can draw a load of 1750 lbs. over the good road, how much could he have drawn over the poor road?

4. If it requires a pull of 200 lbs. to draw a load of a ton over the poor road and $\frac{1}{8}$ as much to draw the same load over a good road, how many tons could a team capable of exerting a pull of 400 lbs. draw over a poor road? Over a good road?

5. (a) How many cubic yards of crushed stone will be required to build a mile of road 18 feet wide, the crushed stone being spread an average depth of 8 inches? (b) What will be the cost of the road at $2.25 per cubic yard?

6. A bridge over a river cost $720,000 and is to be paid for with tolls which average $150 per day. How many years will it take to pay for the bridge, counting 30 days to the month?

7. For a recent year the Federal government distributed $121,875,000 for highway improvement in the United States and Hawaii. If the average cost of paving roads is $15,000 per mile, how many miles would the total distributed pave?

8. Tennessee's share in the appropriation referred to in Problem 7 was $2,692,800. How many miles of surfaced roads could be built in the state if the average cost is $7,200 per mile?

9. A certain state has 42,000 miles of public roads of which 840 miles are improved. (a) What per cent of the roads are improved? (b) How many miles are unimproved?

10. If the tax on gasoline is 5¢ per gallon for roads, how much tax would a car owner pay who runs his car 12,000 miles per year, getting 15 miles per gallon of gasoline?

11. (a) If a set of tires on a car that is driven over bad roads will run 4,000 miles before replacement is necessary, how many sets of tires will be needed to run a car 12,000 miles? (b) What will be the cost of the tires at $75 a set?

12. If a set of tires on a car that is driven over a good road will run 12,000 miles before replacement is necessary, how much more in cost of tires at $75 per set will it take to furnish a car with tires for 12,000 miles travel over poor roads than over good roads?

13. The owner of a farm located on an improved highway sold it for $18,500. Before the improved highway was built the farm had sold for $14,000. What was the per cent of increase in the sale of the farm?

14. How much would it cost to concrete a road 20 feet wide the distance of 2 miles at 7¢ a square foot?

15. How long would it take a road roller 8 feet wide, moving at the rate of 2 miles an hour, to roll a road 24 feet wide and 6 miles in length?

16. If it costs 15¢ a square yard to build a road bed, how much will it cost to build the bed of a road 18 feet wide and 3 miles in length?

17. If an automobile owner pays $30 a year tax for gasoline, and saves on the upkeep of his car by having good roads $50 per year, his net saving is what per cent of his tax?

160. Review Questions

1. What will you need to know in order to compute the per cent of attendance in your room to-day? Illustrate with numbers, and solve.

2. John misspells 5 words in his spelling lesson. Can you compute his grade? If you need more information, supply it and compute the grade.

3. An agent made $300.00 as commission for his services in selling a house. What did the house sell for? If you need more data, supply them and solve.

4. A man bought an article and then sold it for $1.50 more than he gave for it. Did he make a good deal or a poor one? Illustrate.

5. What is the advantage of paying accounts promptly?

6. A department store advertises goods on sale at "$\frac{1}{3}$ off." What does it mean? How can you find the cost to purchasers of the various items?

7. What is meant by "*receipted bill*?"

8. If Bennie buys newspapers at $2\frac{1}{2}¢$ and sells them at $5¢$ each, what per cent is he earning on the money he invests?

9. If Bennie pays $5¢$ each for his papers and has to sell them at $2\frac{1}{2}¢$ each, how much is he losing on the money he invests?

10. Interpret these formulas:

(a) $p = b \times r$.
(b) $r = p \div b$.
(c) $b = p \div r$.

State and solve a problem under each.

161. SIMPLE INTEREST

1. Mr. Harper borrows 100 bu. of wheat from his neighbor in October. He sows the wheat and from the crop which he harvests he returns in October next, 100 bu. of wheat and 6 bu. in addition. These additional 6 bu. we call rent—grain rent. Had Mr. Harper given his neighbor \$6.00 instead of 6 bushels of wheat, in addition to 100 bu. of wheat, what kind of rent would we call the \$6.00? What would Mr. Harper have had during the year in exchange for the \$6.00?

2. Mr. Lewis borrows \$100.00 from his neighbor in October, 1919. A year later he pays his neighbor \$100.00 and \$6.00 in addition. The transaction is similar to those above, but the kind of property involved is different. What does Mr. Lewis get in exchange for the \$6.00?

Money paid for the use of money is called interest.

The sum of money for which the interest is paid is called the **principal.**

The per cent which the interest for one year is of the principal is called the **rate of interest.**

The **time** involved in a money transaction will determine jointly with the rate the amount of the interest.

The sum of the principal and interest is called the **amount.**

3. Refer to Ex. 2 above. Point out the principal, the rate, the interest, the time, the amount.

4. In the beginning of the topic of Percentage, we met problems of this type: What is 6% of \$100.00?
What name did we give the \$100.00? the 6%? the \$6.00? What operation with \$100.00 and 6% gives the \$6.00?

5. Compare the following with the problem just discussed: What is the interest on \$100.00 at 6% for 2 years? If the interest is \$6.00 for 1 year, how much would it be for 2 years? What is the amount for 2 years?

6. Which element, or term, in the subject of Interest corresponds to **base** in the subject of Percentage? to **rate**? to **percentage**? What additional element enters into the topic of Interest?

Note the following formulas:

(a) $\mathbf{P=B\times R.}$

(b) $\mathbf{I=P\times R\times T.}$

When problems are solved by the use of (b), remember that R should always be expressed decimally or as a common fraction and that T is always expressed in years or fractions of years.

When the **amount** (A) is wanted, it is found by use of the following formula:

$$\mathbf{A=P+I.}$$

1. Find the interest on \$400. at 5% for 3 years.

Solution: $I=P\times R\times T=\$400.\times\frac{5}{100}\times 3=\$60.$

2. Find the interest on \$1280. for 2 years and 5 months at $6\frac{1}{2}\%$.

Solution: $I=P\times R\times T=\$1280.\times\frac{6\frac{1}{2}}{100}\times 2\frac{5}{12}$

Simplifying fractions, $I=\$\cancel{1280}^{\,8}.\times\frac{13}{\cancel{200}_{\,5}}\times\frac{29}{\cancel{12}_{\,3}}$

$$I=\frac{\$3016}{15}=\$201.07.$$

3. Find the interest on \$600. for 1 year, 3 mo., 24 da. at 6%.

Solution: $I=P\times R\times T=\$600.\times\frac{6}{100}\times\frac{474}{360}.$

$I=\frac{2\,8\,7}{5}=\$47.40.$

NOTE: 360 days make a common interest year. Each interest month is counted 30 days. 1 yr. 3 mo. 24 da. = 474 da.

By use of the appropriate formula, find the interest on the following principals for the given time and at the given rate:

(Solve without your pencil when possible.)

4. \$150, 3 yr., 6%.
5. \$900 $2\frac{1}{2}$ yr., 5%.
6. \$1200, 6 yr., $7\frac{1}{2}$%.
7. \$2400, 10 yr., 4%.
8. \$1000, 20 yr., 5%.
9. \$800, $3\frac{1}{4}$ yr., 4%.
10. \$4500, $2\frac{1}{6}$ yr., 6%.
11. \$750, 4 yr. 6 mo., 4 %.
12. \$600, 3 yr. 4 mo., $6\frac{1}{2}$%.
13. \$700, 1 yr. 3 mo., 7%.
14. \$640, 7 yr. 2 mo., 6%.
15. \$960, 2 yr. 1 mo., $3\frac{1}{2}$%.
16. \$1200, 4 yr. 8 mo., 6%.
17. \$780, 1 yr. 9 mo., 7%.
18. \$670, 2 yr. 10 mo., 6%.
19. \$675, 1 yr. 11 mo., 8%.
20. \$25, 16 yr. 8 mo., 6%.

Find the interest on:

21. \$1000 for 1 yr. 6 mo. 15 da. at 6%.
22. \$4500 for 2 yr. 5 mo. 18 da. at 8%.
23. \$3600 for 3 yr. 4 mo. 24 da. at 10%.
24. \$720 for 1 yr. 2 mo. 12 da. at 7%.
25. \$600 for 2 yr. 3 mo. 6 da. at $7\frac{1}{2}$%.
26. \$100 for 16 yr. 8 mo. at 6%.
27. \$100 for 20 yr. at 5%.
28. \$100 for 10 yr. at 10%.
29. \$100 for 12 yr. 6 mo. at 8%.
30. \$150 for 3 yr. 2 mo. 27 da. at 6%.

By use of the appropriate formula, find the amount of:

1. \$800 for 3 yr. at 6%.

Solution: $A = P + I = \$800.00 + \$800.00 \times \frac{6}{100} \times 3$
$A = \$800.00 + \$144 = \$944.00.$

2. \$85.45 for 4 yr. at 8%.
3. \$150 for 2 yr. at $7\frac{1}{2}$%.
4. \$75.60 for 1 yr. at 6%.
5. \$1000 for 6 mo. at 8%.
6. \$1260 for 3 yr. at 5%.
7. \$600 for 1 yr. at 10%.
8. \$8700 for 3 yr. at 4%.
9. \$920 for 9 mo. at 3%.
10. \$1720 for 6 yr. at 5%.
11. \$1200 for 1 yr. 6 mo. at 6%.
12. \$725.50 for 2 yr. 9 mo. at 6%.
13. \$985 for 1 yr. 8 mo. at $4\frac{1}{2}$%.
14. \$600 for 5 yr. 10 mo. at $4\frac{1}{4}$%.
15. \$100 for 15 yr. at $4\frac{3}{4}$%.
16. \$780 for 6 yr. 2 mo. at $7\frac{1}{2}$%.
17. \$400 for 2 yr. 1 mo. 15 da. at 8%.
18. \$180 for 1 yr. 11 mo. at 6%.
19. \$680 for 2 yr. 4 mo. at 8%.
20. \$500 for 3 yr. $5\frac{1}{2}$ mo. at 6%.

21. Mr. Smith borrows \$750.00 from Mr. Wilson for 9 mo. and agrees to pay 6% interest. How much will Mr. Smith owe Mr. Wilson at the end of 9 months?

22. Find the amount needed to pay a loan at $7\frac{1}{2}$% of \$400.00 which was borrowed Jan. 1, 1920, and paid July 1, 1920.

23. A bond issued by a certain city pays $4\frac{3}{4}$% interest. Find the amount a \$100.00 bond will be worth in 10 years if the interest remains unpaid.

24. John borrows \$45.00 from his father March 1, to buy two calves. He sells the calves December 1, for \$90.50. If he paid \$30.15 for feed and paid his father what he owed him, including interest at 6%, how much did John have left?

1. How long is it (a) from Jan. 1, 1920, to July 1, 1920? (b) from Jan. 1, 1920, to March 1, 1921? (c) from Jan. 31, 1920, to July 16, 1920?

2. Make the subtractions indicated: (30 da. = 1 common interest month; 12 mo. = 1 yr.)

		Year	*Month*	*Day*
(a)	From	1920	7	1
	Take	1920	1	1
	Answer			
(b)	From	1921	3	1
	Take	1920	1	1
	Answer			
(c)	From	1920	7	16
	Take	1920	1	31
	Answer			

NOTE: January is the first month, July the seventh, March the third.

What month is December? February? October? November? September? June? August? April? May?

3. Find the amount due on a loan of $600.00 made Jan. 15, 1919, and paid March 15, 1920, if it bears interest at 6%.

4. Mr. Sowder borrowed $1000.00 from Mr. Black March 10, 1919, and agreed to pay 8% interest. How much does Mr. Sowder owe Mr. Black Sept. 30, 1919?

5. Find the amount due on $90.00 which I borrowed July 31, 1919, and paid March 1, 1920, with interest at 8%.

6. Find the amount due on a note for $75 dated May 1, 1919, and paid Jan. 16, 1920, interest 5%.

7. Find the amount of a note for $100 at 6% for 16 yr. 8 mo.

162. The Six Per Cent Method

This method counts 12 months of 30 days each a year.

Interest on \$1 at 6% for 1 yr. = \$0.06 (\$1 × .06 = \$0.06).

Interest on \$1 at 6% for 1 mo. = \$0.005 ($\frac{1}{12}$ of \$0.06), or 5 mills.

Interest on \$1 at 6% for 1 da. = \$0.000$\frac{1}{6}$ ($\frac{1}{30}$ of \$0.005), or $\frac{1}{6}$ mill.

1. Find the interest on \$600.00 at 6% for 8 mo. 18 da.

Solution: Interest on \$1 for 8 mo. = 8 × \$0.005 = \$0.04
Interest on \$1 for 18 da. = 18 × \$0.000$\frac{1}{6}$ = \$0.003
Interest on \$1 for 8 mo. 18 da. = \$0.043

Then, Interest on \$600 for 8 mo. 18 da. = 600 × \$0.043 = \$25.80.

The interest on \$1 at 6% for any number of years will be \$0.06 multiplied by the number of years.

The interest on \$1 at 6% for any number of months will be \$0.005 multiplied by the number of months; that is, 5 mills multiplied by the number of months.

The interest on \$1 at 6% for any number of days will be \$0.000$\frac{1}{6}$ multiplied by the number of days; that is, $\frac{1}{6}$ mill multiplied by the number of days.

The interest on any principal at 6% for any number of months and days will be the interest on \$1 for the given time multiplied by the number of dollars in the principal.

2. Find the interest on \$540.00 at 6% for 1 yr. 8 mo. 9 da.

Solution: 1 yr. 8 mo. = 20 mo. (The years are often reduced to months.)
Interest on \$1 for 20 mo. = 20 × \$0.005 = \$0.10
Interest on \$1 for 9 da. = 9 × \$0.000$\frac{1}{6}$ = \$0.0015
Interest on \$1 for 1 yr. 8 mo. 9 da. = \$0.1015
Interest on \$540 for 1 yr. 8 mo. 9 da. = 540 × \$0.1015 = \$54.81.

By the use of the 6% method, find the interest at 6% on:

3. \$3600 for 9 mo. 12 da.

4. \$1000. for 1 yr. 6 mo. 18 da.

5. $480 for 1 yr. 5 mo.
6. $600 for 2 yr. 7 mo.
7. $1200 for 11 mo. 24 da.
8. $100 for 3 yr. 6 mo. 15 da.
9. $1800 for 7 mo. 10 da.
10. $175.60 for 3 mo. 6 da.
11. $150.20 for 90 da.
12. $300 for 18 mo.
13. $80.25 for 60 da.
14. $720 for 1 yr. 4 da.
15. $900.80 for 1 yr. 11 mo.
16. $600.50 for 1 yr. 20 da.
17. $750.21 for 66 da.
18. $4500 for 30 da.
19. $150 for 15 da.
20. $900 for 96 da.

163. Applying the Six Per Cent Method Where the Rate is Not Six Per Cent

1. What part of 6% is 7%?
2. What part of 6 % is 5%?
3. What part of 6% is $7\frac{1}{2}$%?
4. What part of 6% is $4\frac{1}{2}$%?
5. Find the interest on $150 at 7% for 3 mo. 15 da.

Solution: Interest on $1 for 3 mo. at 6% = 3 × $0.005 = $0.015
Interest on $1 for 15 da. at 6% = 15 × 0.000\frac{1}{6}$ = $0.0025
Interest on $1 for 3 mo. 15 da. at 6% = $0.0175
7% is $\frac{7}{6}$ of 6%.

Hence, Interest on $150 for 3 mo. 15 da. at 7% = 150 × $\frac{7}{6}$ × $0.0175
= $3.0625.

Find by the six-per-cent method interest on:

6. $200 for 6 mo. at 5%.
7. $1500 for 90 da. at 4%.
8. $120 for 11 mo. at 7%.
9. $150 for $3\frac{1}{2}$ mo. at 8%.

164. Exact Interest

The United States Government and some banking houses count 365 days to the year, and in computing interest they count the exact number of days between two dates. Interest so computed is called **exact interest.**

1. Find the interest on $120.00 from Jan. 15, 1919, to June 1, 1919, at 6%.

Solution: From Jan. 15, 1919 to June 1, 1919, there are:

16 days in Jan.
28 days in Feb.
31 days in Mar.
30 days in Apr.
31 days in May
1 day in June
137 days

$$\text{Interest} = \$120 \times \tfrac{6}{100} \times \tfrac{137}{365} = \$2.70.$$

Below is a table which shows the exact number of days between any day of any month and the same day of any other month within a year. In the event of a leap-year if Feb. 29 falls between the two dates, add 1 day to the number found from the table. This table is used by government and bank employees for easy and quick reference.

To find the number of days between the same date of any two months, find the number at the crossing of the row and column corresponding to the two months. If the two dates are not the same, make the correction by adding or subtracting as the case may require.

Illustration: From May 13 to Aug. 13, there are 92 da. From Sept. 10 to June 20, there are 273 da.+10 da., or 283 da. From Feb. 23 to Apr. 15, there are 59 da.−8 da., or 51 da., except for leap year, 52 da.

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
January	365	31	59	90	120	151	181	212	243	273	304	334
February	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	31	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
August	153	184	212	243	273	304	334	365	31	61	92	122
September	122	153	181	212	242	273	303	334	365	30	61	91
October	92	123	151	182	212	243	273	304	335	365	31	61
November	61	92	120	151	181	212	242	273	304	334	365	30
December	31	62	90	121	151	182	212	243	274	304	335	365

Find the exact interest on:

2. \$800 from April 1 to Jan. 1 at 7%.
3. \$1200 from June 15 to May 20 at 6%.
4. \$2400 from Aug. 13 to Feb. 28 at 5%.
5. \$720 from July 25 to Dec. 10 at $4\frac{1}{2}$%.
6. \$600 from Jan. 15 to Aug. 10 at 6%.
7. \$1500 from Feb. 24 to Dec. 19 at 4%.
8. \$1000 from March 31 to July 20 at $4\frac{1}{4}$%.
9. \$250 from Nov. 18 to Dec. 17 at 6%.
10. \$300 from Jan. 1 to Jan. 1 at 8%.

165. Finding the Principal

1. What principal at 6% will yield \$12.00 interest in 2 yr.?

Solution: We know that P×R×T=I. Now, since we know that the product of three factors, (P×R×T), is I, and since we have in our problem two of them given, (R and T), we may find the third, (P), by dividing I by the product, R×T. That is, P=I÷ (R×T).

Hence, $P=\$12\div(\frac{6}{100}\times 2)=\$12\times\frac{100}{12}=\$100$.

Fill in the blanks below:

	Principal	*Rate*	*Time*	*Interest*
2.	--------	9%	40 da.	\$5.00
3.	--------	10%	45 da.	7.50
4.	--------	6%	60 da.	6.00
5.	--------	4%	9 mo.	12.00
6.	--------	5%	2 yr.	45.00
7.	--------	7%	1 yr. $4\frac{1}{2}$ mo.	60.75
8.	--------	8%	1 yr. 9 mo.	36.20
9.	--------	$4\frac{1}{2}$%	10 yr.	90.00
10.	--------	$4\frac{1}{2}$%	15 yr.	42.50

166. Finding the Time

1. How long will it take \$500 to yield \$22.50 interest at 6%?

Solution: Since we again have two, (P and R), of the three factors, (P, R, and T) and the product of the three, (I), in the formula,

P×R×T=I,

therefore, we will divide the product of the three factors, (I), by the product of the known factors, (P×R), to find the third factor, (T).

That is, $T = I \div (P \times R)$

Hence, $T = \$22.50 \div (\$500 \times \frac{6}{100}) = .75 \text{ yr.} = \frac{3}{4} \text{ yr.} = 9 \text{ mo.}$

Fill in the blanks below:

	Principal	*Rate*	*Time*	*Interest*
2.	$250	6%	------	$22.50
3.	600	$5\frac{1}{2}\%$	------	135.00
4.	125	9%	------	33.75
5.	800	6%	------	256.00
6.	950	5%	------	35.625
7.	300	$2\frac{1}{2}\%$	------	50.00
8.	450	7%	------	42.00
9.	360	8%	------	48.64
10.	680	4%	------	60.36

167. Finding the Rate.

1. At what rate will $300 yield $27 interest in 3 yr.?

Solution: Since we have two, (P and T), of the three, (P, R, and T), factors, and the product of the three, (I), in the formula, $P \times R \times T = I$, we will divide I by the product of $P \times T$ to find R.

That is, $R = I \div (P \times T)$.

Hence, $R = \$27 \div (\$300 \times 3) = \$27 \div \$900 = .03 = 3\%$.

Fill in the blanks below:

	Principal	*Rate*	*Time*	*Interest*
2.	$1000	------	1 yr. 6 mo.	$135.00
3.	150	------	6 mo.	4.50
4.	500	------	2 mo.	1.25
5.	400	------	3 mo.	3.60
6.	1000	------	2 yr. 6 mo.	100.00
7.	180	------	1 yr. 9 mo.	17.50
8.	300	------	60 da.	5.00
9.	300	------	1 yr. 3 mo.	12.50
10.	400	------	2 yr. 6 mo.	25.00

168. Promissory Notes

One who borrows money gives a written agreement, or promise, to repay the money at a given time. This written agreement is called a **promissory note.** Unless the interest is paid in advance, the note shows the rate of interest.

Sometimes payment for goods purchased, or services rendered, is made with a promissory note. Thus, a farmer may give his note, promising to pay money later, in exchange for his seeds and fertilizers; or a man, unable to pay his doctor bill promptly, may make a note promising to pay his physician at some later date.

The following is a common form of a promissory note:

\$312.50 JACKSON, MISS., May 1, 1920.

Three months after date, I promise to pay to the order of

J. B. BROWN

Three Hundred Twelve and 50/100 Dollars,

for value received, with interest at 6%.

S. H. HAMILTON.

Note that there are two parties concerned in the note. Mr. Hamilton is called the **maker,** and Mr. Brown is called the **payee.** The sum borrowed is called the **face** of the note.

1. When will the above note fall due?
2. Find the interest due at maturity.
3. Find the amount due at maturity.
4. Who pays this amount? Who receives it?

5. Who holds the note before it matures?

6. What will become of the note after it is paid?

If the payee wishes to sell the note before it has matured, he writes his name on the back of the note. This is called **indorsing** the note, and means that the person indorsing guarantees the payment of the note when due.

Write notes having the following data, using your own post-office and the date of writing as the heading:

	Maker	*Time*	*Face*	*Payee*	*Interest rate*
1.	Geo. Mathes	6 mo.	$300	Fred Jones	8 %
2.	Earl Cloyd	1 yr.	750	Yourself	6 %
3.	W. G. Smith	60 da.	90	First Nat'l. Bank	8 %
4.	Yourself	90 da.	1000	Your teacher	5 %
5.	S.C.Brown&Co.	2 mo.	1500	Bank of Commerce	$7\frac{1}{2}$%

6. Find the date of maturity for each of the above notes.

NOTE: When the time is given in days, count exact time; when in months or years, count calendar time. For example, 90 days from March 15 would be June 13; while 3 mo. from March 15 would be June 15.

7. How much will you owe your teacher on note No. 4, above, when it matures?

8. Mr. Smith borrowed $1800, giving three notes of $600 each, with interest at 6%, and maturing in one, two, and three years, respectively. How much was due at the end of each year? Find the total interest on the three notes.

9. How much will Geo. Mathes owe Fred Jones (Problem 1) when the note becomes due?

10. How much will W. G. Smith owe the First National Bank (Problem 3) when the note becomes due?

11. Write a note, supplying all the data, compute the interest and the amount due at maturity, and hand to your teacher for inspection.

169. INFORMATION ABOUT THE UNITED STATES POSTAL SAVINGS SYSTEM

(Extracts from Form PS4. U. S. Postoffice Department.)

Purpose

The United States Government accepts interest-bearing postal savings deposits from the public and guarantees to repay them on demand.

Who May Deposit

Any person 10 years old or over may open a postal savings account in his or her own name by depositing one or more dollars in any post office authorized to accept postal savings deposits.

Amount That May Be Deposited

A person may deposit any number of dollars, and at any time, until the balance to his credit amounts to $2,500, exclusive of accumulated interest.

Opening Accounts

A person desiring to open a postal savings account should apply at the post office, where full instructions will be given. If for any good reason an intending depositor can not apply at the post office, a representative may be sent who will be instructed how to proceed. A person residing at a post office not authorized to accept postal savings deposits may open an account at a depository office by mail, through his local postmaster, who will give full instructions on application.

Deposits

After a postal savings account has been opened, deposits may be made either in person, by a representative, by money order, or by registered mail if the money-order service is not available.

Postal savings deposits are acknowledged by postal savings certificates which are made out in the name of the depositor and serve as receipts. These certificates are not negotiable or transferable. If certificates are lost, stolen, or destroyed, new certificates may be issued.

Withdrawals

A depositor may at any time withdraw all or any part of his postal savings deposits, upon demand, from the post office where the deposits were made. Withdrawals may be made in person, through a representative, or by mail.

Interest

Postal savings certificates bear simple interest at the rate of 2 per cent a year. Interest begins on the first day of the month following the month in

which the certificate is issued and becomes due and payable at the expiration of each full year from the day interest begins as long as the principal remains on deposit. If the deposit is withdrawn, interest will be paid to the end of the quarter year last preceding.

Postal Savings Cards and Stamps

Amounts less than $1 may be saved by purchasing postal savings cards and postal savings stamps at 10 cents each. A savings card with nine savings stamps affixed will be accepted as a deposit of $1 either in opening a postal savings account or in adding to an existing account, or it may be redeemed in cash.

Postal Savings Bonds

A depositor may exchange the whole or a part of his deposits for registered or coupon United States postal savings bonds, bearing $2\frac{1}{2}$ per cent interest, issued in denominations of $20, $100, and $500.

1. If Harry deposits $10 on Dec. 28, 1923, when will interest begin? How much will the interest on Harry's deposit amount to Jan. 1, 1925? (See paragraph on Interest on preceding page.)

2. John deposits $5 Jan. 3, 1920, and $15 July 20, 1920. He withdraws all his deposits Dec. 20, 1920. How much does he get? (See Withdrawals and Interest.)

3. If John should leave his deposits in until Aug. 1, 1921, how much would he get?

4. Mary saves 10¢ a week. How long will it take her to get a savings card worth $1? How may she make use of her savings card? (See Postal Savings Cards and Stamps.)

5. Edwin deposits $20 a quarter. How long will it take him to save enough to buy a U. S. postal savings bond of $100? How much interest will the deposit earn by the time he has saved enough to buy the bond?

6. How much will a $100 bond earn in interest per year? (See Postal Savings Bonds.)

7. How much more does the bond earn per year than the $100 kept in the Postal Savings Bank?

170. Review Problems

1. If you know the amount of a note, the interest, and the time, how do you find the rate? Illustrate with a problem.

2. If you know the face of a note, the amount, and the time, how do you find the rate? Illustrate with a problem.

3. If you know the face of a note, the rate, and the amount, how do you find the time? Illustrate with a problem.

4. What is meant by net profit? by gross profit? Illustrate.

5. Name some items that enter into the total, or gross, cost of goods handled by a grocer.

6. If a grocer makes a gross profit of $33\frac{1}{3}\%$ on the cost of staples, is he likely making a fair profit? Why? Answer as to fruits, bananas, perishable products, etc.

7. Which is the better investment, \$100 loaned at 4%, or \$100 invested in a bond drawing interest at $4\frac{1}{2}\%$, interest payable annually?

8. Harry bought some chicks one day old at 10¢ each. He found that it cost him 20¢ per month to feed each chick. He kept them 3 months and sold them at 85¢ each. What was his net profit on each? What per cent was the net profit of the gross cost? Of the selling price?

9. An iceman pays \$6.00 per ton (2000 lb.) for ice, and retails it at 60¢ per 100 lb. If it costs him \$3.00 per ton to deliver it, and if the loss by waste is 20%, how much is his net profit on each ton? What per cent is his net profit of the gross cost? Of the selling price?

10. Find the interest on a note for \$500 for 3 yr. 5 mo. 18 da., at $5\frac{1}{2}$ per cent.

171. REVIEW

In order to use such expressions as 6%, $12\frac{1}{2}\%$, $16\frac{2}{3}\%$, etc., in the operations of multiplication and division, they should be written either as common fractions or decimals. For example, to find $16\frac{2}{3}\%$ of \$720, $16\frac{2}{3}\%$ should be written as $\frac{16\frac{2}{3}}{100}$, $= \frac{50}{300}$ $= \frac{1}{6} = .16\frac{2}{3}$. Explain the reduction.

Sometimes it is more convenient to use one of the forms and sometimes another. In the illustration above, the third form, $(\frac{1}{6})$, is the most convenient. Why? What is $\frac{1}{6}$ of \$720? If we were finding $16\frac{2}{3}\%$ of \$900, which of the four forms would be the most convenient? Why? Which is the most convenient form for finding $16\frac{2}{3}\%$ of 279?

What per cent of 40 is 8?

To answer this question, we think first, **What part of 40 is 8?** We think successively: $\frac{8}{40}$, $\frac{1}{5}$, $\frac{20}{100}$, 20%. In this process we employ reduction in changing $\frac{8}{40}$ to $\frac{1}{5}$ and $\frac{1}{5}$ to $\frac{20}{100}$. Then, since hundredths means per cent, we have 20%.

What per cent of 45 is 7?

Again, we think, 7 is what part of 45, which we find is $\frac{7}{45}$; but since reduction of this fraction to a common fraction with 100 as a denominator is not convenient, we change the fraction to decimal form, carrying it to hundredths, thus:

$$
\begin{array}{r}
0.15\frac{5}{9} \\
45)\overline{7.00} \\
\underline{45} \\
250 \\
\underline{225} \\
25
\end{array}
$$

Hence, we have $15\frac{5}{9}\%$.

Write the following in at least two other forms, taking care to see that the per cent symbol, (%), appears in one of the forms.

Illustration: $\frac{2}{5} = \frac{40}{100} = 40\%$.

1.	$.62\frac{1}{2}$	11.	$\frac{1}{8}$	21.	.375
2.	$1\frac{3}{4}\%$	12.	$.00\frac{3}{8}$	22.	$1\frac{1}{3}\%$
3.	$.7\frac{1}{2}$	13.	.02	23.	$.02\frac{1}{2}$
4.	$\frac{5}{8}$	14.	.02%	24.	$\frac{5}{6}$
5.	62.5%	15.	$\frac{3}{4}\%$	25.	$62\frac{1}{2}\%$
6.	$2\frac{1}{2}$	16.	$\frac{3}{4}$	26.	2.6
7.	$\frac{1}{2}\%$	17.	$2\frac{1}{2}\%$	27.	$\frac{1}{8}\%$
8.	1.25	18.	$16\frac{2}{3}\%$	28.	$1\frac{1}{3}$
9.	$37\frac{1}{2}\%$	19.	$\frac{1}{5}$	29.	$\frac{1}{6}$
10.	100%	20.	$12\frac{1}{2}\%$	30.	125%

172. Food Problems

Experts say that a well-balanced diet for a man for one day should be 100 grams of protein (lean meat, cheese, eggs, etc.); 115 grams of fat; and 500 grams of carbohydrates (cereals, etc.)

1. What per cent of the total diet is protein? fat? carbohydrates?

Dr. Loyal Meeker, Commissioner of Labor Statistics, U. S. Dept. of Labor, Washington, says that a child 3 years old and under eats 15% as much as a man; a child 4 to 6 years, 40%; a child 7 to 10 years, 75%; a child 11 to 14 years, 90%; a child 14 years and over, 100%; and a woman 90%.

2. Find the number of grams of (a) protein, (b) fat, (c) carbohydrates, a child of 3 years and under should have per day.

3. Answer the question for a child of 4 to 6 years.

4. Compute similar results for a child of 7 to 10 years.

5. Answer as to a child of 11 to 14 years.

173. War Problems

Adjt. Gen. Harris announced in February, 1920, the final result on casualties during the World War, as follows:

Killed in action (including 382 men at sea,) 34,844; died of wounds, 13,960; died of disease, 23,738; died from accident and other causes, 5,102; wounded in action (over 85 per cent returning to duty) 215,423; missing in action (not including prisoners released and returned) 3. Total of 293,070.

1-6. Find the per cent (correct to two decimal places) of the total casualties (a) killed in action; (b) died of disease; (c) died from accident and other causes; (d) wounded in action; (e) died of wounds.

7. The World's Almanac for 1920 gives the total number of soldiers sent abroad during the war to be 2,079,880. What per cent of these is the total of the casualties?

8. What per cent of the number sent abroad died from various causes?

9. What per cent of the total force was wounded?

10. What per cent of those wounded died of their wounds?

11. In a certain county, out of 750 men of draft age 300 were pronounced physically unfit for military service. What per cent of the men were physically fit?

12. In a certain county, 500 men enlisted for military service. Of this number, 225 were sent overseas. What per cent of them were sent overseas?

13. Of the number of men sent overseas in Problem 12, there were killed or missing 20%. How many men were killed or missing?

14. In a certain town, out of the whole number of enlisted men $37\frac{1}{2}\%$ were killed in battle. If there were 72 killed, what was the number of enlisted men?

174. TAXES

1. Where did the money come from with which your school building was erected?

2. Who pays your teacher's salary?

3. Who pays for the roads, bridges, the court house, etc., in your county?

4. Where does the state get the money to build asylums for the insane, schools for the blind, reformatories for the wayward, etc.?

The town, city, county, and state governments incur very large expenses for such purposes as those mentioned, as well as for many others.

Most of these expenses are provided for by the public, and the money which the people pay into the treasuries of the town, city, county, and state governments is called **taxes.**

The amount of each person's tax is fixed by the proper officials, and the payment is enforced by law.

For purposes of taxation, property is divided into two classes:

I. **Real Property;** such as land, buildings, railroads, and any property regarded as immovable.

II. **Personal property;** such as money, notes, jewelry, and household effects.

In addition to property tax, most states impose a tax upon all voters within certain age limits. This is known as **poll tax.**

The amount of poll tax varies in different states and in different counties of the same state, ranging from 50¢ to $3.

The rate of taxation is stated as so many cents on the hundred dollars, as so many mills on the dollar, or as so many per cent of the assessed value of the property.

The assessed valuation of property is made by legally authorized officials and becomes the basis for taxing purposes.

1. If the tax rate is 15 mills ($0.015) on the dollar, or $1\frac{1}{2}\%$, what is the tax on a house with an assessed valuation of $7500?

2. If the tax rate is $2.25 on the hundred dollars, what is the tax on a farm assessed at $12,500?

3. What is the rate per cent in Problem 2?

Find the taxes on property with the indicated assessed valuation and rates:

	Assessed valuation	*Rate*	*Tax*
4.	$ 6,000	12 mills on $1	------
5.	$12,000	$7\frac{1}{2}$ mills on $1	------
6.	$ 7,500	10 mills on $1	------
7.	$ 8,400	$3.25 on $100	------
8.	$ 5,400	$1.85 on $100	------
9.	$16,600	$1.91 on $100	------
10.	$11,000	19 mills on $1	------

Certain officials of a town, county, or other government are legally authorized to fix tax rates for various purposes. They first make up a budget of the expenses that are likely to be met. Next, they get an assessed valuation of all the property that is subject to taxation. Then they assess each owner at a certain rate for each item of expense in the budget.

11. In a certain county in Tennessee, the assessed valuation of all taxable property is $4,520,000. The county court wishes to build a new court house to cost $226,000. What tax rate will raise sufficient money to build the court house, if paid for in one year?

12. What amount will Mr. Carr contribute to the new court house if he owns a farm assessed at $6,500?

13. If the county court should distribute the court house tax equally over a period of ten years, what would the annual rate be, not considering interest? What would Mr. Carr's annual court house tax be?

14. Under the provisions of Problem 13, how much property is Mr. Chase paying taxes on if his court house tax amounts to $23.15 per year?

15. The tax rate in a certain town is $2.25 per $100 for all purposes. Mr. Parsons pays one poll tax of $2, and has property assessed at $9,600. Find the total amount of his taxes.

16. Find out the tax rate in your town or county. How much is the poll tax? Find the tax on a house in your community assessed at $3,000.

17. If the amount of tax on a farm in your county is $60 at the tax rate of your county, find the assessed valuation of the farm.

Hint: Just as $p \div r = b$, tax ÷ rate = assessed valuation.

18. Ascertain the assessed valuation of property and the tax rate in your county, and find the total taxes produced.

19. Is the official who collects taxes in your county paid a commission or a salary?

20. If he is paid a commission, inquire the rate and find his income for collecting taxes. If he is paid a salary, find the rate per cent cost of collecting the taxes.

21. The value of all taxable property in a certain county in Tennessee is $55,562,179.25. The state levies a tax of 15¢ on $100 and the county levies $37\frac{1}{2}$¢ on $100 for the support of the elementary schools. How much money does the county have for its elementary schools?

22. If the county referred to in Problem 21 levies a tax of $9\frac{1}{2}$¢ on $100 for the support of its high schools, how much money does this assessment raise for high schools?

23. Mr. Crabtree, who is a citizen of this county, pays taxes on property with an assessed valuation of $12,500. How much does Mr. Crabtree contribute to the support of the elementary schools? Of the high schools?

175. Revenues

The taxes paid by the people upon real estate and personal property, including poll taxes, go to the support of the state and smaller units of local government. None of these taxes go to the support of the national government.

Our national government is supported chiefly from three sources:

I. Tariffs, duties, or customs, levied as tax upon certain imported articles.

II. Internal revenue taxes, levied upon tobacco products, certain kinds of drugs, soft drinks, certain documents, etc.

III. Income tax, levied upon profits and incomes which exceed certain fixed amounts.

Under I., some imported articles are taxed **ad valorem,** according to value; some are subject to **specific duty,** or so much per article, per pound, etc., and some are subject to both ad valorem and specific duties.

The table below shows the rates of duty, or tariff, on twenty-four imported articles, as fixed by a recent Tariff Law:

Articles	Rates	Articles	Rates
Automobiles, over $2000	45%	Lace	60%
Butter	$2\frac{1}{2}$¢ per lb.	Lemons	$\frac{1}{2}$¢ per lb.
Carpets and rugs, woolen	50%	Macaroni	1¢ per lb.
Cement	10%	Maple sugar	3¢ per lb.
Cheese	20%	Olive oil	30¢ per gal.
Cigars and cigarettes	$4.50 per lb. and 25%	Pens, not including gold	8¢ per gross
Clocks and watches	30%	Perfumery	60%
Cotton clothing, ready made	30%	Pineapples	$5 for 1,000
Drugs	10%	Silk, spun	35%
Furs, evening apparel	50%	Spectacles and glasses	35%
Grape sugar	$1\frac{1}{8}$¢ per lb.	Toys	35%
Honey	10¢ per gal.	Woolen blankets	25%

1. Find the duty paid by an importer in New York on $1500 worth of drugs.

2. An importer bought 2 dozen clocks at $18 each and 3 dozen watches at $37.50 each. Find the total cost, including the duty and an additional charge of $12.60.

3. Find the duty on a set of furs which cost $105.

4. Find the total cost to an importer of 600 lb. of lemons if the purchasing price is 10¢ per lb. and shipping charges 1¢ per lb.

5. Find the total cost of 300 lb. of grape sugar if the purchasing price is 25¢ per lb. and additional charges 1¢ per lb.

6. A tobacco dealer imported 12 boxes of cigars, weighing $1\frac{1}{2}$ lb. to the box, at a price of $2 per lb. Find the total cost. Find the profit in dollars if there are 50 cigars to the box, and if the dealer sells the cigars at 25¢ each. What per cent profit on the gross cost is he making?

7. Find the duty on 100 gross of imported pens.

8. A merchant sells imported maple sugar at 50¢ per lb. What per cent profit is he making on the total cost if he pays 30¢ per lb. for the sugar and the cost of delivery is 2¢ per lb.?

9. In a recent year, the spun silk imported into the United States amounted to 1,555,069 lb. with an invoice value of $6,085,377. Find the amount paid in duties.

10. A manufacturer used 2000 lb. of spun silk valued at $4 per lb. What is the tariff charge?

11. A New York merchant imported 638 pineapples. Find the duty on them.

12. The same dealer imported the following: 100 lb. butter, 200 lb. cheese, at 30¢ per lb., 50 gal. honey, 100 lb. macaroni, 16 gal. olive oil, and 75 pineapples. Find the duty on the entire bill.

13. A dealer in Philadelphia imported the following: carpets and rugs valued at $1500, cotton clothing valued at $850, lace valued at $904.50, and woolen blankets valued at $1160.40. Find his total tariff charges.

14. An automobile dealer imported three automobiles valued respectively at $2500, $2850, $2900. Find the duty on the three automobiles.

15. If the toys imported for the year of 1918-'19 were valued at $4,560,000, find the duty paid by the people of the United States on toys.

16. A clothier imported $25,780 worth of furs, and $750 worth of woolen blankets. Find the amount of the duty he had to pay.

17. If the clothier in Problem 16 sold his furs at a profit of 45% above cost and duty and the blankets at 20% profit above cost and duty, what was his profit on both purchases?

18. Find the duty on 750 lb. of grape sugar.

19. Find the duty on (a) 2500 pineapples; (b) $400 worth of lace.

20. Find the duty on 200 lb. of cigars whose value is $1250.

21. If the duty at 30% was $426, what was the value of the imported goods?

22. Find the duty on a ton of cheese valued at 60¢ a pound; 20 watches at $50 each; 100 gal. of olive oil; 300 lb. of lemons.

23. Find the duty on $6375 worth of cement; $2250 worth of toys; 45 gross of pens.

176. The Income Tax

An income tax is a tax levied by the United States Government upon the incomes of individuals and corporations when such incomes reach certain specified amounts. The money derived from the taxing of incomes is used to help defray the expenses of the National Government.

NOTE: Since the details and the rates of the Income Tax Law are frequently changed by Congress, only the simplest provisions of the law are here outlined. Income tax calculations, involving the surtax and certain other technicalities, are purposely omitted from the problems given below. It is deemed sufficient for the pupil to learn in a general way why and how such revenue is obtained.

Two provisions of a recent Revenue Act are:

I. On incomes of \$5000. or less, a normal tax of 4% on the remainder after allowing an exemption of \$1000. for single persons or \$2500. for married persons, plus \$400. for each dependent under 18 years of age or if mentally or physically incapable of self-support at any age.

II. On incomes of more than \$5000. the exemption is \$1000. for single persons or \$2000. for married persons, plus \$400. for each dependent as described in Section I. The normal tax is 4% on the first \$4000. in excess of the exemption allowable plus 8% on the remainder.

1. A bachelor without dependents has a net income of \$3376. Find his income tax.

2. A married man, with four dependent children, has a net income of \$4750. What is the amount of his income tax?

3. An unmarried woman's annual income, after the legal exemption was made, amounted to \$2850. How much income tax was she due to pay?

4. An unmarried man, with no dependents, has an annual income of \$5850. What income tax will he be required to pay?

5. A widow, with four dependent children, has a net income of \$3150. Find the amount of her income tax.

177. MEAT PRODUCTS

Farm animals when sold for meat are sold either alive at a given price per pound live weight, or slaughtered at a given price per pound dressed weight. The value of an animal per pound live weight depends upon the number of pounds it will make dressed weight as well as upon the quality of the meat. If a farmer is growing beef cattle, he will want to grow the type that has the highest percentage of valuable high-priced cuts, with the smallest percentage of low priced cuts, offal and waste. The dressed weight of the best grade of beef cattle ranges from 58% to 69% of live weight, while the dressed weight of the low-grade or coarse beef cattle varies from 40% to 50% of live weight, and the meat is of poorer quality.

1. If two beeves weighing 1400 pounds each were sold on the market and the dressed weight of one was 65% of the live weight and the other 40% of the live weight, how much more was one worth than the other if beef sold at an average price of 20¢ a pound?

2. A coarse beef weighed on foot 900 pounds. A high-grade beef weighed on foot 1200 pounds. If the dressed weight of the coarse beef was 42% of its live weight, and the dressed weight of the high-grade beef was 58% of its live weight, which beef would be the more profitable for a butcher to buy, the coarse beef at 6¢ a pound live weight or the high-grade beef at 8¢ a pound live weight, provided the butcher sells the beef of each animal at the same price?

Hint: Find the cost per pound dressed weight of each.

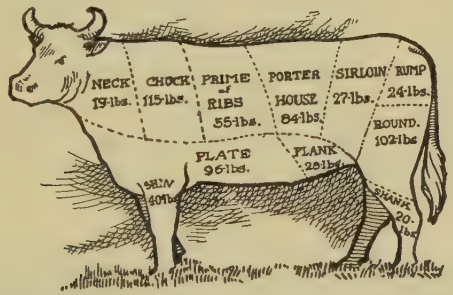
3. What is the value of a coarse beef whose live weight is 950 pounds, if its dressed weight is 50% of its live weight, and the beef sells at an average of 20¢ a pound?

4. Is it better to sell by dressed weight at 20¢ a pound or by live weight at 10¢ a pound, a high-grade beef weighing 1200 pounds whose dressed weight is 58% of the live weight?

5. Which would be better to sell a low-grade beef weighing 850 pounds at 7¢ a pound live weight or 16¢ a pound dressed weight, provided the dressed weight is 44% of the live weight.

6. What is the value of a beef weighing 675 pounds at 18¢ a pound dressed weight, provided the dressed weight is 48% of the live weight?

This picture shows the cuts of a beef weighing 1000 lbs. live weight. The number of pounds in the different cuts varies with the grade of the animal.



7. A butcher bought a beef weighing 1250 pounds live weight, at 8¢ a pound. If the beef's dressed weight is 60% of its live weight, for how much a pound must the butcher sell to gain 50% of the cost?

8. Find the dressed weight of the beef as shown in the picture. What per cent is the dressed weight of the live weight?

9. What per cent of the dressed weight in the above picture is chuck? round steak? porter house? neck? plate? shank? sirloin?

10. Find out from your butcher the prices of the different cuts and calculate the value of each; the total value of the dressed beef.

11. The live weight of a beef was 1000 pounds. If its dressed weight was 550 pounds, what per cent was the dressed weight of the live weight?

High-grade hogs usually dress 75 per cent to 82 per cent of the live weight.

12. What is the dressed weight of a hog whose live weight is 280 pounds if the dressed weight is 80% of the live weight?

13. The live weight of a hog was 325 pounds; its dressed weight was 260 pounds. What per cent of the live weight was the dressed weight?

14. Which is better, and how much, to sell 5 hogs averaging 275 pounds each at 10¢ a pound live weight or at 15¢ a pound dressed weight, if the loss in dressing the hogs is 22%?

15. A farmer sold a hog weighing 350 pounds live weight at 10¢ a pound. Had the farmer sold the hog for the same amount dressed, how much would he have received a pound if the loss in dressing was 20%?

178. COTTON

1. James planted 3 acres in cotton. It cost to cultivate and gather his crop $200. He made $\frac{3}{4}$ bale (bale = 500 lb.) to the acre, which he sold at $200 a bale. How much did he make on his cotton above the cost of production?

2. A manufacturer bought the cotton in Problem 1 and manufactured it into hose. If a pound of cotton makes 12 pairs of hose which the manufacturer sells at 25¢ a pair, what is the value of the hose made from the cotton?

3. If 12 pairs of hose are made from 1 lb. of cotton worth 40¢ a pound, what is the value of the cotton in 1 pair of hose?

4. What is the difference in the cost of the cotton in a pair of hose and the cost of the hose at 25¢ a pair? (See Ex. 3)

5. How much more does the cotton cost in a pair of hose when cotton is selling at 40¢ a pound than it would cost when cotton is selling at 10¢ a pound? (See Ex. 3)

6. If we consider only the actual cost of cotton, how much more would a dozen pairs of hose cost when cotton is selling at 40¢ a pound than when selling at 10¢ a pound?

7. A manufacturer bought 2 bales of cotton at 40¢ a pound which he manufactured into gingham. If a pound of cotton makes 5 yards of gingham, which the manufacturer sells at 40¢ a yard, how much are the 2 bales of cotton worth when manufactured into gingham?

8. How much more valuable is a bale of cotton manufactured into gingham at the rate of 5 yards to a pound and worth 60¢ a yard than if manufactured into hose at the rate of 12 pairs to the pound and worth 20¢ a pair?

9. If a pound of cotton worth 40¢ a pound makes 5 yd. of gingham and the cost of manufacturing is twice the cost of the cotton, for how much should a yard of gingham be sold so as to make a profit of 40% on total cost?

PICKING COTTON

10. The first prize, consisting of a gold medal and $1000 given by The Farm Journal in a cotton production contest in

1919, was won by J. O. M. Smith of Commerce, Georgia, who grew an average of 1360 lb. of lint cotton per acre, on 5 acres.

(a) How many bales did he grow?

(b) What was the value of his cotton at $200 a bale?

(c) If the cost of cultivating and gathering his crop was $200 per acre, what was the net profit per acre?

(d) The average yield in Georgia for a certain year was 165 lb. of lint per acre. How much more did Smith produce per acre than was produced on the average acre for the state?

11. At the rate of 12 pairs of hose to the pound, how many Georgia boys could get a pair of hose out of the 5 acres of cotton grown by J. O. M. Smith?

12. S. J. White, Sumter, S. C., who won the second prize of $500 in the cotton production contest, grew an average of 1197 lb. to the acre on 5 acres.

(a) What was the value of his crop at $205 a bale?

(b) If the cost of production was 40% of the value of the cotton, what was the cost of production?

(c) If made into gingham at the rate of 5 yd. to the pound, how many yards of gingham would White's 5 acres of cotton make, and what would the gingham be worth at 35¢ a yard?

13. William Peterson, Clinton, N. C., who won the third prize of $250, grew an average of 1040 lb. to the acre on 5 acres.

(a) He sold the lint and seed per acre for $493.84. If he received 40¢ a pound for the lint, how much did he get per acre for the seed?

(b) The value of the seed was what per cent of the value of the lint per acre?

(c) The average value of cotton per acre produced in North Carolina in a certain year was $133.72. What per cent was that of the yield produced by William Peterson?

PART III.

179. PARTIAL PAYMENTS

Sometimes a man who has bought property and who has given his note in payment for it reserves the option of making part payments on the debt from time to time.

These are called **partial payments.** The amount and date of each payment are written on the back of the note.

The **United States Rule** of partial payments is the legal rule in nearly all states. It is really a decree of the Supreme Court of the United States. The rule follows:

I. Find the amount of the principal to the time of the first payment, subtract the payment, consider the remainder as a new principal, find the amount to the time of the second payment, subtract the payment, and thus continue until the final settlement is made.

II. If any payment does not equal or exceed the interest, then find the interest up to the time when two or more payments equal or exceed the interest.

1. A note for $1800 given April 1, 1918, showed the following payments: April 1, 1919, $708; June 1, 1920, $672. Find the amount due on the note April 1, 1921, interest at 6%.

Solution:

1919	4	1	Interest on $1800 at 6% for 1 yr.	=$108
1918	4	1	Amt. of $1800 at 6% for 1 yr.	=1908
1	0	0	Payment	= 708
			New principal	=1200
1920	6	1	Interest on $1200 at 6% for 1 yr. 2 mo.	=$ 84
1919	4	1	Amt. of $1200 at 6% for 1 yr. 2 mo.	=1284
1	2	0	Payment	= 672
			New principal	= 612

1921	4	1	Interest on $612 at 6% for 10 mo.	=$30.60
1920	6	1	Amt. of $612 at 6% for 10 mo.	=642.60
	10	0		

2. Mr. Crouch gave his note for $600 to Wm. Carter, reserving permission to make payments at any time. The note was dated Jan. 15, 1923, and was to run for 1 year and draw 8% interest. On the back of the note were the following indorsements:

March 15, 1923, received $6. Wm. Carter.
July 1, 1923, received $50. Wm. Carter.
Dec. 1, 1923, received $50. Wm. Carter.

NOTE: Since the payment on March 15 was not sufficient to pay the interest then due, the principal from Jan. 15 to March 15 will not be reduced. It will save time to find the amount of $600 from Jan. 15 to July 1, and subtract the sum of the two payments from this amount to find the new principal.

3. A school bought a piano on the instalment plan for $600 agreeing to pay $200 cash, and $20 each month plus the interest at 6% upon the sum left unpaid from month to month. How much will the first monthly payment be? How many payments will have to be made to pay for the piano in full?

4. How does Problem 3 differ from Problems 1 and 2? In Problem 3, find the second payment; the third payment.

5. FOR SALE: Five-room cottage, East Lake; large lot; $1800, $300 cash, balance $25 per month, 6%.

WAGNER & Co., Main 2604.

6. Make up five of the best problems you can from this advertisement, which was copied from a newspaper, and solve them. Ask such questions in the problems as you think a purchaser might want to know.

180. The Merchant's Rule

The United States Rule for computing the amount due on a note where partial payments are made provides that unless the payment at any particular time equals or exceeds the interest up to that time the principal remains the same until such time as the sum of the payments equals or exceeds the interest. The effect of this rule frequently inclines the maker of the note to postpone making any payments until the date of settlement.

A rule that seems fairer where partial payments are made on a note is called the **Merchant's Rule.** It follows:

(1) Find the amount of the face of the note from the date of the note to the date of settlement.

(2) Find the amount of each payment from the date of the payment to the date of settlement.

(3) Deduct the sum of the amounts of the payments from the amount of the face of the note.

(4) The remainder is the amount due.

Problems

1. $150.00 Asheville, N. C., May 4, 1920.

Six months after date I promise to pay Burton Bros., or order, One Hundred Fifty Dollars, value received, with interest at 6%. J. H. HAMILL.

Payments: June 1, 1920, $20; July 4, 1920, $30; Sept. 7, 1920, $25.

How much was due at the date of maturity under the Merchant's Rule?

Solution:

6 mo. from May 4, 1920, is Nov. 4, 1920.

1920	11	4	
1920	5	4	
	6	0	Amt. of $150 for 6 mo.....$154.50

1920	11	4	
1920	6	1	
	5	3	Amt. of $20 for 5 mo. 3 da.....$ 20.51
1920	11	4	
1920	7	4	
	4	0	Amt. of $30 for 4 mo.....$ 30.60
1920	11	4	
1920	9	7	
	1	27	Amt. of $25 for 1 mo. 27da.....$ 25.24

Credits on payments and interest.....$ 76.35
Amt. due Nov. 4, 1920 ($154.50—76.35).....$ 78.15

2. $360.00 Augusta, Ga., Nov. 1, 1919.

Ten months after date I promise to pay Miller & Co. Three Hundred Sixty Dollars, for value received, with interest at 6%.

Payments: Jan. 1, 1920, $100; March 3, 1920, $50; May 10, 1920, $60; Aug. 1, 1920, $30.

Find the amount due at the date of maturity under the Merchant's Rule.

3. $25. Monroe, La., April 15, 1920.

Ninety days after date I promise to pay Sterchi Co. Twenty-five Dollars, for value received, with interest at 6%.

Payments: May 1, 1920, $5; June 10, 1920, $7.50; July 1, 1920, $6.50.

Find the amount due at maturity under the Merchant's Rule, counting exact number of days.

4. Find the amount due under the Merchant's Rule on the first problem under the United States Rule for Partial Payments.

How much difference do you find between the results?

181. INSURANCE

In some communities, property owners organize for the purpose of sharing losses and damages that are likely to happen to property by reason of fires, storms, etc. Each member of the organization pays into its treasury, at regular intervals or when called upon, a certain amount of money to compensate persons sustaining loss by fire, or otherwise. The person suffering the loss is paid for his loss an amount fixed by the proper officials of the organization. With such an organization, no one owner suffers any very great loss, and all have the protection afforded by cooperation.

Out of this principle, applied to both property and life, has grown one of the greatest business institutions of the world—the insurance business. The insurance company secures the membership of a large number of people, usually from a wide area, and charges each member a comparatively small sum for the protection given.

Policy—The written agreement signed by the proper officials of the company and by the person seeking the protection is called a policy, or an insurance policy.

Face of Policy—The amount specified in the agreement to be paid in case of loss or damage is called the face of the policy. Property is not usually insured at its full value.

Premium—The cost of the insurance, or the fee paid at fixed intervals, is called the premium.

The rate of premium, or insurance rate, may be stated as so much on \$100, or \$1,000, or a certain per cent of the face of the policy; and the premiums may be paid at any one of several specified intervals, as annually, once every three years, etc.

182. Fire Insurance

1. Mr. Morris has an insurance policy with a face value of \$5,000 on his home. The premium is \$75 for three years. What per cent of the face value is the premium for one year?

2. What will it cost Mr. Morris to insure his household goods for \$1,350, at \$.75 per \$100 for one year?

3. A shipping vessel was valued at \$120,000, and its cargo at \$75,000. Find the cost of insuring each at $\frac{3}{4}$ of its value, if the premium rate is $\frac{1}{2}\%$.

4. A stock of hardware was insured for \$7,500, the premium being \$50 per year. Find the rate.

5. A farmer insured his horses for \$1200 at \$1.50 per \$100 for one year. Find the premium.

6. Find the cost of insuring \$60,000 worth of goods at $\frac{4}{5}$ of their value, the premium rate being $1\frac{1}{8}\%$.

7. Mr. Smith and Mr. Black conduct the same kind of business in the same block. Mr. Smith occupies an old wooden building and Mr. Black a brick building. Each insures his stock of goods and his building for \$8,500. Which ought to pay the higher rate? Why? Mr Smith pays $4\frac{1}{4}\%$ for 2 years, and Mr. Black pays $4\frac{1}{4}\%$ for 3 years. Find the annual cost to each and the difference.

8. How does the kind of fire protection affect insurance rates in towns and cities? Mr. Jones lives in a town which supports a good fire department, while Mr. Ellis lives in a town which maintains no fire department. Each of them insures his home for \$6,000. Which should pay the larger premium?

Mr. Jones pays \$60 for 3 years and Mr. Ellis \$90 for 2 years. Find the difference in annual rates in per cent.

9. Sometimes an insurance policy is said to be open, which means that the amount to be paid in case of loss or damage is to be determined after the loss. Mrs. Moss insured her home for \$3,000. A fire damaged her home to the extent of $\frac{1}{4}$ of its value, as estimated by the company. She had paid \$15 for two years' insurance, and found that she was able to repair the damage for \$670. How much above the cost of repair and insurance did the company pay her?

10. In a certain town where a three-year rate is twice the rate for one year, a residence is insured for $6,000. Find the cost of insurance at the three-year rate when the annual rate is 85¢ per $100.

11. In a recent year, the fire losses in the United States amounted to $290,959,885. The population of the United States in 1920 was 107,321,377. Find the per capita fire loss on this basis in this country. In Europe, the average fire loss per capita is about 35¢ per year. What is the difference in per capita loss in the two countries?

12. The rates of fire insurance for a building depend upon: (1) the location, (2) the construction, (3) the occupancy, (4) the exposure. Explain what is meant by each of these terms.

13. Where does the greater danger from fire exist, in the city or in the country? Why? Where is the better protection found? Why?

14. Inquire rate of premium and face of policy at your home, or elsewhere, and make up two problems based upon the information you secure. Solve the problems.

183. Life Insurance

Most people who are properly concerned about their financial condition in old age and the welfare of those dependent upon them carry insurance upon their lives as a matter of protection against possible want. This protection is afforded the insured and dependents before the death of the insured, or the dependents after the death of the insured, according to the terms of the policy.

The following are the principal kinds of life insurance policies:

I. **The ordinary life insurance policy** guarantees the payment of a fixed amount of money at the death of the insured, the premiums to be paid for life.

II. **The twenty-payment life policy** makes the face of the policy payable at death, the annual premiums to be paid for twenty years unless insured dies within that time.

III. **The endowment policy** provides for the face of the policy to be paid to the insured if living at the end of the endowment period, or to his estate if death occurs before that time, premiums to be paid until the end of the endowment period, or until death if the insured dies before the end of the endowment period.

IV. **The term policy** requires payment of premiums for a specified period of time, and for the payment of the face of the policy only in the event of the death of the insured within the term of insurance. The cost of term insurance is much less than any of the other kinds for the reason that the company may not have to pay anything on the policy.

There are other kinds of insurance policies which differ only slightly from one or another of the four forms described.

The cost of life insurance is expressed as so much per year on each $1,000 of insurance. The cost depends upon the age of the insured and upon the kind of policy desired. The term policy is least expensive of the four kinds described above. The ordinary life policy is less expensive than the twenty-payment or the endowment policies. Why?

The cost per $1,000 varies with the different insurance companies, but the table below is a fair average.

184. Actual Annual Premiums per $1,000 Insurance, as Issued by An Insurance Company

Age	Ordinary Life	20-Payment Life	20-Year Endowment	20-Year Term
20	$ 18.95	$ 27.64	$ 45.98	$ 12.48
25	21.14	30.05	49.35	13.34
30	23.96	32.98	50.74	14.61
35	27.63	36.62	51.59	15.70
40	32.48	41.18	53.69	20.15
45	39.02	47.09	56.74	25.85
50	47.79	54.98	61.75	35.00
55	60.33	65.81	70.02	------
60	77.48	81.09	------	------

185. Problems

Refer to the table for rates.

1. Find the premium on an ordinary life policy with a face value of \$6,000 for a man of 30.

2. Find the premium on each of the other kinds of policy for the same amount at the same age as in Problem 1.

3. A man at the age of 25 took out a 20-year endowment policy for \$4000. He died after having paid five premiums. Not counting interest on premiums paid, how much more did the beneficiary named in the policy receive than the insurance cost?

4. The first annual premium on a \$9000 20-payment life policy was \$592.29. What was the age of the insured?

5. A man at the age of 45 took out a 20-payment life policy for \$3000. He died after having made 10 annual payments. What per cent of the face of the policy was paid in premiums?

6. What is the ratio of the cost of premiums on a 20-year term policy to a 20-year endowment policy for a man 50 years of age?

7. The premium is what per cent of the face of a 20-year endowment policy if the insured is 30 years of age?

8. What is the difference in cost of first premiums paid on an ordinary life policy for a man 20 years old and for a man 60 years old?

9. Many insurance companies pay their policy holders annual dividends, thus reducing the cost of insurance. If the dividend averages $16\frac{2}{3}\%$ of the annual premium, what is the total amount saved on a \$6000 ordinary life policy for a period of 10 years if the insured is 35 years of age?

10. How do you account for the difference in cost of the different kinds of policies?

186. FOOD PROBLEMS

The table below shows the retail prices on ten articles of food in Chicago on June 15 of the years indicated. Work out and fill in the increase blanks, the per cent column to be correct to two decimal places:

	Article	*1914*	*1919*	*Increase over 1914* *Cents*	*Per cent*
1.	Round steak (lb.)	21.8¢	34.6¢	-----	------
2.	Pork chops (lb.)	19.0"	37.5"	-----	------
3.	Bacon (lb.)	31.2"	59.7"	-----	------
4.	Milk (qt.)	8.0"	14.0"	-----	------
5.	Butter (lb.)	30.8"	57.5"	-----	------
6.	Lard (lb.)	15.0"	38.6"	-----	------
7.	Eggs (doz.)	24.4"	51.5"	-----	------
8.	Flour (lb.)	2.9"	7.2"	-----	------
9.	Corn meal (lb.)	2.8"	5.7"	-----	------
10.	Sugar (lb.)	5.0"	9.9"	-----	------

The prices below are comparative wholesale prices from Bradstreet's Journal. Fill in the blanks:

	Article	*1914*	*1919*	*Increase over 1914* *Cents*	*Per cent*
11.	Corn (bu.)	$0.83	$2.11	-----	------
12.	Wheat (bu.)	1.215	2.365	-----	------
13.	Beeves (100 lb.)	10.90	18.25	-----	------
14.	Hogs (100 lb.	7.25	23.00	-----	------
15.	New Orleans Molasses (gal.)	0.50	0.76	-----	------

187. THE BUSINESS OF FARMING

Within recent years scientific farming has been responsible for improved methods along many lines. Some of these improvements are: better treatment of the soil, economy of labor by the use of improved machinery, road building, cooperative organizations and industries for farmers, better buying and marketing conditions, and lower rates of interest on loans necessary to carry on farming operations on the proper scale.

Farming has always been the most important industry in the South, and while scientific methods have been somewhat delayed there are indications that the South is soon to become the most important agricultural section of the country. Almost three-fourths of the land in the South is still unimproved. As more of these unimproved lands are placed under cultivation and methods of farming are improved we may expect the South to become more nearly self-supporting in agricultural productions. The Secretary of Agriculture reported recently that the average farm in one of the Southern states produced less than 2 eggs a week, about $\frac{2}{3}$ pint of milk per day, $\frac{1}{8}$ of a hog per year, $\frac{1}{12}$ of a beef per year, and $\frac{1}{100}$ of a sheep per year. So low is the production along these and other lines that the annual cotton crop is not sufficient to pay for the food that is shipped into the South from other sections. There are large importations of wheat, corn, and oats into a number of the Southern states. Three of these states annually import almost $100,000,000 worth of these grains.

On the next page are the acreages and values of farms, the crop acreages, and the average crop values per year during a period of six years in twelve Southern states.

1. Rank these 12 states with respect to each of the four columns, numbering the state having the highest rank, 1; the one next highest, 2; etc., to the one having lowest rank, 12.

	Acreage of farms	Value of farms	Crop Acreage	Average crop values per yr. for six years
Alabama	20,732,000	$ 370,138,000	7,205,239	$219,332,000
Arkansas	17,416,000	400,089 000	5,376,484	214,904,000
Florida	5,253,000	143,183,000	1,223,078	62,766,000
Georgia	26,953,000	580,546,000	9,662,383	351,860,000
Louisiana	10,439,000	301,220,000	3,586,348	171,092,000
Mississippi	18,557,000	426,314,000	6,158,719	226,443,000
North Carolina	22,439,000	637,716,000	5,737,037	228,384,000
Oklahoma	28,859,000	918,198,000	11,921,570	209,545 000
South Carolina	13,512,000	392,128,000	5,152,847	237,429,000
Tennessee	20,041,000	612,520,000	6,365,143	200,521,000
Texas	112,435,000	2,218,645,000	18,398,092	568,795,000
Virginia	19,495,000	652,065,000	4,256,226	193,579,000

Suggestion: Write the states in the order found in the table. Rule four columns, as in the table, heading them properly. Refer to the table in the text-book, scan the first column, "Acreage of farms," and write the figure 1 in the column opposite the state having the largest acreage; the figure 2 opposite the state having the second largest acreage, etc. Do this for each of the four columns.

2. Do the various states maintain the same rank through the four columns? Add the ranks horizontally for each state. Which state has the lowest total of ranks? Which state would average highest in rank for all of the four columns? Which next highest? Which lowest?

3. Find the average value per acre of the farms for your state.

4. Find the per cent of farm acreage cultivated in crops (crop acreage) for your state.

5. What is the average crop value per year for your state for the six years indicated above?

6. Compare your state as to average crop production per acre with the average for Florida, which ranks first among the 12 states.

188. (Preserving Soil Fertility)

Note to teacher: This section may be omitted without interrupting the sequence. (Data from Mayne and Hatch's Agriculture.)

Soil that has been often cropped usually needs certain plant elements in some form to replace the elements which have been exhausted by the crops from year to year. The most important of these elements are nitrogen, phosphoric acid, and potash. Average barn-yard manure or fertilizer contains these elements in considerable proportions. It has been found that every 1,000 lb. of live weight farm animals and fowls will produce fertilizer as follows:

Animal	No. of tons in 1 year	Value
Horse	12	$40
Cows	14	39
Sheep	9	45
Calves	14	40
Pigs	18	80
Fowls	4	68

This table means that if two calves together weigh 1,000 lb. they will produce fertilizer at the rate of 14 tons a year, and the 14 tons are worth $40.

It has been computed that 2 T. of barnyard fertilizer a year are sufficient for 1 acre of ordinary crops.

1. For how many acres will two horses weighing 1,200 lb. each furnish fertilizer?

2. If a farmer keeps a herd of 8 cows averaging 950 lb. each, what is the value of the fertilizer for 1 yr.? How much land will it fertilize?

3. If the farmer also has 30 sheep averaging 60 lb. each, find the value of the fertilizer and the number of acres it will keep productive each year.

4. If the farmer has 100 hens averaging $4\frac{1}{2}$ lb. each, find the value of the fertilizer.

5. Find the value of the fertilizer produced by 12 pigs with an average weight of 150 lb. How many acres of land will it fertilize?

6. Mr. Smith lives in a town where he has a garden 100 ft. by 108.9 ft. How many chickens averaging $3\frac{1}{8}$ lb. each will Mr. Smith need to keep to supply his garden with the proper amount of fertilizer? What is the value of the fertilizer?

7. Make up a problem similar to the above based on all the farm animals on your own premises or on those of some one you know. Solve the problem.

189. Crops and Soil Fertility

Note to teacher: This section may be omitted without interrupting the sequence.

The farmer may buy fertilizers that are manufactured with any required proportion of the three principal plant foods—nitrogen, phosphoric acid, and potash.

Since nitrogen can be supplied by planting the land in legumes such as clover, or cowpeas, or soy beans, or alfalfa, and turning them under, the plant foods which the farmer usually wishes to buy in commercial fertilizers are phosphoric acid and potash.

The table below shows the number of pounds of each of the three principal plant foods in a ton of twenty-four farm products.

Products—1 ton	No. lb. nitrogen	No. lb. phosphoric acid	No. lb. potash
1. Bran, corn	35.8	20.2	12.4
2. Bran, wheat	49.2	53.8	39.4
3. Butter	2.4	0.8	0.8
4. Buttermilk	12.8	3.4	3.2
5. Corn, cob	7.8	1.2	12.0
6. Corn, grain	33.0	14.2	11.4
7. Corn, stover	12.2	7.6	21.8
8. Cotton seed	58.4	21.0	21.8
9. Cotton-seed meal	145.0	60.8	31.6

	No. lb. nitrogen	No. lb. phosphoric acid	No. lb. potash
10. Cotton-seed hulls	13.4	8.6	20.8
11. Hay, alfalfa	46.8	12.2	35.9
12. Hay, clover	39.4	11.0	37.4
13. Hay, cowpeas	28.6	10.4	29.4
14. Hay, timothy	18.8	6.6	28.4
15. Manure (estimated)	10.1	5.5	9.3
16. Milk	11.6	3.8	3.4
17. Milk, skim	10.0	4.2	4.0
18. Oats	36.4	15.6	9.6
19. Ox, fat	46.6	31.0	3.6
20. Pig, fat	35.4	13.0	2.8
21. Soy Beans	107.2	20.8	25.2
22. Straw, oat	11.6	6.0	35.4
23. Straw, rye	10.0	5.0	17.2
24. Straw, wheat	10.0	4.4	12.6

It is customary with fertilizer dealers and farmers to refer to commercial fertilizers as, "2—8—2", "$\frac{1}{2}$—11—1," etc.; meaning, "2% nitrogen, 8% phosphoric acid, 2% potash," "$\frac{1}{2}$% nitrogen, 11% phosphoric acid, 1% potash," etc. Sometimes the order of the formula is changed to "2—2—8," etc., meaning, "2% nitrogen, 2% potash, 8% phosphoric acid," etc. Pupils should make inquiries as to local custom. Unless otherwise indicated, the order in this text will be nitrogen, phosphoric acid, potash, this being the order used by experiment stations.

If the fertilizer has only phosphoric acid in it, as is true in much that is used in certain sections, it is usually called, "14% acid," or "16% acid," etc., depending upon the per cent of phosphoric acid contained.

1. If the values of the three chief food plants for fertilizing purposes are 25¢ per lb. for the nitrogen, 8¢ per lb. for the phosphoric acid, and 30¢ per lb. for the potash, find the value of 1 ton of a brand of fertilizer with the formula 1—12—2.

NOTE: The values of nitrogen, phosphoric acid, and potash fluctuate from time to time, depending upon the supply and demand.

Use the prices in Problem 1, and solve the following:

2. Find the value of 1 ton of fertilizer with the formula 2—12—0.

3. Which is more expensive, and by how much, a ton of 2—8—2, or a ton of 1—12—2?

4. What is the value of a ton of 16% acid fertilizer?

5. Find the value of a ton of 0—10—2 fertilizer.

6. A farmer cuts and sells 12 tons of clover hay. If the fertilizer value of nitrogen is 25¢ per lb., phosphoric acid 8¢ per lb., and potash 30¢ per lb., what is the value of the soil fertility taken off?

7. If the hay was sold at $30.00 per ton, how much more was received for the hay than the value of the fertilizers taken off?

8. How many tons of manure will be required to replace the phosphoric acid taken off with the hay? Find its value at $5.00 a ton.

9. It is estimated that 160 lb. of clover hay will produce 10 lb. gain in a two-year-old steer. Find the beef value of the clover hay when fed to two-year-old steers, if beeves are worth $12\frac{1}{2}$¢ per lb.

10. It is estimated that for each 100 lb. of clover hay fed steers, 75 lb. of manure are produced. If the manure is worth $4.00 per ton, find the value of the manure saved for the farm by feeding the 12 tons of clover hay to steers.

11. A farmer sold 300 bu. of corn. What was the value of plant food sold off his farm? (Use prices and data on page 276.)

12. How much more valuable is the plant food in a ton of soy beans than in a ton of cow peas? (Use prices and data on page 277.)

13. At an Alabama Experiment Station, a comparison of cowpea hay and wheat bran for dairy feed was made with two lots of cows, as follows:

LOT I	Daily ration per cow	Average daily yield per cow	
		Milk	Fat
Cowpea hay	7.8 lb. @ $36 per ton	17.3 lb.	1.13 lb.
Cotton seed	4.8 lb. @ 50 per ton		
Wheat bran	2.4 lb. @ 60 per ton		
Cotton-seed meal	2.4 lb. @ 65 per ton		
Cotton-seed hulls	9.6 lb. @ 20 per ton		
LOT II.			
Wheat bran	6.1 lb. @ $60 per ton	16.0 lb.	1.02 lb.
Cotton seed	4.8 lb. @ 50 per ton		
Wheat bran	2.4 lb. @ 60 per ton		
Cotton-seed meal	2.4 lb. @ 65 per ton		
Cotton-seed hulls	9.6 lb. @ 20 per ton		

Find the cost of the daily ration per cow for Lots I and II. Which is the greater cost and how much?

14. If the butter fat is separated from the milk and sold at 60¢ per lb., and the skim milk sold to a cheese factory at 3¢ per lb., what is the daily money return per cow for each lot?

15. What is the daily profit per cow for each lot? Which is more economical as dairy feed, cowpea hay or wheat bran?

16. Which is more economical as fertilizer, cotton-seed meal at $65 per ton or cotton-seed hulls at $20 per ton? (See p. 276.)

17. How much more of each kind of plant food is consumed in growing one ton of alfalfa hay than in growing one ton of timothy hay? (See page 277.)

18. How much does it cost in soil fertility to grow a ton of alfalfa hay? (Use prices in Problem 1.)

19. How much more is gained in soil fertility from feeding a ton of alfalfa hay than a ton of timothy hay? (Use prices in Problem 1 and data on page 277.)

20. Inquire prices of alfalfa hay and timothy hay at a feed store, or note the prices in the market reports of the newspapers, and find which is more profitable per ton, considering the soil fertility taken out, a crop of alfalfa hay or a crop of timothy hay? How much?

190. Farming by Machinery

Much of the farm work that was done by hand twenty-five years ago is now done by machinery.

1. Mr. Cox employed three men to do his fall plowing, paying them \$1.25 per day. He now uses a tractor plow operated by one man, whom he pays \$2.00 a day. If the one man can do as much plowing per day as the three formerly did, what is the saving in labor per day?

2. If the fall plowing requires 36 work days, what is the money saving in labor for all the plowing?

3. If the three teams can be used at a cost of \$3.00 per day each, what is the saving on teams and labor for 40 days work, if the cost of operating the tractor is \$6.00 per day?

4. A tractor cost \$500, and the saving on labor and teams was \$120 at the close of the season. What per cent of the cost of tractor was the saving on labor and teams?

5. If the tractor when properly cared for depreciates 20% of its original cost the first year of use; 10% of the reduced value the second year; and 10% of the third value the third year, what will its value be at the end of the third year?

191. INTEREST

1. Find the interest on \$600 at 8% for 2 yr. 6 mo.

Solution: I (Interest) = \$600. × .08 × $2\frac{1}{2}$ = \$120.

2. How many elements are given in this problem? How many are required?

3. Let P represent *principal*; R, *rate*; and T, *time*. Write the formula, or equation, that shows how to find I (interest) when P, R, and T are given.

4. How is R written when used in multiplying or dividing? How is T expressed?

NOTE: Both R and T are considered as abstract numbers.

5. The type of problem above is called the most important in the topic of Interest. Why?

6. Find the interest on \$360 at 6% for 90 days.

7. Find the amount of \$720 at 3% for 60 days.

8. Find the interest on \$150 at $7\frac{1}{2}$% for 1 yr. 6 mo.

9. Which is more, interest on \$300 at $4\frac{1}{2}$% for 1 yr. or interest on \$100 at $4\frac{1}{2}$% for 3 yr.?

10. How much more is the amount of \$300 at $4\frac{1}{2}$% for 1 yr. than the amount of \$100 at $4\frac{1}{2}$% for 3 yr.?

192. Borrowing Money

Men frequently borrow money to carry on their business through a dull or non-productive season. For example, a shoe dealer usually finds the summer a dull season for his business, but the fall and winter seasons require a full stock of goods which must be bought during the summer or preceding winter. He may prefer to borrow money and pay cash for his stock, for the reason that cash discounts on the goods amount to more than interest on the borrowed money. Or, a farmer may need to borrow money in the spring in order to buy seed, fertilizers, etc., for the coming crops. In the case of the shoe

dealer, the borrowed money is paid back with interest after he has sold part or all of his winter shoes, and in the case of the farmer the money is paid back with interest after he has harvested and sold his crops.

1. Mr. Taylor borrows $1,200 at the bank on July 1 at 8%. With the borrowed money he buys $1,200 worth of shoes which are delivered on July 1, "Terms 2/10, n/30," that is, 2% discount for cash, net up to 30 days, or 6% interest after 30 days. On Oct. 1, he pays the bank the borrowed money with interest at 8%. If Mr. Taylor had allowed his shoe bill to remain unpaid until Oct. 1, the wholesaler would have collected interest at the rate of 6% from July 1. How much does Mr. Taylor save by borrowing and paying cash instead of buying on credit?

Solution:

Bill of goods	$1200	
Interest at 6% for 2 mo.	12	
Amount of bill if bought on credit		$1212
Bill of goods	$1200	
Discount at 2%	24	
Amount of bill if bought for cash	$1176	
Interest paid bank at 8% on $1200	24	
Amount of bill if discounted		$1200
Amount saved		$12

2. Mr. Clark borrows $300 from Mr. Cox Feb. 10, 1919, and pays it back Sept. 1, 1919, with interest at 8%. How much did Mr. Clark have to pay?

3. Mr. Clark used the money on Feb. 10 to buy seeds, fertilizers, and implements for his farm. By paying cash he got a discount of 5%. How much did Mr. Clark save by borrowing the $300 instead of buying on credit and paying 6% interest to the seed and implement dealer to Sept. 1st?

4. In finding interest at 6%, some bankers *multiply the principal by the number of days and divide by 6,000.* For

example, in finding the interest on \$720 at 6% for 75 days, the work is as follows:

$$\frac{\$720\times 75}{6000}=\frac{\$54000}{6000}=\$9.$$

This is sometimes called the *Bankers' Method.*

Show that the method is a short cut for the usual longer solution: $\$720\times\frac{6}{100}\times\frac{75}{360}=\$9$.

An easy way to divide by 6000 is to point off three more decimal places and divide by 6. Show why.

5. If the rate of interest in the problem were 5% instead of 6% what part of \$9 would you take to get the interest? What would the answer be? Show how you would correct \$9 for a rate of 7%; of $4\frac{1}{2}$%.

Find by the Bankers' Method the interest at 6% on:

6. \$1200 for 30 days.
7. \$636 for 60 days.
8. \$1600 for 34 days.
9. \$350 for 63 days.
10. \$1706 for 90 days.

Find as above and correct for 8%:

11. \$1600 for 60 days.

Solution: $\frac{\$1600\times 60}{6000}\times\frac{8}{6}$. (Multiplication by $\frac{8}{6}$ corrects for 8%.)

12. \$3200 for 33 days.
13. \$2000 for 117 days.
14. \$ 400 for 93 days.
15. \$ 200 for 63 days.

Find as above and correct for 5%:

16. \$ 650 for 36 days.
17. \$ 800 for 90 days.
18. \$6000 for 114 days.
19. \$ 190 for 120 days.
20. \$ 725 for 96 days.

21. *In order to find the interest on any principal at 6% for 60 days, divide the principal by 100, or move the decimal point two places to the left.*

Illustration: Int. on \$735 for 60 da. at 6% = \$7.35.

This method is sometimes called the *60-day method.*

Show that this method is a short cut for the usual solution:

$\$735 \times \frac{6}{100} \times \frac{60}{360} = \$7.35.$

22. Find the interest on \$960 for 75 days at 6%.

Solution: 75 da. = 60 da. + $\frac{1}{4}$ of 60 da. (15 da.)

Interest for 60 da. = \$9.60

Interest for 15 da. = \$2.40 ($\frac{1}{4}$ of \$9.60)

Interest for 75 da. = \$12.00

Find by the 60-day method the interest at 6% on:

23. \$360 for 70 days.

24. \$720 for 90 days.

25. \$800 for 78 days.

Hint: 78 da. = 60 da. + 12 da. + 6 da.

26. \$3000 for 50 days.

Find as above and correct for 9%:

27. \$2400 for 85 days.

28. \$1230 for 45 days.

29. \$ 780 for 30 days.

30. \$ 680 for 110 days.

193. DOING BUSINESS AT THE BANK

A **bank** is an institution that receives and loans money, and buys and sells commercial papers. There are various forms of banks, such as: **Federal Reserve Banks, National Banks, State Banks, Savings Banks,** and **Private Banks.** In most states a **Trust Company,** which by law may act as administrator, executor, and receiver, is also authorized by law to carry on a banking business.

The newest of the above named kinds of banks are the Federal Reserve Banks. These are the large central banks of our national banking system, whose business it is to issue bank

notes (paper money) and loan money to other banks that are members of the Federal Reserve system.

In order to organize a bank, a group of persons, called **stockholders,** furnish the **capital stock,** or money, necessary to begin with. Usually a large number of persons join in furnishing the capital stock, which is ordinarily divided into **shares** of $100 each. The stockholders elect a **Board of Directors,** which board elects the officers of the bank—the **President, Cashier,** etc. The officers and directors have active charge of the business affairs and policies of the bank. The capital stock is held as a guarantee to **depositors** that money received as **deposits** will be paid when demanded. The deposits constitute the large part of the money in a bank, and it is the loaning of the deposits that usually makes the banking business profitable.

194. **Opening an Account**

FOURTH AND FIRST NATIONAL BANK
NASHVILLE, TENN.

No. 1 Apr. 15, 1920

PLEASE LIST EACH CHECK SEPARATELY	DOLLARS	CENTS
CURRENCY	200	
GOLD		
SILVER	7	50
CITY CHECKS	75	85
DO.	95	10
DO.		
DO.		
DO.		
DO.		
OUT-OF-TOWN CHECKS		
DO.	150	25
DO.		
TOTAL	528	70

CREDIT J. S. Holmes

SEE THAT ALL CHECKS AND DRAFTS ARE ENDORSED

When a person wishes to begin doing business with a bank, he usually opens an account by filling out a **deposit slip,** as shown above, and gives it together with the deposit to the teller or cashier.

1. Who is the depositor? (See deposit slip on page 285.) How much does he deposit? When?
2. What is the name of the bank? Where is it?
3. What does "currency" mean?

The depositor is usually asked to write his name and address on a card or in a signature book, so that the bank may have his signature on file for purposes of identification.

A **bank book,** or **pass book,** is then given the depositor. This book contains the amount and date of the deposit, and should be presented with every subsequent deposit in order that records of deposits may be made. The book serves as the bank's receipt to the depositor for the deposits.

A blank **check book** is also given the depositor to use for withdrawing money from the bank. A check should be written plainly in ink, and the number of dollars should be written on the "Dollars" line as far to the left as possible and the remainder of the space filled with a heavy line. These are precautions against a possible change of the amount by a dishonest holder. Below is a sample check.

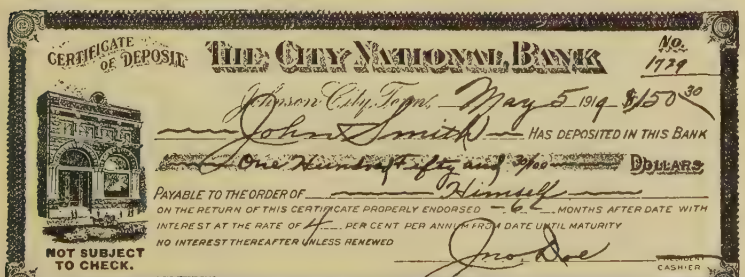
The person who writes the check is the **drawer** or **maker,** the one to whom the money is to be paid is the **drawee** or **payee.**

1. Who is the drawer in the sample check on preceding page? the payee?

2. For how much is the check drawn? What bank is to pay the amount of the check? Where is the bank?

The words "or order" make it possible for the payee to transfer the check to another instead of taking it to the bank himself. In transferring it to another, the payee must write his name on the back of the check. This is known as **indorsing a check,** and means that the payee guarantees the payment of the check. Such indorsement is also required by the bank which pays cash for the check.

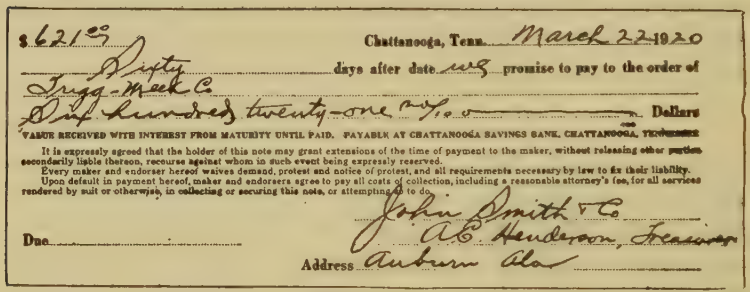
If one does not wish to open an account with a bank, but wishes to deposit his money for safe-keeping, he is given a **certificate of deposit** as evidence of the deposit, instead of a pass book. Such deposit is not subject to check, but may be withdrawn upon presentation of the certificate. If left for a specified period of time, such deposits usually draw interest. Below is a sample of a certificate of deposit.


CERTIFICATE OF DEPOSIT
THE CITY NATIONAL BANK
No. 1729
Johnson City, Tenn. May 5 1914 $150.30
John Smith HAS DEPOSITED IN THIS BANK
One Hundred Fifty and 30/100 DOLLARS
PAYABLE TO THE ORDER OF Himself
ON THE RETURN OF THIS CERTIFICATE PROPERLY ENDORSED 6 MONTHS AFTER DATE WITH
INTEREST AT THE RATE OF 4 PER CENT PER ANNUM FROM DATE UNTIL MATURITY
NO INTEREST THEREAFTER UNLESS RENEWED
NOT SUBJECT TO CHECK.
Jno. Doe
CASHIER

1. To whom is this certificate payable? When is it payable?

2. Would this be called a **demand** certificate of deposit or a **time** certificate of deposit? (In the one case, the certificate is payable upon the demand of the depositor; in the other, at a specified time.)

A **promissory note** is a written agreement to pay a specified amount of money at a specified time or upon demand. It is evidence of a loan, or of an unpaid remainder on a purchase. The **face of the note** is the amount to be paid. The parties to the agreement are the **maker** and **payee**, as in checks; or, **borrower** and **lender**. Below is a sample note.



$621.00 Chattanooga, Tenn. March 22 1920

Sixty days after date we promise to pay to the order of

Trigg-Meek Co.

Six hundred twenty-one 00/100 Dollars

VALUE RECEIVED WITH INTEREST FROM MATURITY UNTIL PAID. PAYABLE AT CHATTANOOGA SAVINGS BANK, CHATTANOOGA, TENNESSEE

It is expressly agreed that the holder of this note may grant extensions of the time of payment to the maker, without releasing other parties secondarily liable thereon, recourse against whom in such event being expressly reserved.

Every maker and endorser hereof waives demand, protest and notice of protest, and all requirements necessary by law to fix their liability.

Upon default in payment hereof, maker and endorsers agree to pay all costs of collection, including a reasonable attorney's fee, for all services rendered by suit or otherwise, in collecting or securing this note, or attempting so to do.

John Smith & Co.

A. B. Henderson, Treasurer

Due

Address Auburn Ala

1. Who is the maker? The payee?

2. When was it written? When is it to be paid?

3. The face of the note usually bears interest at a specified rate. Does this one?

4. A note must show that something of value has passed from the lender to the borrower. What words show it in the note above?

5. May this note be transferred to another? What words show it?

6. Name six essential features of a promissory note.

Checks and notes that may be transferred and collected by some other person or institution than the payee or lender are called **negotiable papers.** Non-negotiable papers are those that must be collected by the payee or lender, and are recognized by the absence of such words as: "the order of," "or order," etc. Indorsement of a negotiable paper is always necessary in order to transfer it. When the payee or lender indorses a check or note, he promises to pay it if the maker does not.

195. Loans and Discounts

When banks loan money for short periods of time, the interest is usually taken out in advance.

Interest paid in advance is called **bank discount,** or **discount.**

The maker of the note, or borrower, receives the face of the note less the interest or discount. This remainder is called the **proceeds.**

1. Mr. Prince borrows \$100 at 8% discount at the City National Bank May 1, 1920, for 3 mo. How much does Mr. Prince get? What is it called? When does he get it? When does the bank get back the loan of \$100? How long has the bank been without its use? How much does the bank make on the loan? When does the bank get the interest or discount?

In some cases interest on a bank loan is not collected in advance, but paid with the face of the note at the date of maturity. If the discount is taken out when the loan is made, the note will not show any rate of discount, but if not paid when due it will begin bearing interest at the legal rate. If interest is collected at the date of maturity, the note shows the rate of interest. A note may be non-interest bearing, in which case it draws no interest unless it remains unpaid at maturity. It then draws the legal rate of interest from the date of maturity.

The term of discount, or the length of time the discount rate

applies, is counted in exact days. Thus, from March 1 to April 1 is 31 days; from April 15 to May 15 is 30 days, etc.

Find the time (term of discount), the discount, and the proceeds on each of the following notes:

	Date of Note	Face	Rate of Discount	Date of Maturity	Time	Discount	Proceeds
1.	Jan. 1, 1920	$ 60	8 %	Mar. 1, 1920			
2.	Feb. 1, 1920	$ 75	6 %	Jun. 1, 1920			
3.	July 1, 1924	$100	$7\frac{1}{2}$%	Aug.15, 1924			
4.	Apr.20, 1920	$150	6 %	Sept. 1, 1920			
5.	May 11, 1923	$300	8 %	July 1, 1923			
6.	Oct. 10, 1919	$160	6 %	Mar. 1, 1920			
7.	April 1, 1920	$175	6 %	July 3, 1920			
8.	Aug. 10, 1919	$200	$7\frac{1}{2}$%	Dec. 1, 1919			
9.	Jan. 12, 1920	$600	8 %	May10, 1920			
10.	Feb.29, 1920	$300	6 %	July 29, 1920			

11. Mr. Black borrows $300 at the bank for 90 days at 8%. How much will he pay the bank at the end of 90 days? How much does the bank get out of the transaction?

12. To whom would Mr. Black make the note for the loan of $300 payable? Could he make it payable to himself and sell it to the bank?

13. J. E. Walker borrows $300 from S. H. Hyder for 90 days, agreeing to pay 6% interest. Mr. Hyder holds Mr. Walker's note, but 30 days before the note is due, Mr. Hyder finds that he must have the money. He takes the note to the First National Bank and offers it for sale. After satisfying itself that Mr. Walker and Mr. Hyder are responsible men, the bank buys the note. That is, the bank discounts the note 30 days before it is due, charging 8% rate of discount. Should the bank pay Mr. Hyder the full amount of the note at the date of its maturity? Note carefully the steps:

(1) The bank finds amt. due at maturity: $\$300 \times \frac{6}{100} \times \frac{90}{360} = \$304.50$.

(2) The bank finds the discount for 30 days on this amt. at 8%: $\$304.50 \times \frac{8}{100} \times \frac{30}{360} = \$2.03$.

(3) The bank subtracts the discount from the amt. = $302.47.

(4) The bank pays Mr. Hyder the remainder, or proceeds.

14. How much would Mr. Hyder have received from Mr. Walker at the end of 90 days? How much did he receive from the bank? When? Whom will Mr. Walker pay? How much? When? How much does the bank make out of the transaction?

NOTE: Banks always discount the maturity value of notes. The length of time from the date of discount to the date of maturity is called the term of discount. The rate of discount is not always the same as the rate of interest specified in the note; it is frequently higher.

15. If you own a note for $600, bearing 6% interest, dated Jan. 1, 1920, due in 1 yr., what will a bank give you for it Oct. 1, 1920, if the rate of discount is 8%?

16. Mr. Jones sells a one-year note for $1000 due Dec. 15, 1923, and bearing 7% interest, to a bank on Sept. 1, 1923, at a discount rate of 8%. How much does Mr. Jones receive?

17. A note for $1500 dated March 1, 1919, to run 90 days at 7%, was discounted April 20, 1919, at 7½%. Find the proceeds.

18. Mr. Cox bought a horse from Mr. Martin, giving in payment his note for $200 dated Feb. 27, 1922, and due in 1 year, with interest at 6%. One month later Mr. Martin discounted the note at the bank at 8%. How much did Mr. Martin receive? How much did the bank make on the deal?

19. The discount at 6% on a 90-day note was $37.50. Can you find the face of the note?

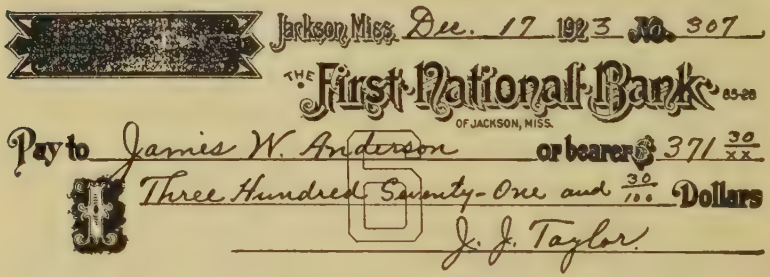
20. A man accepts a note for $850, dated May 1, at 5% interest. The note is due in 90 days. The man discounts the note May 10 at 6%. Find the proceeds.

196. Making Remittances

If a person wishes to make a payment to another person living in some distant place, he may do so by sending the money in a letter, registered or unregistered. Even if registered, it may be lost by accident, as by fire, in which case the government would be responsible only up to the value of $50. It may be stolen by robbers or employees, in which case the government would be responsible for amounts over $50 to the extent only that it is able to apprehend the guilty person and force him to make good the loss.

There are five commonly used ways of making such payments.

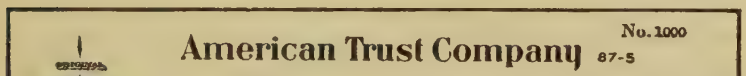
(a) The **check** is a written order drawn by a person, or firm, on a bank directing the payment of a specified amount to the payee named.


Jackson, Miss. Dec. 17 1923 No. 307
THE First National Bank
OF JACKSON, MISS.
Pay to James W. Anderson or bearer $ 371 30/xx
Three Hundred Seventy-One and 30/100 Dollars
J. J. Taylor

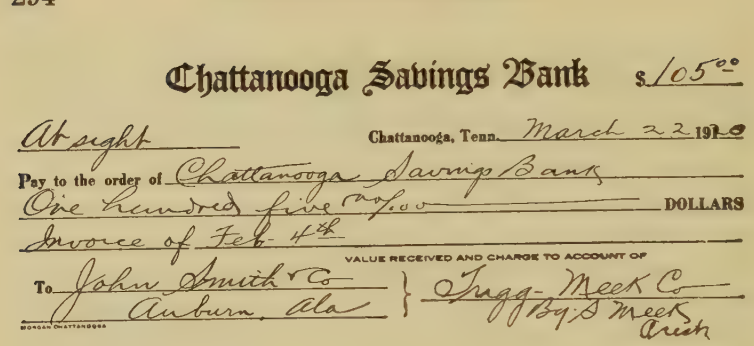
If J. L. Clark of Corinth writes a check on his home bank in favor of C. L. Horton of Cincinnati, Mr. Horton may take it to some bank in Cincinnati and leave it for collection. If the Cincinnati bank finds the check good, it will cash the check if Mr. Horton can be identified, charging him **exchange**, a small sum for investigating and collecting the check. If Mr. Horton is a depositor or customer of a bank in Cincinnati, that bank may

accept the check for collection and, if found good, credit Mr. Horton's account with the amount of the check, or pay him the cash, without making any exchange charge.

(b) A **banker's draft** is an order, or check, drawn by one bank upon another bank directing the payment of a certain sum to the payee named. Usually a bank maintains credit, or keeps deposits, with other banks, called its correspondents, in some larger cities like New Orleans, Atlanta, Chicago, and New York. It sells drafts on those deposits and charges a small exchange depending in amount upon the face of the draft. The form of the banker's draft is shown below:



A **commercial** or **individual draft** is an order, or check, drawn by one person, firm, or company, called the drawer, upon another person, firm, or company, called the drawee, directing the payment of a certain sum to the payee named in the order. The commercial draft is a very common means of collecting overdue accounts. If the Trigg-Meek Co. of Chattanooga has an overdue account against John Smith & Co., of Auburn, Trigg-Meek Co. may draw a draft on John Smith & Co. in favor of Trigg-Meek Co.'s bank in Chattanooga. The form of the draft is shown below:


Chattanooga Savings Bank $105.00

At sight Chattanooga, Tenn. March 22 1910

Pay to the order of Chattanooga Savings Bank

One Hundred Five 00/100 DOLLARS

Invoice of Feb. 4th

VALUE RECEIVED AND CHARGE TO ACCOUNT OF

To John Smith & Co
Auburn, Ala

Trigg-Meek Co
By S Meek
Asst

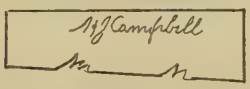
The commercial draft is made payable at sight, or at some number of days after sight. It is deposited in Trigg-Meek Co's. local bank. The Chattanooga bank sends it to an Auburn bank which presents it to John Smith & Co. If it is a sight draft and John Smith & Co. pays it to the Auburn bank, the Auburn bank sends the amount, less a fee for collecting, to the Chattanooga bank, which bank then pays Trigg-Meek Co. the money or credits it to their account. If the draft is a time draft and if John Smith & Co. agrees to pay it at the time specified, he writes the word "Accepted" and his name and the date across the face of the draft. The Auburn bank presents the draft again at the indicated time for payment, and remits to the Chattanooga bank. Sometimes Trigg-Meek Co., upon receiving an order for goods from John Smith & Co. may bill the goods to themselves—Trigg-Meek Co.—at Auburn, and send the bill of lading and a draft for the amount of the goods to an Auburn bank for collection. If John Smith & Co. pays the draft, the bank will give him the bill of lading with which he may get the goods from the freight agent. In this way Trigg-Meek Co. would run no risk in collecting from an unreliable or unknown customer.

197. Indorsements

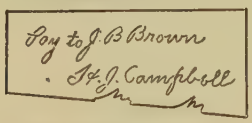
When a person wishes to get the cash on a check, he writes his name on the back. If he writes only his name, it is called a **blank indorsement**; if he gives direction as to whom payment is to be made and signs his name, it is called an **indorsement in full**; if the indorser is not willing to become responsible for payment, that is, pay himself in case the drawer fails to pay, he writes the words **without recourse,** and signs his name.

Below are samples of the three kinds of indorsements mentioned:

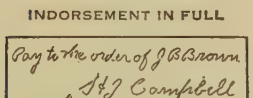
BLANK INDORSEMENT



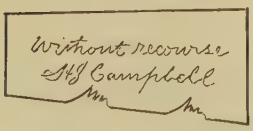
INDORSEMENT IN FULL



INDORSEMENT IN FULL



WITHOUT RECOURSE



To transfer a draft, a check, or a note, any one of the above forms of indorsement may be used.

(c) The **postal money order** is drawn by one postmaster on another and directs payment to a designated payee who must be identified in order to get the money. The fees charged for the money order are paid to the postmaster issuing it and are as follows:

For orders for sums not exceeding	$ 2.50	3¢
For orders over $2.50 and not exceeding	5.00	5¢
For orders over 5.00 and not exceeding	10.00	8¢
For orders over 10.00 and not exceeding	20.00	10¢
For orders over 20.00 and not exceeding	30.00	12¢
For orders over 30.00 and not exceeding	40.00	15¢
For orders over 40.00 and not exceeding	50.00	18¢
For orders over 50.00 and not exceeding	60.00	20¢
For orders over 60.00 and not exceeding	75.00	25¢
For orders over 75.00 and not exceeding	100.00	30¢

It is not necessary that the money paid to one postmaster actually be sent to the other postmaster. The United States government merely receives the money at one post office and pays at another.

(d) The **express money order** is an order from one of the company's agents to another to pay a specified amount to a designated payee. The fees are approximately the same as those given for post-office money orders.

(e) The **telegraphic money order** is an order by wire from one of the company's agents to another to pay a specified amount to a person named in the telegram. The fees depend upon the distance between the offices involved. This method of sending money is used when haste is desired, and the fees are higher than for other kinds of drafts and orders.

198. Problems

1. What is the premium, or fee, at $\frac{1}{4}\%$ on a New Orleans draft for $500? Find the total cost to the purchaser.

2. The draft referred to in Problem 1 was bought by J. R. Austin in Corinth, and sent by him to the Alkahest Lyceum System in Atlanta. Why should Mr. Austin send a draft instead of a check? Would the Corinth bank have a record of the check? Would it have a record of the draft? Would the Atlanta bank to which the Alkahest System presents the check know whether Mr. Austin had $500 in the Corinth bank? How would it find out?

3. A 60-day draft for $750 dated May 1, 1920, was accepted May 10, 1920. Find the discount at 6% from the time it was accepted until due.

NOTE: Remember that when a time draft is accepted it really becomes a promissory note, and discount is computed upon its face value unless interest is specified.

4. Miller Bros., Little Rock, drew a 30-day draft on John Doe & Co., Bellville, Ark., in favor of the First National Bank, Little Rock, for $860.50, dated March 2, 1920. Write out the draft in proper form.

5. Accept the draft for the proper party, as of March 5, 1920.

6. The First National Bank discounts the draft March 5 at 6%. Find the proceeds. Who gets the proceeds?

7. Find the fees on the following postal money orders: (a) $16.50; (b) $87.25; (c) $43.10; (d) $1.25.

8. Find the cost, including the fee, of a New York draft for $600, premium 0.1%.

9. If you wish to send $75.00 in payment for some goods, which method would you use if the premium on a draft is 0.1%, and if you do not wish to send a check?

10. H. R. South & Co., Montgomery, sold a bill of goods amounting to $740.50 to C. N. Smith & Bro., Tuscumbia, on March 2, 1920. Terms 2/10; n/30. On April 1, H. R. South & Co., not having had a remittance, drew a 30-day draft on C. N. Smith & Bro. for the amount of the bill including interest at 6% from April 1. It was accepted April 3.

(a) Write out the proper form for the draft, making the City National Bank the payee. (b) Make the proper notation to show acceptance.

11. Find the proceeds if H. R. South & Co. discounted the draft, April 4, at the City National Bank at 6%.

199. STOCKS AND BONDS

W. J. Souder wishes to have a creamery in his community. He considers three ways of operating it. First, he might put in all the money needed to provide the buildings, grounds and equipment, and manage the business himself, hiring whatever help he needs at so much per day. Second, he might persuade one or two of his neighbors to join him, the two or three men to be **partners,** each paying in a part of the money needed to equip and operate the plant. For example, if Mr. Cox and Mr. Sanders join Mr. Souder in the business, the three men would be partners and form a **partnership.** The firm name might be Souder, Cox & Sanders. Third, Mr. Souder might get several of his neighbors to join him and elect a board of directors for the transaction of the business. These directors would secure a charter under the laws of the state, which would empower the organization to transact business as an individual. Such an organization is called a **corporation.**

Let us suppose that $6,000 will be necessary to equip the plant and start the creamery. This amount is called **capital.** If Mr. Souder furnishes all the capital and manages the business alone, he would be entitled to all the profits; and if the creamery failed to succeed, Mr. Souder would be liable for all the debts.

If Souder, Cox & Sanders go into the business on an equal basis, each would furnish $2,000 of the capital, and each would share $\frac{1}{3}$ of the profits, or $\frac{1}{3}$ of the debts if the business failed. The law also provides that all the partners are bound by the acts of any one of them in connection with the business, and that in case of a failure if some of the partners are unable to pay their share of the debts, the member or members of the firm able to pay must pay the debts of the entire firm.

If Mr. Souder persuades 29 of his neighbors to join him in the creamery, the 30 men would probably organize themselves into a stock company or corporation according to laws prescribed

by the state. A few of the men would be elected **directors** to have active charge of the creamery.

The $6000 **capital stock** might be divided into 60 equal **shares.** Each share would have a **par value** of $100. The net profits of the business would be divided among the 30 **stock-holders** as dividends, and in case of a failure in business the law provides that no stock-holder shall be liable for more than the face value of his stock.

If Mr. Martin, one of the stock-holders, wishes to sell his stock after the business has been in operation for some time, he might get more or less than $100 per share, depending upon the degree of success of the business and the consequent size of the dividends likely to be **declared.** That is, the stock might sell **above par** or **below par.**

200. Problems

1. Souder, Cox, and Sanders form a partnership, each putting in $2000. If the profits above expenses are $900, how much should each of the partners receive in profits?

2. If Souder puts in $3000, Cox $2000, and Sanders $1000, how much should each receive of the $900 profits?

3. Suppose Souder, Cox, and Sanders should put in $2500, $2000, and $1500 respectively, how should $800 in profits be divided among them?

4. A, B, C, and D formed a partnership. A put in $1200; B, $1500; C, $1300; and D, $2500. If the profits are $1000, how much of the profits should each partner get?

5. What are the advantages of the stock company over the individual or partnership method of carrying on a business which depends upon co-operation for its success?

6. If Mr. Mart sells two shares to Mr. Smith, who already owns two shares, what part of the dividends should Mr. Smith then get, if the capital stock is $6000?

7. Suppose Mr. Souder had found 120 persons who were interested in the creamery and had divided the $6000 capital stock into 120 equal shares, what would the par value of each share be?

8. Find the par value of 10 shares if there are 240 shares of the entire capital stock of $6000.

9. (a) Suppose the creamery should make a net profit of $500 above all expenses, what per cent of the capital stock is the net profit?

(b) Find the dividend that should be paid a stock-holder who owns 1 share with a par value of $50; 6 shares; 10 shares.

If one person carries on a business of any kind alone he is responsible for all the debts, and is entitled to all the profits of the business. If two or more persons engage in a business together they may form a partnership, which is usually known by a firm name. The partners have an agreement as to the services, money, etc., contributed by each, and as to the share of the profits each is to receive. The business acts of any one member of the firm bind all members of the firm, and each is personally liable for the entire indebtedness of the firm.

If the persons engaging in a business wish to limit their financial responsibility to the extent of their investment only, and if a large number of persons wish to make investments in money but not in personal services, a **stock company** or **corporation** is organized. This organization usually carries on business on a larger scale than a partnership, and requires a larger capital. The organization is formed under state laws, and has the same legal rights and responsibilities as an individual. It may buy and sell property, sue and be sued for debts, etc.

The **Articles of Incorporation,** which must be available to the public, state the purposes of the corporation, the **capital stock,** the number of **shares** into which it is divided, the **par**

value of each share, the number of shares each **stock-holder** owns etc. **Certificates of stock** are issued to the stock-holders as evidence of the number of shares owned and the amount paid into the treasury. The owner of each certificate is entitled to participate in the profits, and to have a voice in the election of **directors,** who in turn elect the **officers** to have active charge of the business.

If the business is prosperous, a part of the profits is usually distributed among the stock-holders as **dividends.** If the business is carried on at a loss, it may be advisable to make an assessment upon each stock-holder in proportion to the number of shares owned. Some stock is by law non-assessable, in which case the holder cannot be assessed. Dividends and assessments are usually expressed as so many per cent of the face value of the stock.

Sometimes a corporation issues **preferred stock,** in which case the other stock is called **common stock.** Holders of preferred stock have first chance on the profits and receive a fixed rate of dividend if that amount is earned. Dividends on common stock are paid out of the profits remaining after preferred dividends are paid, if any such profits remain.

When the dividend rates amount to a higher rate per cent on the face value of the stock than the prevailing rates of interest, the stock will sell for more than its face value. The stock is then said to be **above par,** or **at a premium.** If the dividend rates are less than the average interest rates, the stock will sell for less than its face value, and is said to be **below par,** or **at a discount.** The market value of stock is usually stated as so many per cent of the face value. It is sometimes stated, however, as so many dollars per share, the face value of a share being $100 unless otherwise specified.

When a corporation wishes to borrow money, it may issue and sell **bonds,** which are really interest bearing notes with the property of the corporation as security. Bonds are issued

similarly by the national government, by states, by counties, by cities, by special school districts, etc., which are recognized by law as corporations. Such bonds bear a specified rate of interest, and the corporation redeems them by means of taxation, usually through a **sinking fund** set apart specifically for the purpose.

The chief differences between shares of stock and bonds are:

(1) A share of stock is evidence of ownership of property, while a bond is a note whose payment is guaranteed by the property;

(2) The income from a share of stock depends upon the earnings of the corporation, while a bond bears a fixed rate of interest;

(3) A dividend on a share of stock is not due until declared so by the officers and directors, while the interest on a bond is due and payable at fixed intervals of time.

When bonds are made payable to the owner or some one to whom he may transfer them, they are registered on the books of the company and are called **registered bonds.** When they are made payable to bearer, the interest is paid upon presentation at the proper time of attached **coupons,** and the bonds are called **coupon bonds.**

Persons who make it their business to keep closely in touch with the market conditions affecting corporation stocks and bonds for the purpose of advising others and of buying and selling stocks and bonds for others are called **brokers.**

The fee charged by the broker is called **brokerage.** It is always a certain per cent of the face value of the stock, and is practically always $\frac{1}{8}\%$, either for buying or selling.

Persons who make their business that of dealing in stocks and bonds usually form an organization known as a **stock exchange.** These exchanges are located in the larger cities, where brokers buy and sell for customers in all parts of the country.

Commercial Club Corporation
NASHVILLE TENNESSEE
AUTHORIZED CAPITAL $65,000.00. Preferred $25,000.00; Common, $30,000.00
This Certifies that John Doe
is the owner of
shares of the par value of $25.00 each of the PREFERRED STOCK of the
COMMERCIAL CLUB CORPORATION, transferable only on the books of the corporation, by the holder hereof, in
$25.00
EACH

Certificate of Stock

Stock quotations may be found in most daily newspapers, in financial journals, and in various commercial reports. A quotation of 125, for example, means that a share of the stock, whose face value is $100, will sell for $125 plus the brokerage which is usually $\frac{1}{8}\%$ of $100.00, or $12\frac{1}{2}¢$. That is, a share of the stock will cost $\$125.12\frac{1}{2}$ if a man is buying, and will bring $\$124.87\frac{1}{2}$ ($\$125 - 0.12\frac{1}{2}$) if a man is selling.

The table below was taken from a daily newspaper, and shows stock sold in New York on a certain date with the highest and lowest prices for that date. All quotations are on stocks of $100.00 par or face value.

Stocks	Highest	Lowest
American Beet Sugar	80	$79\frac{7}{8}$
American Sugar	$124\frac{3}{4}$	124
Baltimore & Ohio	37	$36\frac{1}{8}$
Coca-Cola Company	$37\frac{1}{8}$	36

Stocks	Highest	Lowest
Cuba Cane Sugar	$41\frac{1}{2}$	$41\frac{1}{4}$
General Motors	$238\frac{1}{2}$	$233\frac{1}{8}$
Great North. pfd.	$78\frac{3}{8}$	$77\frac{1}{2}$
Missouri Pacific	30	29
Norfolk & Western	$94\frac{1}{2}$	93
Okla. Prod. & Rfg.	$5\frac{1}{2}$	$5\frac{1}{8}$
Seaboard Air Line	9	9
Seaboard Air Line pfd.	$18\frac{3}{8}$	18
Southern Railway	$25\frac{7}{8}$	25
Southern Railway pfd.	57	$56\frac{3}{4}$
Tennessee Copper	10	$9\frac{7}{8}$
Texas Company	$177\frac{3}{4}$	173
United States Steel	$95\frac{3}{8}$	$94\frac{3}{8}$
United States Steel pfd.	111	$110\frac{1}{4}$

201. Problems in Stocks

Find the cost of the following stocks, par value \$100, including $\frac{1}{8}\%$ brokerage, at highest price quoted in the table above:

1. 75 shares American Beet Sugar.

2. 60 shares Baltimore and Ohio.

3. 120 shares Coca-Cola Company.

4. 300 shares Seaboard Air Line pfd. (preferred).

5. 800 shares Tennessee Copper.

6. If Mr. Martin bought 600 shares of American Beet Sugar at the lowest quotation ($79\frac{7}{8}$), how much did they cost him, including brokerage at $\frac{1}{8}\%$?

Solution: Cost $= 600 \times (\$79\frac{7}{8} + \$\frac{1}{8}) = 600 \times \$80 = \$48{,}000$.
The brokerage is always computed on the face value of the stock.
Brokerage $= \frac{1}{8}\%$ of $\$100 = 0.00\frac{1}{8} \times \$100.00 = \$\frac{1}{8}$, on each share.

7. If Mr. Martin sold the 600 shares at the highest price, how much did he receive after paying the brokerage of $\frac{1}{8}\%$?

8. Did Mr. Martin gain or lose on the two transactions? How much?

9. How much did the brokers make for buying and selling the 600 shares?

10. A man bought 100 shares of Coca-Cola Company stock at the lowest price and sold it at the highest; how much did he make after paying the brokerage of $\frac{1}{8}\%$ for purchases and sales?

11. A man bought 50 shares of the Southern Railway pfd. stock at the highest quotation. Find the cost including brokerage at $\frac{1}{8}\%$.

12. If the Southern Railway pfd. stock pays an annual dividend of 6%, what per cent is the man who bought the stock at the highest price quoted making on his investment?

NOTE: A dividend of 6% means 6% on the face value of the stock. How much would this amount to on a $100 share? What was the investment (cost) per share?

13. What is the cost of 50 shares of Southern Railway, brokerage $\frac{1}{8}\%$, if bought at the highest price?

14. If the Southern Railway stock pays an annual dividend of 3%, what is the per cent profit on the investment?

15. How much does a man receive who sells 100 shares of Southern Railway at the highest price, after his broker is paid?

16. The capital stock of a company is $500,000. One-fourth of the capital stock is preferred and guarantees 6% dividends if the earnings amount to that much. $22,500 is set apart as dividends. Find the rate of dividend paid on the common stock.

17. A creamery company with a capital stock of $15,000 declares a dividend of $4\frac{1}{2}\%$. What is the amount of the dividend? How much will a man receive in dividends if he owns 12 shares of $100 each?

18. If Seaboard Air Line pfd., bought at highest price, pays 6%, and the common stock pays 3%, which is the better investment? How much better?

The table below gives quotations on some bonds in New York on a given date:

New York Bonds

U. S. 2's, reg.	$100\frac{1}{2}$	L. & N. 4's	$81\frac{1}{2}$
U. S. 2's, coup.	$100\frac{1}{2}$	M. K. & T. 1st, 4's	$57\frac{1}{2}$
U. S. 4's, coup.	106	Nor. Pac. 4's	$74\frac{1}{2}$
U. S. 4's, reg.	106	Southern Railway 5's	$84\frac{5}{8}$
Anglo-French 5's	$97\frac{1}{2}$	Tex. Co. 6's	$100\frac{1}{4}$
C. of Ga. Csl. 5's	81	Union Pacific 4's	82
Cent. Leath. 5's	$94\frac{1}{2}$	U. S. Steel 5's	$97\frac{1}{4}$
City of Paris 6's	$90\frac{1}{2}$	N. C. & St. L. 5's	93

Problems

1. How much interest does a bond of U. S. 2's, reg. bear?

NOTE: U. S. 2's, reg. means United States registered bonds, bearing 2% interest. U. S. 2's, coup. means United States coupon bonds, bearing 2% interest

How much interest does a bond of U. S. 4's, reg. bear?

2. Which is the better investment at the quotations above, U. S. 2's, reg. or U. S. 2's, coup.?

3. Which is the better investment at the quotation above, U. S. 4's, reg. or U. S. 4's, coup.?

4. Which is the better investment at the prices quoted, a U. S. 2's, reg. or a U. S. 4's, reg., brokerage not considered?

Hint: The income for the first is \$2 on \$100.50; for the second \$4 on \$106.

5. How much will 10 bonds of C. of Ga. Csl. 5's cost at the quotation in the table, brokerage $\frac{1}{8}\%$?

6. What is the annual income on each of the C. of Ga. Csl. 5's bonds? What per cent is this income on the investment?

Suggestion: $r = \$5 \div \$81\frac{1}{8}$.

7. Find the cost of 5 bonds of So. Ry. 5's. Find the per cent of interest on the investment.

8. Which is the better investment, Cent. Leath. 5's or L. & N. 4's?

9. Compare likewise L. & N. 4's and U. S. Steel 5's.

10. How many U. S. 4's, reg. can be bought for \$1273.50, brokerage $\frac{1}{8}\%$?

202. Liberty Bonds

There were five issues of government bonds during the Great War. These were known as Liberty Loans or Liberty Bonds. All issues bear simple interest payable semi-annually. Some of the bonds are coupon bonds and others are registered. Coupon bonds were issued in denominations of various sizes from \$50 to \$1000, and registered bonds in sizes from \$100 to \$10,000.

The table below shows year of issue, year of maturity, and rate of interest of each of the five loans.

Loan	Year of Issue	Year of Maturity	Rate of Interest
First cvd	1917	1947	$4\frac{1}{4}\%$
Second	1917	1942	4%
Third	1918	1928	$4\frac{1}{4}\%$
Fourth	1918	1938	$4\frac{1}{4}\%$
Victory	1919	1923	$4\frac{3}{4}\%$

NOTE: The First Loan Bonds called for a rate of $3\frac{1}{2}\%$, with provision for conversion into $4\frac{1}{4}\%$ bonds later. First cvd, above, means First Loan Bonds converted into the $4\frac{1}{4}\%$ bonds.

1. Find the interest per year on a bond of \$100 for each of the five loans.

2. If the coupons each represent the interest for a half year, how many coupons will each of the five different bonds have attached to it?

3. How much interest will each coupon of the various bonds call for?

4. If none of the coupons of a Victory Bond of $100 denomination were detached and cashed until the bond matured, how much was it worth?

5. Is there any reason for a Victory Bond selling higher than any other of the same size?

Below are the quotations on the various Liberty Bonds on a certain day in New York, with the term in years each issue is to run:

Loan	Quotation	Term in yrs.
First cvd $4\frac{1}{4}$s	98.27	30
Second 4s	97.16	25
Third $4\frac{1}{4}$s	98.31	10
Fourth $4\frac{1}{4}$s	98.74	20
Victory $4\frac{3}{4}$s	100.00	5

6. If Mr. Simpson sold 6 bonds of the Third Loan, each $100 in size, how much did he get for them at the price quoted, brokerage $\frac{1}{8}\%$?

7. Which is a better investment at the prices quoted above, a bond of the Second Loan or one of the Fourth Loan?

8. Which investment would pay better, a Fourth Loan or Texas Co. 6's at $100\frac{1}{4}$? Which is safer? Why?

9. Can you explain why Liberty Bonds, as well as bonds in general, fluctuate less in market prices than shares of stock in corporations?

10. Find the rate of income on the investment in a bond of the Fourth Loan.

11. Find the rate of income on a First Loan investment.

203. NATIONAL FARM LOAN ASSOCIATIONS

Organization

1. By the Federal laws a national farm loan association may be organized in any community where ten or more citizens owning land desire to borrow money from the government equal to a total amount not less than $25,000. Individual loans may be as small as $100 or as large as $25,000.

2. Each citizen who joins must pay as a subscription to the stock of the association 5% of the loan he secures. This is held by the local association as security until the farmer pays his debt, when the 5% subscription is returned to him. All members of the association must be applicants for loans.

3. Federal land banks have been established throughout the country for the purpose of carrying the law into effect.

4. The following table shows the number designating each Federal land bank district, the states included in the district, and the cities in which the banks are located.

Dist. No.	STATES	Federal land bank at—
1	Maine, Massachusetts, Vermont, New Hampshire, Rhode Island, Connecticut, New Jersey, and New York.	Springfield, Mass.
2	Pennsylvania, West Virginia, Maryland, Delaware, Virginia, and District of Columbia.	Baltimore, Md.
3	North Carolina, South Carolina, Georgia, and Florida	Columbia, S. C.
4	Indiana, Ohio, Kentucky, and Tennessee	Louisville, Ky.
5	Louisiana, Mississippi, and Alabama	New Orleans, La.
6	Illinois, Missouri, and Arkansas	St. Louis, Mo.
7	North Dakota, Minnesota, Wisconsin, and Michigan	St. Paul, Minn.
8	Wyoming, Nebraska, South Dakota, and Iowa	Omaha, Neb.
9	New Mexico, Kansas, Colorado, and Oklahoma	Wichita, Kans.
10	Texas	Houston, Tex.
11	California, Nevada, Utah, and Arizona	Berkeley, Calif.
12	Idaho, Washington, Montana, and Oregon	Spokane, Wash.

Persons desiring to get in touch with any of these banks should address them in this style: "Federal Land Bank of Louisville, Louisville, Ky." The banks are equipped with all the necessary literature to accommodate inquirers.

204. Limitations

1. No loan may be made except upon the security of first mortgages.

2. The amount of the mortgage cannot exceed one-half the appraised value of the land and 20% of the permanent improvements thereon, which must be insured.

3. The proceeds of the loan must be used for the extinguishment of pre-existing indebtedness or for productive purposes which include the purchase of live stock, fertilizers, equipment, and improvements.

4. Every mortgage must contain an agreement to pay off the debt (principal and interest) in fixed annual or semi-annual instalments.

5. The amount of each instalment may be fixed by the borrower, but cannot be less than sufficient to pay off the debt in 40 years, nor greater than to pay it off in 5 years.

6. The rate of interest charged any borrower cannot exceed 6% per annum.

7. The borrower cannot be called upon to pay the debt except by the instalments he originally fixes unless he defaults, but after five years he may pay off the whole or any portion at his option at any instalment period. Under the amortization plan, the term of the loan and the amount of each instalment are relative; determining one fixes the other.

205. Amortization

1. Amortization is the term applied to the process of paying off an indebtedness by instalment payments of a fixed amount, which includes interest and a part of the principal, throughout a period of years. Amortization means *killing mortgages.*

2. The Federal Farm Loan Board has encouraged the Federal Land Banks in establishing a standard 35-year loan. It has done this because this sort of loan is one which simplifies

the bookkeeping, makes the matter of payments and amortization plain to everybody, and gives every borrower ample opportunity to adjust his business affairs. It spreads the privilege of payment over a generation in time, and meets every want of almost every borrower.

3. The following amortization tables have been approved by the Federal Farm Loan Board for the use of the land banks and for the guidance of farmers who borrow under this system.

TABLE I.

(A loan of $1,000 at 5½ per cent interest payable in 35 years by means of annual instalments of $65, which includes interest and part of the principal.)

Payment Number	Instalment	Interest	Applied on principal	Principal still unpaid	Payment Number	Instalment	Interest	Applied on principal	Principal still unpaid
1	$65 00	$55.00	$10.00	$990.00	19	$65.00	$38.79	$26.21	$678.98
2	65.00	54.45	10.55	979.45	20	65.00	37.35	27.66	651.32
3	65.00	53.87	11.13	968.32	21	65.00	35.82	29.18	622.14
4	65.00	53.26	11.74	956.58	22	65.00	34.22	30.78	591.36
5	65.00	52.61	12.39	944.19	23	65.00	32.52	32.48	558.88
6	65.00	51.93	13.07	931.12	24	65.00	30.74	34.26	524.62
7	65.00	51.21	13.79	917.33	25	65.00	28.85	36.15	488.47
8	65.00	50.45	14.55	902.78	26	65.00	26.87	38.13	450.34
9	65.00	49.65	15.35	887.43	27	65.00	24.77	40.23	410.11
10	65.00	48.81	16.19	871.24	28	65.00	22.56	42.44	367.67
11	65.00	47.92	17.08	854.16	29	65.00	20.22	44.78	322.89
12	65.00	46.98	18.02	836.14	30	65.00	17.76	47.24	275.65
13	65.00	45.99	19.01	817.13	31	65.00	15.16	49.84	225.81
14	65.00	44.94	20.06	797.07	32	65.00	12.42	52.58	173.23
15	65.00	43.84	21.16	775.91	33	65.00	9.53	55.47	117.76
16	65.00	42.68	22.32	753.59	34	65.00	6.48	58.52	59.24
17	65.00	41.45	23.55	730.04	35	62.50	3.26	59.24	-------
18	65.00	40.15	24.85	705.19					
						2,272.50	1,272.50	1,000.00	-------

Illustration: Mr. A borrows $1000 which he wishes to repay, including interest, in 35 yr. by annual instalments of $65. The first year he pays $65, of which $55 is applied on interest and $10 on the principal, which reduces the principal from $1000 to $990 for the second year, etc., until the thirty-fifth year, when he pays $62.50, of which $3.26 is applied on interest and $59.24 on the principal, which reduces the principal to nothing.

TABLE II.

(A loan of $1,000 at 5½ per cent interest repayable in 34½ years by means of semi-annual instalments of $32.50 which includes interest and part of the principal.)

Payment Number	Instalment	Interest	Applied on principal	Principal still unpaid	Payment Number	Instalment	Interest	Applied on principal	Principal still unpaid
1	$32.50	$27.50	$5.00	$995.00	36	$32.50	$19.58	$12.92	$699.02
2	32.50	27.36	5.14	989.86	37	32.50	19.22	13.28	685.74
3	32.50	27.22	5.28	984.58	38	32.50	18.86	13.64	672.10
4	32.50	27.08	5.42	979.16	39	32.50	18.48	14.02	658.08
5	32.50	26.93	5.57	973.59	40	32.50	18.10	14.40	643.68
6	32.50	26.77	5.73	967.86	41	32.50	17.70	14.80	628.88
7	32.50	26.62	5.88	961.98	42	32.50	17.29	15.21	613.67
8	32.50	26.45	6.05	955.93	43	32.50	16.88	15.62	598.05
9	32.50	26.29	6.21	949.72	44	32.50	16.45	16.05	582.00
10	32.50	26.12	6.38	943.34	45	32.50	16.01	16.49	565.51
11	32.50	25.94	6.56	936.78	46	32.50	15.55	16.95	548.56
12	32.50	25.76	6.74	930.04	47	32.50	15.09	17.41	531.15
13	32.50	25.58	6.92	923.12	48	32.50	14.61	17.89	513.26
14	32.50	25.39	7.11	916.01	49	32.50	14.11	18.39	494.87
15	32.50	25.19	7.31	908.70	50	32.50	13.61	18.89	475.98
16	32.50	24.99	7.51	901.19	51	32.50	13.09	19.41	456.57
17	32.50	24.78	7.72	893.47	52	32.50	12.56	19.94	436.63
18	32.50	24.57	7.93	885.54	53	32.50	12.01	20.49	416.14
19	32.50	24.35	8.15	877.39	54	32.50	11.44	21.06	395.08
20	32.50	24.13	8.37	869.02	55	32.50	10.86	21.64	373.44
21	32.50	23.90	8.60	860.42	56	32.50	10.27	22.23	351.21
22	32.50	23.66	8.84	851.58	57	32.50	9.66	22.84	328.37
23	32.50	23.42	9.08	842.50	58	32.50	9.03	23.47	304.90
24	32.50	23.17	9.33	833.17	59	32.50	8.38	24.12	280.78
25	32.50	22.91	9.59	823.58	60	32.50	7.72	24.78	256.00
26	32.50	22.65	9.85	813.73	61	32.50	7.04	25.46	230.54
27	32.50	22.38	10.12	803.61	62	32.50	6.34	26.16	204.38
28	32.50	22.10	10.40	793.21	63	32.50	5.62	26.88	177.58
29	32.50	21.81	10.69	782.52	64	32.50	4.88	27.62	149.80
30	32.50	21.52	10.98	771.54	65	32.50	4.12	28.33	121.50
31	32.50	21.22	11.28	760.26	66	32.50	3.34	29.16	92.34
32	32.50	20.91	11.59	748.67	67	32.50	2.54	29.96	62.38
33	32.50	20.59	11.91	736.76	68	32.50	1.72	30.78	31.90
34	32.50	20.26	12.24	724.52	69	32.47	.87	31.60	-------
35	32.50	19.92	12.58	711.94		$2,242.47	$1,242.47	$1,000.00	-------

1. How does the amount applied on interest vary from year to year?

2. How does the amount applied on the principal vary from year to year?

3. How does the principal vary from year to year?

4. Explain each of these variations.

206. Making Additional Payments

1. The Federal Farm Loan Act gives the farmer the privilege of paying any amount he wishes, from $25 up, to be applied on the principal of his loan after the lapse of five years, on the date of any payment called for by his contract. Table III shows how such payments reduce the amount of the loan and shorten the term.

TABLE III.

(A 35-year, amortization loan of $1,000 at 5½ per cent interest, but with the privilege of repaying succeeding sums that would be regularly applied on the principal.)

Payment No.	Instalment	Interest	Applied on principal	Principal still unpaid	Additional payment	Principal still unpaid
1	$65.00	$55.00	$10.00	$990.00		
2	65.00	54.45	10.55	979.45		
3	65.00	53.87	11.13	968.32		
4	65.00	53.26	11.74	956.58		
5	65.00	52.61	12.39	944.19	$41.41	$802.78
6 (9)	65.00	49.65	15.35	887.43		887.43
7	65.00	48.81	16.19	871.24		871.24
8	65.00	47.92	17.08	854.16		854.16
9	65.00	46.98	18.02	836.14	39.07	797.07
10 (15)	65.00	43.84	21.16	775.91		775.91
11	65.00	42.68	22.32	753.59		753.59
12	65.00	41.45	23.55	730.04	107.90	622.14
13 (22)	65.00	34.22	30.78	591.36		591.36
14	65.00	32.52	32.48	558.88	70.41	488.47
15 (26)	65.00	26.87	38.13	450.34		453.44
16	65.00	24.77	40.23	410.11		410.11
17	65.00	22.56	42.44	367.67	92.02	275.65
18 (31)	65.00	15.16	49.84	225.81		225.81
19	65.00	12.42	52.58	173.23	55.47	117.76
20 (34)	65.00	6.48	58.52	59.24		59.24
21	62 50	3.26	59.24			
	$1,362.50	$768.78	$593.72		$406.28	

2. The advantages of using this method of paying off ahead of time are perfectly plain to any borrower. The amount of interest paid is reduced from $1,272.50, under the regular amortized 35-year loan, to $768.78, or a saving of $503.72 interest.

207. Questions and Problems

1. How many citizens are necessary to form a national farm loan association? What size loans may be secured? (See Organization, 1.)

2. How much does it cost a farmer to join a national farm loan association? What is done with this fee? (See Organization, 2.)

3. In which district is your state located? Where is the Federal Land Bank for your district? (See Organization, 4.)

4. What is meant by first mortgages?

5. How long does the farmer have to repay his loan? (See Limitations, 5.)

6. What does amortization mean? (See Amortization, 1.)

7. What length loan does the Federal Farm Loan Board prefer the farmers to make? (See Amortization, 2.)

8. Mr. Simpson wishes to secure a loan of $2000. At how much must his land and 20% of the permanent improvements, valued at $2500, be appraised? (See Limitations, 2.)

9. How much will it cost Mr. Simpson to join this national farm loan association? When will he get this back? (See Organization, 2.)

10. If Mr. Simpson wishes to repay this loan in 35 years, how much will his annual instalment be? If he wishes to pay semi-annually, how much will each payment be? (See Tables 1 and 2.)

11. How much will Mr. Simpson pay altogether in annual payments? In semi-annual payments?

12. Mr. Laws wishes to borrow $1500. What will his membership stock in the local association amount to?

13. What will his semi-annual instalment be?

14. How much will he still owe at the end of 17 years?

15. Suppose a farmer borrows $1000 on a straight loan at $5\frac{1}{2}\%$. What will the interest be per year? How much more would his annual instalment be under the amortization plan?

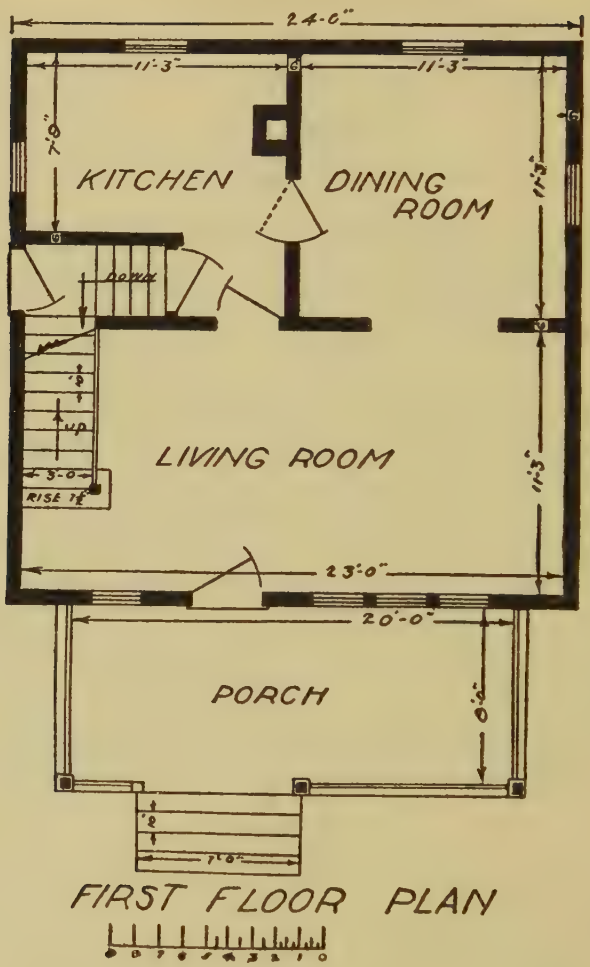
208. HOUSE PLANS

Front Elevation

1. The height of the house is 30′ from the ground to the ridge-pole. The width of the house is 24′.

2. Measure these dimensions and find how many feet are represented by 1 in. Express the result thus: Scale: 1 in. =8 ft.

This is called the **scale** for the drawing.



3. Apply the scale in Problem 2 to the various dimensions In the First Floor Plan to find whether the same scale was used for the Front Elevation and the First Floor Plan.

4. By use of the scale answer the following:

(a) What is the height of the front door? the width?

(b) What are the dimensions of the lower windows? the upper? the ones in the dormer?

(c) How high are the porch columns?

(d) What is the width of the door between the living room and dining room? between the dining room and kitchen?

5. How many square feet of siding are required for the front of the house, making full deductions for all doors, windows, and brick foundation, allowing $\frac{1}{5}$ for waste, and disregarding door and window casings?

6. Find the cost of the lumber required to floor the dining room with 1 inch flooring at \$90 per M board feet, allowing $\frac{1}{5}$ for waste.

7. Find the cost of the linoleum for the kitchen at \$1.25 a square yard.

8. Find the cost of picture molding around the walls of the living room at 20¢ a foot.

9. Find the cost of concreting the porch at \$1.85 per square yard.

10. There are 16 steps of the stairway; find the height of the second floor above the first.

11. Draw a floor plan for a room 16 ft. by 20 ft., using the scale 1 in. = 4 ft.

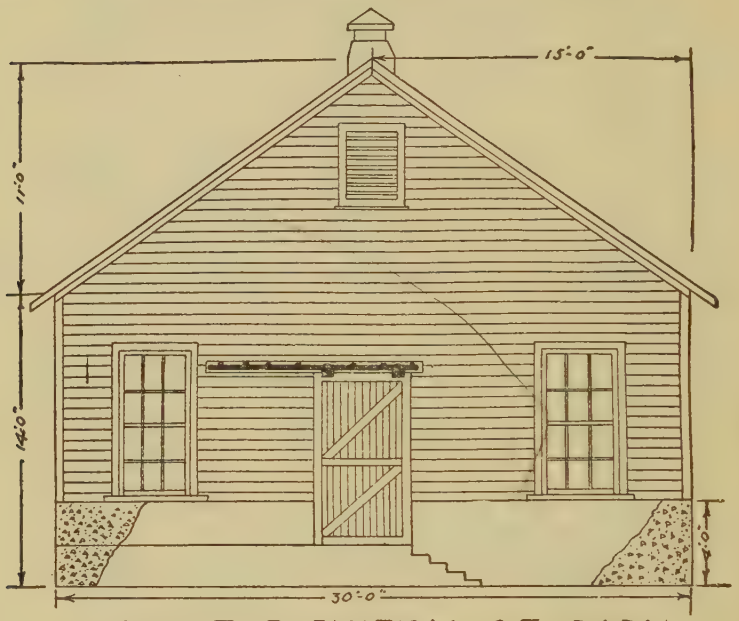
12. A baseball diamond is 90 ft. square, and the pitcher's box is 60 ft. from the home plate. Draw a plan of the diamond to the scale 1 in. = 15 ft., locating the bases and pitcher's box.

13. Draw the gable end of a barn in the shape of a triangle 40 ft. wide, with the ridge pole 20 ft. above the joists, using the scale 1 in. = 8 ft.

14. Find the number of board feet of lumber 1 in. thick for the gable in Problem 13, allowing $\frac{1}{5}$ for waste.

15. Find the cost of the lumber at \$47.50 per M board feet.

209. PROBLEMS ON BARN CONSTRUCTION.



FRONT ELEVATION OF BARN

1. How wide is the barn? How high are the walls?

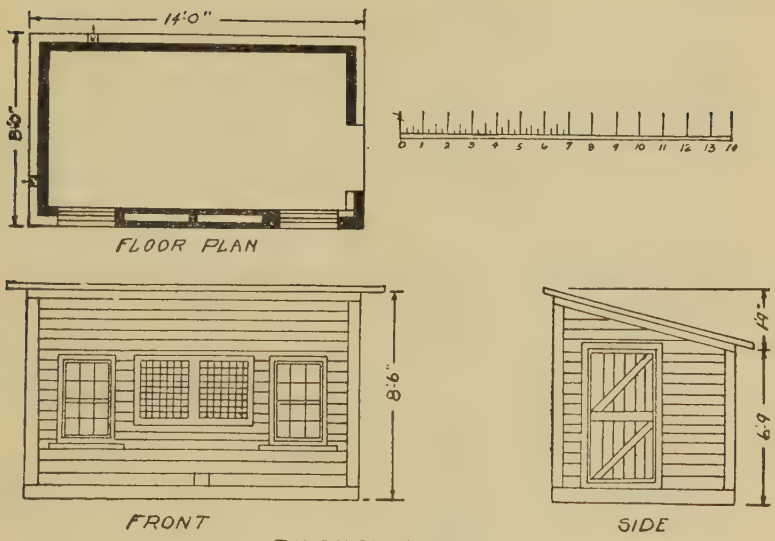
2. What is the height of the ridge pole above the eaves?

3. The barn is 40 ft. long. Find the total amount (board feet) of lumber 1 in. thick needed for all the walls, including the two gables, making no allowances for doors or windows.

4. The length of the rafters is 20 ft., including a projection of 18 in. at the lower ends.

5. Find the number of squares (100 sq. ft.) of roofing needed for the roof.

6. Find the cost of the material needed for walls and roof if the lumber costs $85 per M and the roofing costs $4.50 per square.



CHICKEN HOUSE

210. Building of Chicken Houses

1. By use of the scale find the dimensions (1) of the double-screen window of the chicken house; (2) of the door.

2. The length of the rafters is 10 ft., including a total projection above and below of 12 in.

3. Find the (1) amount of 1 in. lumber needed for the four walls of the chicken house, no deductions for openings or waste; (2) number of squares of roofing needed for the roof.

4. Find the cost of the material for the chicken house if the lumber costs \$70 per M and the roofing costs \$3.75 per square.

5. It is estimated that 5 sq. ft. of floor space are needed for each head of poultry. How many head of poultry will this house accommodate?

211. FOUNDATION WALLS AND BASEMENT FLOORS

Cellar and basement walls are usually extended below the surface of the ground to a depth depending upon the height of the building. Such walls are called **foundation walls.** Walls that are intended principally to withhold a bank of earth are called **retaining walls.**

Materials for these walls may be brick, stone, or concrete. Brick is used extensively in dry soils and for partition walls. Concrete is used in damp soils because it withstands the deteriorating effects of water.

The thickness of foundation walls depends upon their depth and the height of the building they support. In large cities the thickness and depth of foundation walls are fixed by law.

For a two-story house, the thickness of a brick foundation wall is usually 12 in. to 16 in. If the construction material is stone or concrete, the thickness is about 20 in.

When we estimate the labor and the number of bricks in the walls of a house, outside dimensions of the walls are usually taken. This means that the corners are counted twice, but because of the extra labor in laying them, this method of computing is considered fair to both employer and employee; and because of wastage by breaking, etc., the allowance for double material in corners is made.

1. A common brick is 8 in. by 4 in. by 2 in. How many cubic inches are there in its volume? How many common bricks could be laid in 1 cu. ft. of space?

2. After allowance is made for the space taken up by the mortar, 22 bricks are counted to 1 cu. ft. of wall. How many bricks are there in a brick wall 40 ft. long, 6 ft. high and 16 in. thick?

3. If the house is 24 ft. wide, 28 ft. long, and if the brick foundation is 12 in. thick and extends 8 ft. below the first floor beams of the house, find the number of bricks in the foundation walls.

4. Find the cost of the bricks at \$16.50 per M.

5. If a brick mason can lay 160 bricks per hour and if he is paid \$1 per hour, how much will the wages of the bricklayers amount to?

6. If the mortar costs \$1.25 per thousand bricks, find the cost of the mortar.

7. Usually one helper (hod-carrier) is needed for each mason and the wages of the helper is one-half the wages of the mason. Find the cost of the helpers.

8. From the results of Problems 4, 5, 6, and 7, find the total cost of the foundation walls.

9. Concrete work is estimated by the cubic foot or cubic yard. Find the cost of the material for the walls described in Problem 3, if constructed of concrete at a cost of \$10.50 per cubic yard including cost of material and labor.

10. Which is cheaper, and how much, the brick foundation or the concrete foundation?

11. Usually the floor of a basement is constructed of concrete. Find the cost of the concrete floor for the basement of the house described in Problem 3, if the concrete is 6 in. thick and covers the entire floor.

Hint: Observe that the walls are 12 in. thick, and that the dimensions of the floor will be less than the outside wall dimensions.

12. Find the cost of excavating for a foundation 5 ft. deep, $3\frac{1}{2}$ ft. wide, and 400 ft. in length, at 75¢ a cubic yard.

13. In problem 12, what will it cost to lay the foundation in concrete at \$12.50 a cubic yard?

14. How many cubic feet in a foundation $3\frac{1}{2}$ ft. deep, $2\frac{1}{2}$ ft. wide and 300 ft. long?

212. LONGITUDE AND TIME

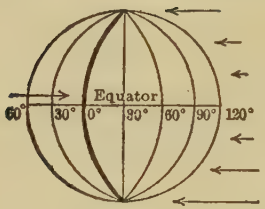


FIG 1.

You have learned in your geography that the earth is a sphere which revolves on its axis like a top and that it makes one complete revolution from west to east in 24 hr. In making a complete revolution, the earth describes a circle. A circle contains 360°. Hence 24 hr. of time corresponds to 360° of longitude.

1 hr. of time corresponds to $\frac{1}{24}$ of 360° $= 15°$.

1 min. of time corresponds to $\frac{1}{60}$ of $15° = \frac{1}{4}° = 15'$

1 sec. of time corresponds to $\frac{1}{60}$ of $15' = \frac{1}{4}' = 15''$.

Thus we see the number of hours, minutes, and seconds of time is $\frac{1}{15}$ of the number of degrees, minutes, and seconds of longitude; or the number of degrees, minutes, and seconds of longitude is 15 times the number of hours, minutes, and seconds of time. Hence, to find the difference in **longitude** multiply the difference in time by 15; to find the difference in **time** divide the difference in longitude by 15.

1. Find the difference in time between two places if the difference in longitude is 30°; 60°; 75°; 105°; 135°; 150°; 50°.

2. The difference in longitude between two places is 35° 45′ 45″. Find the difference in time.

Solution: 15)35° 45′ 45″

2 hr. 23 min. 3 sec.

Find the difference in time between two places when the difference in longitude is:

3. 48° 34′ 30″ **4.** 120° 37′ 15″ **5.** 97° 17′ 45″

6. 175° 49′ **7.** 48′ 45″ **8.** 160° 7′ 45″

9. 115° 20′ 45″.

10. A tourist started from a certain point and traveled until he found his watch 2 hr. 15 min. 20 sec. too slow. Over how many degrees of longitude did he travel?

Solution:

2 hr.	15 min.	20 sec.
		15
33°	50′	00″

Find the difference in longitude between two places if the difference in time is:

11. 5 hr. 48 min. 46 sec.

12. 11 hr. 37 min. 20 sec.

13. 9 hr. 20 min. 35 sec.

14. 8 hr. 25 min. 45 sec.

15. 4 hr. 17 min. 32 sec.

16. When it is noon at Louisville, what time is it 15° west? 15° east? 45° west? 60° east? 75° west? 90° east? 90° west? 105° west? 120° west? 165° east? 165° west?

17. Would you change your watch backward or forward in traveling west? Why?

213. Measuring Longitude

In measuring longitude you must have some meridian for a starting point, and number all other meridians east and west of that **principal meridian.** The meridian passing through Greenwich, in England, has been accepted by astronomers of different countries as the **prime meridian.** In Figure **1,** the heavy shaded meridian represents the prime meridian.

If James lives on 5th St. 4 blocks north of Main St., and William lives on 5th St. 8 blocks north of Main St., how many blocks are they apart?

If Henry lives on 5th St. 3 blocks south of Main St., how far does he live from James? William?

How would you find the difference in longitude between two places located on the same side of the prime meridian? On opposite sides?

What is the difference in **solar time** (sun time) between places located on the following meridians:

1. 30° E. and 45° E. long.? **2.** 15° E. and 15° W. long.?

3. 15° W. and 75° W. long.? **4.** 45° W. and 15° E. long.?

5. 60° W. and 60° E. long.? **6.** 90° E. and 30° E. long.?

7. 90° E. and 90° W. long.? **8.** 65° E. and 35° E. long.?

9. 75° W. and 75° E. long.? **10.** 140° W. and 35° W. long.?

11. 30° E. and 10° W. long.? **12.** 26° W. and 24° E. long.?

13. When it is noon at Indianapolis, is it before or after noon at Denver? Boston? San Francisco? Key West? Washington? St. Louis? Baltimore?

14. When it is 10:30 A. M. at Boston whose longitude is 71° 3′ 30″ W., what time is it at San Francisco whose longitude is 122° 26′ 15″ W.?

Solution: Since both places are on the same side of the meridian, subtract to find difference in longitude. Divide difference in longitude by 15 to get difference in time.

Since the difference in time is 3 hr. 25 min. 31 sec. and since we want to know the hour in San Francisco when it is 10:30 A.M. in Boston, we have only to subtract the difference in time from the time in Boston, 10:30 A.M.

122° 26′ 15″
71° 3′ 30″
51° 22′ 45″ Difference in longitude

15)51° 22′ 45″
3 hr. 25 min. 31 sec.
Difference in time.

	hr.	min.	sec.
	10	30	00
	3	25	31
Time in San Francisco	7	4	29

or we would say 4 min. 29 sec. past 7 if we looked at the clock. Why did we subtract to find the difference in time?

15. If the longitude of Washington is 77° 3′ 6″ W. and the longitude of New Orleans is 90° 3′ 28″ W., what is the time at New Orleans when it is 1:30 P.M. (13 hr. 30 min.) at Washington?

NOTE: Our method of counting the time divides the day into two 12-hour periods. In finding the difference in time between two places, we often use the number of the hour, counting from midnight, for convenience. Thus, 2 P.M. is the 14th hour, 8 P.M. the 20th hour, etc.

Table of Longitudes

Boston	71° 3′ 30″ W.	Paris	2° 20′ 20″ E.
Berlin	14° 36′ E.	Philadelphia	75° 10′ 5″ W.
London	0° 5′ 48″ E.	New York	74° 0′ 3″ W.
Albany	74° 2′ W.	Petrograd	30° 19′ 21″ E.
Calcutta	88° 20′ E.	Nashville	86° 48′ W.
New Orleans	90° 3′ 28″ W.	Washington	77° 3′ 6″ W.
Denver	104° 58′ 00″ W.	Manila	120° 58′ 6″ E.
San Francisco	122° 26′ 15″ W.	Rome	12° 27′ 14″ E.
St. Louis	90° 15′ 15″ W.	Montreal	73° 25′ W.

What is the difference in solar time:

1. Between Boston and Paris?
2. Between Nashville and Berlin?
3. Between Calcutta and Petrograd?
4. Between New York and San Francisco?

5. Between Rome and St. Louis?

6. Between Calcutta and Philadelphia?

7. Between London and Washington?

8. Between New Orleans and San Francisco?

9. When it is 2:30 P.M. at Denver, what is the time at Boston?

10. When it is 5 o'clock 15 min. 10 sec. A.M. at St. Louis, what is the time in Albany?

11. If the difference in time between Paris and New York is 5 hr. 5 min. 21 sec., what is their difference in longitude?

12. If a ship sends a wireless message at 10 A.M. and it is received by another vessel at 5 A.M., how far away in longitude is the receiving ship? (Consider transmission of message instantaneous.)

13. A vessel in longitude 84° 15′ 45″ W. sent at 3:45 P.M. a wireless to another vessel east of it which received it at 5:30 P.M. What was the longitude of the receiving vessel?

14. A tourist, who had not changed the time of his watch, found on reaching San Francisco his time was 3 hr. 15 min. 30 sec. too slow. How far, and in what direction, had he traveled?

214. International Date Line

If you were to start westward from Washington to travel around the world, and should keep correct solar time, you would set your watch back 1 hr. every time you passed over 15°. You would find on reaching Washington at the completion of your journey that you had set your watch back 24 times. Hence, you would be 24 hr., or 1 da., behind the calendar. If you had traveled eastward, you would have set your watch forward 24 times, and on your return you would be 1 da. ahead of the calendar.

To adjust the calendar, a somewhat irregular line called the

International Date Line has been established. This line follows approximately the 180th meridian.

A vessel going westward changes its calendar forward 1 da. in crossing the line. If the vessel is going eastward, the calendar is set back 1 da. on crossing this line.

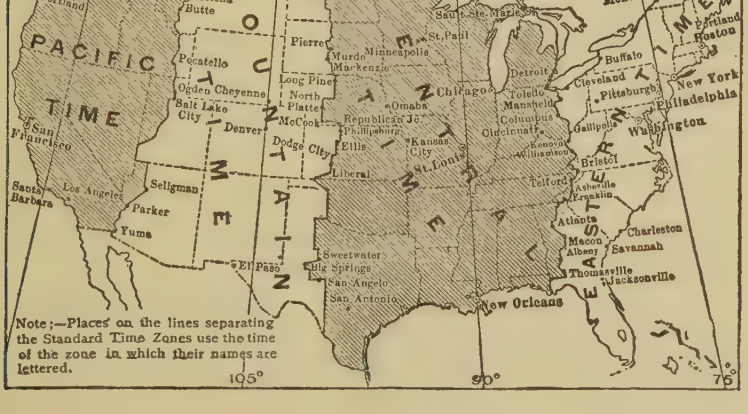
215. Standard Time

For the convenience of the railroads, **standard time** was established in 1883. According to the agreement, the United States is divided into four divisions or time belts.

The **Eastern** time is the solar time of the 75th meridian; the **Central** time of the 90th meridian; the **Mountain** time of the 105th meridian; the **Pacific** time of the 120th meridian.

These meridians pass approximately through the center of each belt. Each time belt represents time 1 hr. earlier than the adjacent belt to the east.

It will be noted that the boundaries of the time belts do not follow exactly the meridians. This is because the boundary lines are influenced by important railroad terminals.



Finding Standard Time

1. When it is 10:15 A.M. at Philadelphia, what time is it at Denver?

2. When it is 7 A.M. at Charleston, S. C., what time is it at San Francisco?

3. If an important event occurs in Boston at 5 o'clock, state how it is possible for it to appear in a 3 o'clock paper at Portland.

4. About what is the difference between the standard time and the solar time at Detroit?

5. In going from Portland to New York, what changes would you make in your watch?

6. Find the difference in standard and solar time at San Francisco.

7. When it is 1 P.M. at Philadelphia, what is the time in Salt Lake City?

8. Where would a city be located to have the same standard and solar time? Find such city on the map.

Problems Involving Time

1. If a horse trots $\frac{1}{4}$ mi. in $\frac{3}{4}$ of a minute, at that rate how long would it take him to go a mile?

2. A man traveled on a train at the rate of 32 mi. an hour. How long did it take him to make a trip of 115 mi.?

3. A train leaves Nashville at 3:55 A.M. and reaches Montgomery 302 mi. distant at 12:10 P.M. What was the average rate per hour made by the train?

4. A train leaves Birmingham at 4 P.M. and reaches Montgomery at 7:30 P.M. The distance between the cities is 96.7 mi. What is the rate of travel per hour?

5. A train leaves Montgomery at 12:30 P.M. and travels at the rate of 32 mi. an hour. What time will it reach New Orleans, which is 318 mi. distant?

216. RATIO AND PROPORTION

1. Note these three ways of expressing the same relation between two numbers:

	a		*b*		*c*
(1)	$12 \div 6$	=	$\frac{12}{6}$	=	12 : 6
(2)	Quotient	=	Fraction	=	Ratio
(3)	$\frac{\text{Dividend}}{\text{Divisor}}$	=	$\frac{\text{Numerator}}{\text{Denominator}}$	=	$\frac{\text{Antecedent}}{\text{Consequent}}$

Read (1), (2), (3), above.

2. All of the terms and expressions above have been used frequently, heretofore, except those under (c). The expression, 12 : 6, is called a **ratio.** It is read, the ratio of 12 to 6. It expresses a relation between two numbers heretofore expressed in the form of a fraction or a division. We may think of these three symbols as indicating division: first, (÷), used between dividend and divisor, as $12 \div 6$; second, (—), used between numerator and denominator, as $\frac{12}{6}$; and third, (:), used between antecedent and consequent, as 12 : 6.

Note that the ratio symbol is made of two dots, the fraction symbol of a dash, and the division symbol of two dots and a dash. It is thought that the symbol for division that was first used consisted of two dots (:).

(1) What is 12 called in the division? in the fraction? in the ratio?

(2) What is 6 called in the division? in the fraction? in the ratio?

Ratio

3. A ratio is made up of two similar numbers, called **antecedent** and **consequent,** and expresses the quotient found by dividing the antecedent by the consequent.

4. Just as in division both dividend and divisor may be multiplied or divided by the same number without changing the quotient, and in a fraction both numerator and denominator may be multiplied or divided by the same number without a change of value; so in a ratio both antecedent and consequent may be multiplied or divided by the same number without changing the value of the ratio.

5. Show why the following expressions are equal:

(1) $12 \div 6 = 6 \div 3 = 2 \div 1$; $2 \div 1 = 6 \div 3 = 12 \div 6$.

(2) $\frac{12}{6} = \frac{6}{3} = \frac{2}{1}$; $\frac{2}{1} = \frac{6}{3} = \frac{12}{6}$.

(3) $12 : 6 = 6 : 3 = 2 : 1$; $2 : 1 = 6 : 3 = 12 : 6$.

Express the following ratios in their simplest form:

1. $36 : 9$.

Solution: Dividing antecedent and consequent each by 9,
$36 : 9 = 4 : 1$.

2. $2\frac{1}{2} : 3$.

Solution: Multiplying antecedent and consequent each by 2,
$2\frac{1}{2} : 3 = 5 : 6$

3. $\frac{1}{8} : \frac{5}{8}$.

Solution: Multiplying antecedent and consequent each by 24,
$\frac{1}{8} : \frac{5}{8} = 8 : 15$.

4. $12 : 15$.

5. $20 : 16$.

6. $15 : 24$.

7. $36 : 12$.

8. $3\frac{1}{8} : 10$.

9. $15\frac{1}{2} : 31$.

10. $6\frac{1}{3} : 12\frac{2}{3}$.

11. $15 : \frac{1}{2}$.

12. $\frac{1}{8} : \frac{7}{8}$.

13. $\frac{2}{3} : \frac{3}{4}$.

14. $20 : \frac{1}{5}$.

15. $3\frac{1}{3} : 1$.

16. $5 : 5\frac{1}{2}$.

17. $16\frac{2}{3} : 100$.

18. $12\frac{1}{2} : 100$.

19. $8\frac{1}{3} : 100$.

217. Applications of Ratio

Here are some average retail prices of food in Chicago for the years mentioned. Find the ratios of the 1914 prices to the 1919 prices, and express the results as common fractions in their lowest terms:

	Food	Quantity	1914 Price	1919 Price	Ratio
1.	Round Steak	lb.	21.8¢	34.6¢	$\frac{21.8}{34.6}=\frac{218}{346}=\frac{109}{173}$
2.	Pork Chops	lb.	19.0	37.5	
3.	Bacon	lb.	31.2	59.7	
4.	Milk	qt.	8.0	14.0	
5.	Butter	lb.	30.8	57.5	
6.	Lard	lb.	15.0	38.6	
7.	Eggs	doz.	24.4	51.1	
8.	Bread	lb.	6.1	10.0	
9.	Flour	lb.	2.9	7.2	
10.	Corn Meal	lb.	2.8	5.7	

1. Note that the ratio of bread prices for the two years may be expressed as a decimal: $\frac{61}{100}=.61$. Could you express all of them as decimals? What is the consequent in a one-place decimal, as .6? in a two-place decimal, as .61? in a three-place decimal, as .061?

2. The side of one square is 6 in. and the side of another is 3 in. (a) What is the ratio of their sides? (b) What is the ratio of their areas?

3. Is it true that the areas of two figures that are alike in form have the same ratio as the square of their like dimensions?

218. Specific Gravity

The table below shows the weight of 1 cu. ft. of various substances, and of 1 cu. ft. of distilled water at a temperature of 60° F. Find the ratio of each to water and express the result as a decimal.

	Substance	Wt. in oz. of 1 cu. ft.	Wt. in oz. of 1 cu. ft. of water	Ratio
1.	Alcohol	814	1000	
2.	Milk	1032	1000	
3.	Cork	240	1000	
4.	Maple	750	1000	
5.	Air	1.29	1000	
6.	Honey	1450	1000	
7.	Marble	2720	1000	
8.	Steel	7830	1000	
9.	Silver	10470	1000	
10.	Lead	11360	1000	
11.	Mercury	13590	1000	
12.	Gold	19360	1000	

The ratio you have just found for each substance in the table is called the **specific gravity** of that substance. That is, the ratio of the weight of any substance to the weight of an equal volume of water is the specific gravity of that substance. Specific gravity answers the question, *How many times as heavy as water?*

By reference to the ratios, or specific gravities, answer these questions:

1. Milk is how many times as heavy as water?
2. Cork is how many times as heavy as water?
3. Honey is how many times as heavy as water?
4. Lead is how many times as heavy as water?
5. Gold is how many times as heavy as water?
6. If you mix water and honey, which will rise to the top?
7. Will alcohol float or sink in water?
8. Will maple float or sink in water?

9. Can you carry a gallon of gold?
Suggestion: A gallon of water weighs $8\frac{1}{3}$ lb.

10. Which is the heaviest substance in the table? Which is the lightest?

11. How much does a gallon of honey weigh? a gallon of milk? a gallon of air?

219. Simple Proportion

An equality of two ratios is called a **proportion,** as $12 : 6 = 14 : 7$.

The proportion may be written $\frac{12}{6} = \frac{14}{7}$. Why? Formerly, it was frequently written $12 : 6 : : 14 : 7$, but this form is rapidly disappearing.

The proportion may be read, *The ratio of 12 to 6 equals the ratio of 14 to 7,* or *12 is to 6 as 14 is to 7.*

The proportion consists of four terms, two antecedents and two consequents. Name the two antecedents and the two consequents in $12 : 6 = 14 : 7$.

The four terms are called in order from left to right: *first, second, third,* and *fourth.* The first and fourth are the **extremes,** and the second and third terms are the **means.**

1. Find the product of the extremes.

2. Find the product of the means.

3. How do the two products compare?

4. Compare the products of extremes and of means in these proportions:

(a) $3 : 6 = 5 : 10$ (c) $18 : 3 = 30 : 5$

(b) $1 : 3 = 7 : 21$ (d) $2\frac{1}{2} : 5 = 4 : 8$

In every proportion, the product of the extremes is equal to the product of the means.

This is called the fundamental principle of proportion. It may be used to test proportions where all of the four terms are proposed.

1. Test these proposed proportions:

(a) $2 : 5 = 3 : 7$. Test: $2 \times 7 = 14$; $5 \times 3 = 15$; hence, the proposed proportion is not true.

(b) $3 : 18 = 5 : 30$. Test: $3 \times 30 = 90$; $18 \times 5 = 90$; hence, the proposed proportion is true.

(c) $3 : 16 = 4 : 21$ (d) $7 : 2 = 10 : 3$

(e) $3 : 4 = 5 : 6$ (f) $3 : 5 = 15 : 25$

(g) $5 : 10 = 6 : 12$ (h) $3\frac{1}{3} : 5 = 6 : 9$

(i) $1\frac{1}{2} : 3 = 7 : 14$ (j) $1.2 : 3.6 = 2\frac{1}{3} : 7$

2. Multiply each term of the first ratio in (h) by 3. Is the resulting ratio still equal to the ratio 6 : 9? Multiply each term of the first ratio in (i) by 2. Is the resulting ratio still equal to the ratio 7 : 14?

3. Explain the results found in Exercise 2.

4. Multiply each term of the first ratio in (j) by 5, and each term of the second ratio by 3. Are the resulting ratios equal?

5. Explain the results you found in Exercise 4. If three of the terms of a proportion are given, how may we find the missing term?

The following steps will help you answer:

(a) $3 : 5 = 12 : n$, $3n = 60$. ($3n$ means $3 \times n$.)

(b) If $3n = 60$, how much is $1n$, or n? Hence, $n = 20$.

(c) Check: (a) now becomes, $3 : 5 = 12 : 20$;

$3 \times 20 = 5 \times 12$

$60 = 60$

Find the missing term and check:

1. $6 : 3 = 16 : n$.

Solution: $6n = 48$ Why?

$n = 8$ Why?

Check: $6 : 3 = 16 : 8$; $6 \times 8 = 3 \times 16$

$48 = 48$

2. $4 : n = 9 : 72$.
3. $6 : 1 = 24 : n$.
4. $n : 7 = 2 : 14$.
5. $5 : 8 = n : 40$.
6. $\frac{1}{3} : \frac{2}{5} = 6 : n$.

Solution: Multiply each term of the first ratio by 15; the proportion becomes, $5 : 6 = 6 : n$

then, $5n = 36$

$n = 7\frac{1}{5}$

Check: $\frac{1}{3} : \frac{2}{5} = 6 : 7\frac{1}{5}$; $\frac{1}{3} \times 7\frac{1}{5} = \frac{2}{5} \times 6$

or, $\frac{1}{3} \times \frac{36}{5} = \frac{2}{5} \times 6$

That is, $\frac{12}{5} = \frac{12}{5}$

7. $2\frac{1}{5} : 11 = n : 25$.
8. $3\frac{1}{3} : n = 6 : 18$.
9. $1\frac{1}{2} : n = 3\frac{1}{4} : 9\frac{1}{2}$.

Solution: Multiplying each term of the first ratio by 2, and each term of the second ratio by 4, the proportion becomes,

$3 : 2n = 13 : 38$

$26n = 114$

$n = 4\frac{5}{13}$

Check: $1\frac{1}{2} : 4\frac{5}{13} = 3\frac{1}{4} : 9\frac{1}{2}$; $1\frac{1}{2} \times 9\frac{1}{2} = 4\frac{5}{13} \times 3\frac{1}{4}$

$\frac{3}{2} \times \frac{19}{2} = \frac{57}{13} \times \frac{13}{4}$

$\frac{57}{4} = \frac{57}{4}$

10. $n : 1\frac{1}{8} = 2\frac{1}{2} : 1\frac{1}{4}$ **13.** $n : 12 = 13 : 26$ **16.** $\frac{1}{2} : \frac{1}{8} = \frac{1}{6} : n$
11. $14 : n = 1 : 3\frac{1}{7}$ **14.** $35 : 7 = n : 4$ **17.** $n : \frac{7}{9} = \frac{5}{6} : \frac{3}{8}$
12. $15 : 6.25 = 1.2 : n$ **15.** $63 : 7 = 6\frac{1}{2} : n$ **18.** $\frac{9}{10} : n = \frac{4}{5} : \frac{1}{2}$

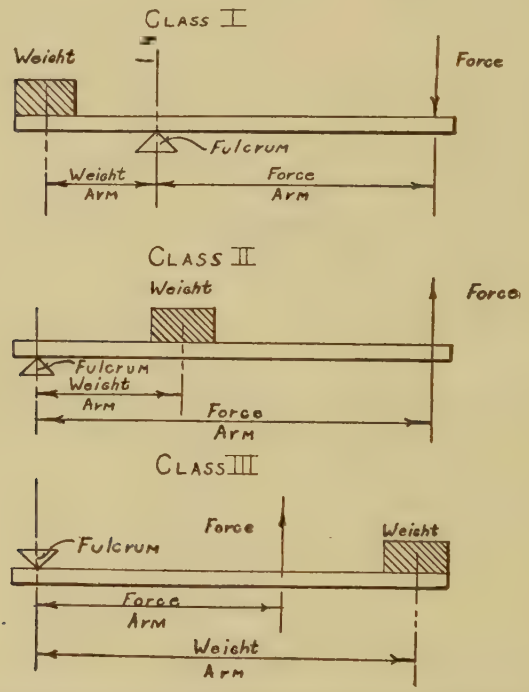
220. Levers

In lifting a weight of 100 lb. without the aid of a machine, a man employs a force of 100 lb. If we think of the weight as **resistance** and the force as **effort,** we may say the resistance is overcome by the effort.

By use of a simple machine, known as a **lever,** the man may reduce the effort necessary to overcome the resistance. A

lever is a rigid bar supported at a point about which it may move freely.

There are three classes of lever:



In these illustrations, the **fulcrum** is the point about which the lever moves freely.

1. Now suppose the man who wishes to raise 100 lb. should fasten the weight on one end of a bar 8 ft. long, support the bar with a fulcrum 3 ft. from the weight, and at the other end of the bar exert the force necessary to balance the lever. He would find that he could balance the weight with a force of 60 lb., and raise the weight with a force of slightly more than 60 lb.

This is called a lever of the first class. (See Class I above). Examples of this class of lever are the crow-bar, used as a pry, the see-saw, etc.

2. If the man had supported the bar at one end with the fulcrum and had fastened the weight 3 ft. from the fulcrum, he could have balanced the lever by a force of $37\frac{1}{2}$ lb., and have raised the weight with a force slightly more than $37\frac{1}{2}$ lb.

This is called a lever of the second class. Examples of this class of lever are the ordinary bar when used as indicated, nut crackers, lemon squeezers, etc.

3. If the man should fix the end of the bar under the fulcrum, place the weight at the other end of the bar and apply the force at a point 3 ft. from the fulcrum, he would have to exert a force of $266\frac{2}{3}$ lb. to keep the bar in balance, and slightly more than that to raise the weight.

This is called a lever of the third class. Examples of this class of lever are the fore-arm raising a weight in the extended hand, the elbow being the fulcrum and the muscles of the fore-arm the force; the door when being pushed near the hinge, etc.

4. In which direction, upward or downward, is the effort exerted in the first class lever? in the second class? in the third class?

The law of the lever is expressed in the following proportion: **F : W = Wa : Fa;** in which **F** stands for force, **W** for weight, **Wa** for the distance from the fulcrum to the point where the weight is applied, and **Fa** the distance from the fulcrum to the point where the force is applied.

5. Use the proportion in [4], and show hov the result in [1] above was found.

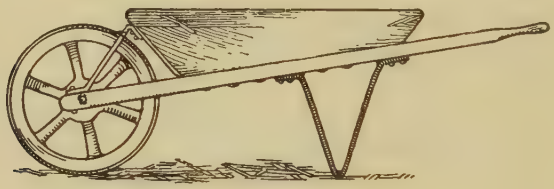
Solution: F : W = Wa : Fa.
F : 100 = 3 : 5
5 × F = 300
F = 60

6. Show how the result of [2] was found.

7. Show how the result of [3] was found.

8. If a see-saw is 10 ft. long and supported by a beam 4 ft. from one end, how much must a boy weigh to balance a 60-lb. boy on the end of the shorter arm?

9. A wheelbarrow is 5 ft. long from the axle to the end of the handles. A load (weight) of 350 lb. is exerted $1\frac{1}{2}$ ft. from the axle. How much force must a man exert to move the load?



Miscellaneous Problems

1. If a boy 5 ft. high casts a shadow 6 ft. long, how high is a tree that casts a shadow 72 ft. at the same time? Draw a figure to show heights and shadows.

Suggestion: Height boy : height tree = shadow boy : shadow tree.

2. Measure the height of a tree or pole in your school yard by the method of Problem 1.

3. If a wheat bin $7\frac{1}{2}$ ft. deep holds 324 bu. of wheat, how many bushels will a bin of the same length and width hold if it is 12 ft. deep?

4. If 6% of \$800 is \$48, find by proportion $7\frac{1}{2}\%$ of \$800.

Suggestion: $6\% : 7\frac{1}{2}\% = \$48 : \$n$.

5. If the interest on \$1200 for a given time at a given rate is \$108, find the interest on \$1750 for the same time at the same rate.

221. Proportional Parts

1. There were 770 pupils attending a certain school. There were 6 boys to 5 girls enrolled. How many were boys? How many were girls?

Since there are 6 boys to 5 girls, out of 11 pupils $\frac{6}{11}$ of them are boys and $\frac{5}{11}$ are girls.

$\frac{6}{11}$ of $770 = 420$.
$\frac{5}{11}$ of $770 = 350$.

This process may also be expressed in this form:

$$11 : 6 : : 770 : (\quad), \frac{6 \times 770}{11} = 420$$

$$11 : 5 : : 770 : (\quad), \frac{5 \times 770}{11} = 350$$

The process of dividing a number into parts, each of which has a ratio to the sum of the parts, is called **partitive proportion.**

2. Divide 39 chestnuts between David and Thomas so that David gets 8 every time Thomas gets 5.

3. If Brown worked 8 da. and Crawford 9 da. and they received \$85 for the work, what was the share of each?

4. Two men rent a pasture for \$1250. If one grazes 200 cattle and the other 300 cattle, what was each man's share of the rent?

5. Divide \$1500 among 3 men so that they shall share in the proportion of 5, 3 and 4.

6. The cost of a basket of fruit was \$2.50. Divide the cost among Ellen, Laura and Bessie in the proportion of 4, 5 and 6.

7. Two men bought a farm of 120 A. One man paid \$5000 and the other \$7000. They divided the farm in proportion to the payment of each. How many acres should each receive?

8. Divide \$65 between two boys so that their shares shall be to each other as $\frac{1}{4}$, $\frac{5}{6}$.

$\frac{1}{4}=\frac{3}{12}$ Change fractions to a common denominator. Since the
$\frac{5}{6}=\frac{10}{12}$ denominators are equal, divide the number into parts pro-
$3+10=13$ portional to numerators.
$\$65\times\frac{3}{13}=\$15$.
$\$65\times\frac{10}{13}=\$50$.

9. Divide 84 into three parts that shall be to each other as 3, 4 and 5.

10. Three men bought a farm for \$20,000, and paid for it in parts proportional to 5, 7, 8. How much did each pay?

11. Two girls bought oranges for 75¢; one paid $\frac{4}{11}$ as much as the other. How much did each pay?

12. Divide 9844 into parts proportional to 14, 15, 17.

13. Divide \$1365 into parts proportional to $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

14. Divide 644 mi. into parts proportional to $\frac{3}{4}$, $\frac{8}{9}$, $\frac{11}{12}$.

15. A worked 18 da. and B worked 24 da. They received \$210. How should it be divided between them?

222. PARTNERSHIP

When two or more men associate themselves together for the transaction of business they form a **partnership.** Each one entering into the agreement is called a **partner.** The partnership is called a **firm, company** or **house.** The amount of money invested is the **capital.** The **gains** and **losses** are shared by all the members of the company in proportion to the amount of money each has invested.

1. Three farmers, A, B and C, bought a tractor engine and plows for \$1350. A paid \$450, B \$500 and C \$400. They received for plowing a field \$108. What is the share of each?

Hint: What part of the whole cost did each pay?

2. Two men rented a pasture. One grazed 6 cows 6 weeks and the other 8 cows 4 weeks. If the pasture costs \$64, how much should each pay?

Hint: 6 cows for 6 weeks is equivalent to 1 cow for 36 weeks or 36 cows for 1 week, etc.

3. Four men formed a company to purchase timber lands at a cost of \$279,000. A put in \$54,000, B \$60,000, C \$75,000, and D \$90,000. At the close of the year they sold the lands at a loss of \$73,000. Find each one's share of the loss.

4. A, B and C engaged in business. A invested \$5600, B \$4500 and C \$5900. They made a profit during the year of \$4000. What was the share of each?

5. Five men jointly rented a plantation for \$6750. A paid of the rent \$1500, B \$1200, C \$900, D \$1000 and E \$2150. They grew \$15,000 worth of cotton, \$2500 worth of corn and \$1500 worth of hay. The cost of labor and material in producing the crop was \$8,500. Divide the net profits of the crop.

6. The total capital invested by two partners was \$15,000. If one's share of the profit was \$450 and the other's share was \$550, what was the amount of capital each had invested?

7. The total gain was \$1250. A's capital was \$6,000, B's capital was $\frac{3}{5}$ of A's and C's was $\frac{8}{9}$ of B's. What was the gain of each?

8. A, B, C and D rented a pasture for \$1500. A pastured 20 horses for 15 da., B pastured 25 horses for 30 da., C pastured 40 horses for 12 da., D pastured 12 horses for 30 da. What amount should each pay for the rent?

9. A and B formed a partnership. A put in \$800 for 10 mo. and B put in \$600 for 12 mo. They gained \$450. What is each one's share of the gain?

Hint: A's \$800 for 10 mo. = \$8000 for 1 month, B's \$600 for 12 months = \$7200 for 1 month.

10. A, B and C form a partnership. A put in \$600 for 5 mo., B put in \$500 for 6 mo. and C put in \$1000 for 4 mo. They gained \$600. What is each man's share of the gain?

223. STOCKS AND BONDS

You have learned that people whose business is to buy or sell stocks and bonds are called *brokers*.

The broker's charge is called *brokerage*. It varies, but the following table shows the brokerage charge on the New York Stock Exchange for differently priced stocks:

Stock Selling	Brokerage
From \$1 to \$10 a share	$7\frac{1}{2}¢$ a share
From \$10 to \$25 a share	$12\frac{1}{2}¢$ a share
From \$25 to \$50 a share	15¢ a share
From \$50 to \$75 a share	$17\frac{1}{2}¢$ a share
From \$75 to \$100 a share	20¢ a share
From \$100 to \$200 a share	25¢ a share

The price at which a stock sells is called its *market* value.

The stock quotations found in the daily papers show the price of a share of stock for a particular day, thus Standard Oil of N. J. $68\frac{3}{4}$, which means that the stock sold at \$68.75 a share.

Stock quotations are given in the daily papers in the following way:

Stock		Shares	High	Low	Close
American Can	4	9,750	$126\frac{3}{4}$	120	$122\frac{5}{8}$
Bendix Aviation	2	4,800	$30\frac{7}{8}$	30	$30\frac{1}{4}$
Coca-Cola	6	1,500	$184\frac{1}{2}$	180	183
Gen. Asphalt	4	300	$42\frac{1}{2}$	40	$42\frac{1}{4}$
N., C. & St. L. R. R.	5	7,000	$95\frac{1}{2}$	95	95
U. S. Rubber		3,800	$18\frac{5}{8}$	$17\frac{3}{4}$	18

To read the stock quotations you would say there were sold of the American Can 4, 9750 shares, high $126\frac{3}{4}$ (\$126.75), low 120 (\$120.00), close $122\frac{5}{8}$ (\$122.625) per share. Read the quotation on each of the other stocks.

1. Find the cost of 10 shares of each stock bought at high, counting brokerage as charged on the New York Exchange.

2. Which is the better investment from the standpoint of income, American Can 4 at 120, or General Asphalt 4 at 40?

Solution: A share of American Can 4 pays a dividend of \$4 a year. The General Asphalt 4 pays a dividend of \$4 per year.

Cost of 1 share American Can \$120.

$\$4 \div \$120 = 3\frac{1}{3}\%$ on investment American Can.

Cost of 1 share General Asphalt \$40.

$\$4 \div \$40 = 10\%$ income General Asphalt.

$10\% - 3\frac{1}{3}\% = 6\frac{2}{3}\%$. General Asphalt is better by $6\frac{2}{3}\%$.

3. Which secures the better income on investment and how much, Bendix Aviation 2 at 30, or Coca Cola 6 at 180?

4. Find the cost of 125 shares U. S. Rubber at $17\frac{3}{4}$, brokerage $12\frac{1}{2}¢$ a share.

5. Compare the per cent of income on investment a share of Southern Railroad Stock at 85, yielding 8% dividend, and a share N. Y. Central Railroad Stock at 190, yielding 9% dividend.

6. Would a stock whose par value is \$100 and yielding 8% dividend likely sell above or below par? Why?

BONDS

You will note that the brokerage on stocks, as shown on the preceding page, varies with the market price of the stocks.

Brokerage on bonds is usually \$2 per \$1,000 of their par value, except government bonds, where it is $\frac{1}{16}\%$ or $\frac{1}{32}\%$ of the par value. Stocks are usually issued in shares of \$100 par value; bonds are most frequently issued in denominations of \$1,000 par value.

The dividend on stock may vary from year to year. The interest on bonds remains the same.

Mr. Smith owns 200 shares Standard Oil of New Jersey and 10 bonds of \$1,000 each and paying 4% interest, issued by the same company. As a stockholder he owns 200 shares, representing his interest in the company; as a bondholder he has really loaned the Standard Oil Company \$10,000 for which they pay

him 4% interest semi-annually till the date the bond matures when the bond is taken up and he is paid $10,000, or par value.

The bonds of a corporation are safer than its stock, because the interest on bonds must be paid before any dividend on stock is declared, and in the event the corporation fails the bond-holders must be paid first.

Find the cost of the following bonds, each having a par value of $1,000:

1. 210 Am. Tel. & Tel. Co. 5's at $107\frac{1}{2}$, brokerage $2.

2. Compare the per cent of income from a 5% $1000 bond at 90 with that from a 6% $1000 bond at 105.

Solution: 5% of $1000 = $50 income.

90% of $1000 = $900 cost or investment

$$\frac{\$50}{\$900} = 5\tfrac{5}{9}\%$$

6% of $1000 = $60 income

105% of $1000 = $1050 cost or investment

$$\frac{\$60}{\$1050} = 5\tfrac{17}{21}\%$$

$5\frac{17}{21}\% - 5\frac{5}{9}\% = \frac{1}{63}\%$ in favor of 6% bond.

3. Compare the per cent of income from a $3\frac{1}{2}\%$ $1000 bond at 80 with that from a $5\frac{1}{2}\%$ $1000 bond at 110.

224. CONTRACTS

When two parties agree to exchange property, or to carry out a business transaction, or the one agrees to render or receive a service from the other for a consideration, a *contract is entered into*. If the transaction involves a large amount of money or if it is to be executed at some future date, the agreement is generally reduced to a concise written form, and is signed by both parties whose signatures are usually witnessed. If one party makes a business proposition in a letter and the other party accepts the proposition by letter, it makes in effect a written contract.

There is no fixed form for written contracts, but they are often written somewhat like the following:

Contract Between John Adams and Henry Smith

This contract entered into this twentieth day of March, 1920, between John Adams of Vicksburg, Mississippi, party of the first part, and Henry Smith, of the same place, party of the second part, witnesseth:

That the said party of the first part rents to said party of the second part a store house and lot on B Street, with all the counters, shelves, furniture and permanent fixtures in said store house for a period of twelve months from April 1st, 1920 to April 1st, 1921.

In consideration of the use of said store house and lot, the said party of the second part agrees to pay said party of the first part fifty dollars ($50) per month, same to be paid in advance on the first day of each month.

In witness whereof we hereunto set our hands this twentieth day of March, 1920.

John Adams
Witness: Henry Smith
James Brown

The limitations and number of conditions that enter into a contract will determine the length of it. It may consist of only a few lines, or it may cover several type-written pages.

1. A packing house contracted to furnish a meat market with 4800 pounds of beef and 2500 pounds of pork each month. On the 25th of the month the meat market had received only 2500 pounds of beef and 1275 pounds of pork. (a) How much more meat was due the market in accordance with the terms of the contract? (b) If the packing house sold the beef at 15¢ a pound and the pork at 16¢ a pound, how much did the meat market owe the packing house on the 25th of the month?

2. A coal dealer contracted with a mine operator for a monthly delivery of 15 cars of coal averaging not less than 35 tons to the car, at a price of $3.50 f.o.b. (free on board) mines. If the freight was $1.25 a ton, how much did the coal dealer make on a ton if he sold it for $6.50?

3. A book agent made a contract with a publishing house to sell books on a commission of 15% or on a straight salary of $200

per month. If he sold $2500 worth of books a month, how much more would he make on commission than on straight salary?

4. John Brown bought a house and lot for $3750. When he signed the contract, he deposited $1500 in the bank with the instruction that it be turned over to the real estate agent when he furnished a satisfactory deed. If the purchaser gave a note secured by mortgage on his place for $2000, how much additional money did he need to pay for his place?

5. Write a contract in which you are the party of the first part and your schoolmate party of the second part.

225. Finding the Net Cost by Taking Off a Single Discount Equivalent to Two Discounts

Instead of taking off two discounts separately, a single discount equivalent to the two may be found first and the two taken off the list price in a single operation.

For example, if the two discounts of 25% and 20% are allowed, it is easily seen that after the first one is taken off, the cost is 75% of the list price; and that after 20% or $\frac{1}{5}$ of 75%, which is 15%, is taken off, 60% of the list price remains as the final cost, or net cost. That is, 100% of the first list price − 25% of the first list price = 75% of the first list price; and, 75% of the first list price − 20% of this remainder = 60% of the list price, or net cost.

NOTE: The order of taking off successive discounts is immaterial.

This process may be shortened by the following rule:

From the sum of the two discounts subtract their product, and take the result from 100%.

That is, $100\% - (25\% + 20\% - 25\% \times 20\%)$

$= 1.00 - (.25 + .20 - .25 \times .20)$

$= 1.00 - (.45 - .05)$

$= 1.00 - .40$

$= 1.00 - .40$

$= .60$, or 60%.

Note that the rates are all changed from the per cent form to the decimal form in order to perform the operations.

Problems

Find a single discount equivalent to:

1.	40% and 10%.	**11.**	40% and 20%.
2.	50% and 10%.	**12.**	28% and $16\frac{2}{3}$%.
3.	20% and 10%.	**13.**	55% and $33\frac{1}{3}$%.
4.	50% and 20%.	**14.**	36% and $12\frac{1}{2}$%.
5.	40% and $33\frac{1}{3}$%.	**15.**	$66\frac{2}{3}$% and 10%.
6.	50% and 25%.	**16.**	$33\frac{1}{3}$% and 10%.
7.	75% and 20%.	**17.**	52% and $8\frac{1}{8}$%.
8.	60% and 20%.	**18.**	50% and 5%.
9.	30% and 10%.	**19.**	60% and 25%.
10.	20% and $12\frac{1}{2}$%.	**20.**	25% and 60%.

21. Find a single discount equivalent to 25%, 20%, and 10%.

Hint: Find the equivalent of the first two by subtracting their product from their sum, and then the equivalent of this result and the third.

22. Which is the larger discount, a single discount of 65% or successive discounts of 60% and 10%? How much?

23. A man is given choice of a single discount of 70% or the successive discounts of 50% and 25% from a piano listed at $600. He chooses the single discount. Did he gain or lose by his choice? How much?

226. Annual Interest

Annual Interest is simple interest on the principal increased by simple interest on the unpaid interest from the date it is due till the date of settlement.

In some states annual interest is permitted, but in others it is prohibited by law. In order to avoid any conflict with the law, a non-interest bearing note is sometimes drawn up for the principal, and also a note for the unpaid interest of each year or interest period. These latter notes are called **interest notes.**

Problems

1. \$600.00 Auburn, Ala., May 4, 1920.

Five years after date I promise to pay John H. Warren, or order, Six Hundred Dollars, value received, with interest at 6%, *due and payable annually.* H. H. Jones.

If the interest remains unpaid until the note matures, how much will then be due?

Solution:

Principal	\$600.00
Simple interest on \$600 at 6% for 5 yr.	180.00
Interest on the first year's interest, \$36 at 6% for 4 yr.	8.64
Interest on the second year's interest, \$36 at 6% for 3 yr.	6.48
Interest on the third year's interest, \$36 at 6% for 2 yr.	4.32
Interest on the fourth year's interest, \$36 at 6% for 1 yr.	2.16
Total amount due at maturity of note	\$ 801.60

Note the shorter way of computing interest on interest:

Principal	\$600.00
Simple interest	180.00
Interest on \$36 at 6% for (4+3+2+1) yr., or 10 yr.	21.60
Total amount	\$ 801.60

Observe that the total amount is made up of three parts: the principal, the **simple interest,** and the **annual interest.**

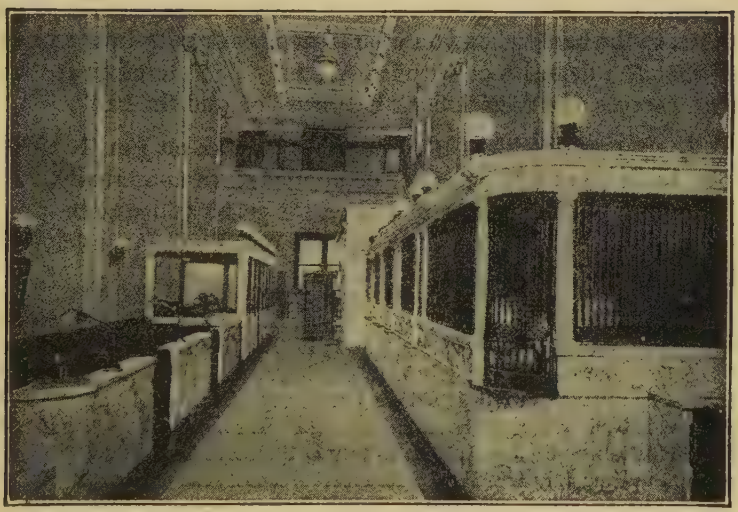
2. A note was dated March 1, 1920, and was to be paid March 1, 1923. If the rate of interest was 8%, the face of the note \$800, and nothing paid until maturity of the note, how much was due, annual interest being allowed?

3. What is the total amount due on the note described in Problem 2 if the interest is due and payable semi-annually?

Hint: There would be twice as many interest periods and the rate for each period will be one-half of the annual rate.

4. Find the total amount due, annual interest being allowed, on a note for \$400 dated Nov. 20, 1917, and due April 20, 1920 with interest at 5%, payable semi-annually.

227. Compound Interest and Savings Banks



Every boy and girl should form the habit of saving money. Have you any money in the savings bank? Did you buy any Thrift Stamps?

Not only are you saving your money when you buy Thrift Stamps or deposit your money in a savings bank, but your money is increasing by the interest it earns. And, better still, not only is the money you actually invest earning more money, but the interest is added to your investment at the end of a certain period of time, thus increasing the interest-bearing principal. This new principal is the sum of the interest and principal, or the amount, at the close of the interest period, and continues to increase similarly with each succeeding interest period. This is really interest on the principal and unpaid interest, combined at given intervals of time. It is called **compound interest.**

You may open an account, and begin your savings bank by

depositing $1 or more. You will be given a book in which the amount of your deposit is written. This book must always be taken to the bank when you wish to deposit or withdraw money so that the deposits or withdrawals may be entered in it. It really serves as a receipt book between you and the bank.

1. James deposits $100 in a savings bank on Jan. 1, 1920. The bank pays 4% interest, compounded semi-annually. If James leaves the interest and allows it to be added to the previous principal every six months, how much will James have in the bank on Jan. 1, 1922?

Solution: Deposit Jan. 1, 1920	$ 100.00
Interest on $100 at 4%, Jan. 1, 1920 to July 1, 1920	2.00
Amount in bank July 1, 1920	$ 102.00
Interest on $102 at 4% July 1, 1920 to Jan. 1, 1921	2.04
Amount in bank Jan. 1, 1921	104.04
Int. on $104.04 ot 4% Jan. 1, 1921, to July 1, 1921	2.08
Amount in bank July 1, 1921	$ 106.12
Int. on $106.12, July 1, 1921 to Jan. 1, 1922	2.12
Amount in bank Jan. 1, 1922	$ 108.24

Rate Per Period

Periods	2%	2½%	3%	4%	5%
1	$1.02000	$1.02500	$1.03000	$1.04000	$1.05000
2	1.04040	1.05063	1.06090	1.08160	1.10250
3	1.06121	1.07689	1.09273	1.12486	1.15763
4	1.08243	1.10381	1.12551	1.16986	1.21551
5	1.10408	1.13141	1.15927	1.21665	1.27628
6	1.12616	1.15969	1.19405	1.26532	1.34010
7	1.14869	1.18869	1.22987	1.31593	1.40710
8	1.17166	1.21840	1.26677	1.36857	1.47746
9	1.19509	1.24886	1.30477	1.42331	1.55132
10	1.21899	1.28008	1.34392	1.48024	1.62889
11	1.24337	1.31249	1.38423	1.53945	1.71033
12	1.26824	1.34489	1.42576	1.60103	1.79586

On the preceding page is a section of a compound interest table, showing the amount of \$1 at compound interest for any number of intervals from 1 to 12.

Most intervals, or periods, are 6 mo., that is, the interest is compounded semi-annually.

To illustrate: \$1 at compound interest for 2 years at 4%, compounded semi-annually, runs through 4 (half-year) periods with a rate of 2% ($\frac{1}{2}$ of 4%) for each period. Hence, under the 2% column and opposite the 4th period we find the amount, \$1.08243.

To find the amount for any number of dollars, multiply the amount of \$1 by the number of dollars. For example, in Problem 1 above, the time and rate were the same as in the illustration just given; hence, to find the amount for \$100, $\$1.08243 \times 100 = \$108.243$, which result agrees with that reached in the problem.

By the use of the table solve the following problems:

2. Find the amount of \$630 for 3 years at 5%, interest payable semi-annually.

3. Mary deposits \$10 in a savings bank on her 12th birthday. The bank pays 4% compound interest, payable semi-annually. How much will it amount to by the time Mary is 17 years old?

4. If Mary had put \$10 in the bank on each of her birthdays from the 12th to the 16th, how much would Mary have in the bank on her sixteenth birthday?

5. Find the difference between the amount of \$100 at simple interest for 10 years at 5%, and the amount of \$100 deposited in a savings bank that pays 5% compound interest, payable annually, for 10 years.

6. What amount will you have in the bank if you deposit \$5 on your 14th birthday, and \$5 every six months thereafter, until you are 20 years old, if the bank pays compound interest at 4%, semi-annually?

228. FARM ACCOUNTS

Every farmer has some idea of the cost of producing farm products and on what particular products he makes the greatest profits, but few keep accurate records that show just exactly the profit or loss on the various farm products.

It is not necessary to apply strict methods of bookkeeping to farm accounts, but sufficient records should be kept to enable the farmer to know definitely how much he is making or losing on a field, a crop, farm animals, or any other farm products.

A convenient form for keeping accounts is a double page for each person, firm, field, crop, etc. On the left-hand page are written the charge items, or what is paid out. On the right-hand page are written the credit items, or what is received. Or, a single page may be ruled so that there is a left-hand side and a right-hand side on the same page. Of course, the double page arrangement gives more room for details concerning each item.

Below is an account a farmer kept with Bob Frazier, a hired man.

BOB FRAZIER

1920			1920		
May 1	Cash	$ 3 00	June 1	1 month's work	$ 30 00
8	Cash	4 50			
15	Paid bill	6 30			
25	Cash	5 00			
		$ 18 80			$ 30 00
	Balance—Cash	$ 11 20			
		$ 30 00			$ 30 00

Problems

Rule a page, as above; open an account with John Collins, hired man, and make the proper entries of these items:

May 1, 1919: Began work.

3	Advanced cash	$ 3.00
15	Paid debt at store	7.50
16	Paid doctor bill	12.50
June 1	Credit for 1 month's work	38.00

Show the balance due John Collins on June 1, record it in the proper place, and close the account.

Below is an account kept by a farmer with 2 A. of potatoes:

2 A. Potatoes (Left-hand Page)

1920						
Mar. 20	Manure—3 T.	@	$ 2.50	$	7	50
21	Seed potatoes—22 bu.	@	1.75		38	50
June 1	Arsenate of lead—10 lb.	@	.15		1	50
July 1	Rent on land—2 A.	@	10.00		20	00
	120 hr. man labor	@	.30		36	00
	100 hr. horse labor	@	.20		20	00
	100 hr. use machinery	@	.10		10	00
				$	133	50
	Gain				72	30
				$	205	80

2 A. Potatoes (Right-hand Page)

1920						
Sept. 1	30 bu. potatoes, sold	@	$ 1.25	$	37	50
10	70 bu. potatoes, sold	@	1.20		84	00
20	30 bu. potatoes, sold	@	1.15		34	50
	12 bu. seed potatoes, sold	@	2.50		30	00
	14 bu. for house use	@	1.20		16	80
	Value of manure left in soil—(estimated)				3	00
				$	205	80
	Results:					
	Cost of production			$	133	50
	Cost per acre				66	75
	Cost per bushel					86
	Gain per acre				36	15

Problems

1. How was the gain ($72.30) found?

2. Find the total number of bushels produced.

3. How was the cost per bushel (86¢) found?

4. How was the cost per acre found?

5. How was the gain per acre found?

6. Rule two pages (left-hand and right-hand) and make the proper record of an account kept by a farmer who rented to Nat Edwards 6 A. of corn land:

1919			
Nov. 30,	Rent of land	@	$ 5.00
	2 T. silage	@	6.00
	45 hr. use of machinery	@	.05
	240 bu. corn	@	1.75
	40 hr. use of horse	@	.20
	5 T. manure	@	2.00

Close the account, and state the results. What is the gain?

7. Make out a double-page account for a 2 A. orchard, showing the items below:

Jan. 10, 1919	1 spray	@	$ 12.00
	1 bu. oats	@	.60
	3 lb. rape	@	.40
June 1,	100 apple barrels	@	.40
Nov. 1,	Use of land	@	per A. 4.50
	Apples sold		250.00
	Man labor		75.00
	Horse labor		30.00
	Use of machinery		20.00
	60 gal. cider	@	.60
	Fruit used at home		40.00
	Freight paid on apples		20.00
	5 apple barrels left over	@	.40

Close the account in proper form. What is the gain?

229. GOVERNMENT LIFE INSURANCE

(Selected from Bulletin L.D. 9, Bureau of War Risk Insurance.)

The Origin of Government Insurance in U. S.

When this country entered the World War, the need was at once felt for some liberal method of protection for the soldiers and their families. Congress realized that a man could fight better if he carried no fear in his heart for his loved ones' future should he die or become so disabled that he could no longer support his family or himself. Congress also realized that many men would come out of the war so impaired in mind or body that they could not pass a life insurance examination. This was the origin of Government Life Insurance in the United States.

The Principle of Life Insurance.

The general principle of insurance of any kind is the grouping together of many people to protect each other against possible loss. This principle can be better understood if we take fire insurance as an example. The risk of a fire may be slight, but the loss, if it does occur, may be very heavy, and if borne by one person alone it might mean financial ruin. When this loss is shared by thousands of others, no individual suffers, for each one pays a premium which from experience has been found to be sufficient to meet the loss by fire on any one or more of the houses insured. This same principle is the foundation of life insurance, though in a more complicated form.

Protection Lost Sight of.

Many who carry life insurance do not seem to realize that, if they fortunately go on living from year to year, they have been just as surely protected against loss through death as by their

fire insurance their houses have been protected against loss through fire, and *where they have only paid the exact cost of their death protection year by year* they are no more entitled to the return of any part of their life insurance premiums than they are entitled to the return of any part of their fire insurance premiums when their houses do not burn.

The premiums for War Term Insurance increase year by year, and if carried for an indefinite period beyond the limit allowed by the Government (of five years after the termination of the war as declared by proclamation of the President) it would not be long before they would reach a point where the men could not pay them.

The United States Government offered opportunity at any time, during the five years following the declaration of peace by proclamation of the President, to convert War Term Insurance to any one or any combination of the six permanent plans of U. S. Government Insurance, with premiums which do not increase, but which will decrease if dividends are earned in the future.

The following are the six forms of U. S. Government Life Insurance:

(1) Ordinary Life.

(2) Twenty-payment Life.

(3) Thirty-payment Life.

(4) Twenty-year Endowment.

(5) Thirty-year Endowment.

(6) Endowment maturing at age 62.

Below are given tables showing premium rates for three of the six forms:

230. PREMIUM RATES ON U. S. GOVERNMENT LIFE INSURANCE

Ordinary Life — **$1,000 Insurance**

Age at conversion	Monthly premium	Quarterly premium	Semi-annual premium	Annual premium
15	$1.04	$3.11	$6.20	$12.29
16	1.06	3.17	6.31	12.52
17	1.08	3.23	6.43	12.76
18	1.10	3.29	6.55	12.99
19	1.12	3.35	6.67	13.23
20	1.15	3.44	6.85	13.58
21	1.17	3.50	6.97	13.82
22	1.20	3.59	7.15	14.18
23	1.23	3.68	7.33	14.53
24	1.26	3.77	7.51	14.88
25	1.29	3.86	7.68	15.24
26	1.32	3.95	7.86	15.59
27	1.35	4.04	8.04	15.95
28	1.39	4.16	8.28	16.42
29	1.43	4.28	8.52	16.89
30	1.47	4.40	8.76	17.36
31	1.51	4.52	9.00	17.84
32	1.55	4.64	9.23	18.31
33	1.60	4.79	9.53	18.90
34	1.65	4.94	9.83	19.49
35	1.70	5.09	10.13	20.08
36	1.76	5.26	10.48	20.79
37	1.81	5.41	10.78	21.38
38	1.88	5.62	11.20	22.33
39	1.94	5.80	11.56	22.92
40	2.01	6.01	11.97	22.74
41	2.09	6.25	12.45	24.69
42	2.16	6.46	12.87	25.52
43	2.25	6.73	13.40	26.58
44	2.34	7.00	13.94	27.64
45	2.43	7.27	14.48	28.71
46	2.53	7.57	15.07	29.89
47	2.64	7.90	15.73	31.19
48	2.76	8.26	16.44	32.60
49	2.88	8.62	17.16	34.02
50	3.01	9.00	17.93	35.56
51	3.15	9.42	18.77	37.21
52	3.30	9.87	19.66	38.98
53	3.46	10.35	20.61	40.87
54	3.63	10.86	21.62	42.88
55	3.82	11.43	22.76	45.13

20-Payment Life **$1,000 Insurance**

Age at conversion	Monthly premium	Quarterly premium	Semi-annual premium	Annual premium
15	$1.63	$4.88	$9.71	$19.25
16	1.66	4.97	9.89	19.61
17	1.68	5.03	10.01	19.85
18	1.71	5.12	10.19	20.20
19	1.73	5.18	10.31	20.44
20	1.76	5.26	10.48	20.79
21	1.79	5.35	10.66	21.14
22	1.82	5.44	10.84	21.50
23	1.85	5.53	11.02	21.85
24	1.88	5.62	11.20	22.21
25	1.91	5.71	11.38	22.56
26	1.95	5.83	11.62	23.04
27	1.98	5.92	11.80	23.39
28	2.02	6.04	12.03	23.86
29	2.06	6.16	12.27	24.33
30	2.10	6.28	12.51	24.81
31	2.14	6.40	12.75	25.28
32	2.19	6.55	13.05	25.87
33	2.23	6.67	13.28	26.34
34	2.28	6.82	13.58	26.93
35	2.33	6.97	13.88	27.52
36	2.38	7.12	14.18	28.11
37	2.44	7.30	14.54	28.82
38	2.50	7.48	14.89	29.53
39	2.56	7.66	15.25	30.24
40	2.62	7.84	15.61	30.95
41	2.69	8.05	16.02	31.78
42	2.76	8.26	16.44	32.60
43	2.83	8.47	16.86	33.43
44	2.91	8.70	17.34	34.38
45	2.99	8.94	17.81	35.32
46	3.08	9.21	18.35	36.38
47	3.18	9.51	18.94	37.56
48	3.28	9.81	19.54	38.75
49	3.38	10.11	20.14	39.93
50	3.50	10.47	20.85	41.34
51	3.62	10.83	21.57	42.76
52	3.75	11.22	22.34	44.30
53	3.88	11.61	23.11	45.83
54	4.03	12.06	24.01	47.68

20-Year Endowment **$1,000 Insurance**

Age at conversion	Monthly premium	Quarterly premium	Semi-annual premium	Annual premium
15	$3.29	$9.84	$19.60	$38.86
16	3.29	9.84	19.60	38.86
17	3.30	9.87	19.66	38.98
18	3.30	9.87	19·66	38.98
19	3.30	9.87	19.66	38.98
20	3.31	9.90	19.72	39.10
21	3.31	9.90	19.72	39.10
22	3.31	9.90	19.72	39.10
23	3.32	9.93	19.78	39.22
24	3.32	9.93	19.78	39.22
25	3.33	9.96	19.84	39.34
26	3.33	9.96	19.84	39.34
27	3.34	9.99	19.90	39.45
28	3.34	9.99	19.90	39.45
29	3.35	10.02	19.96	39.57
30	3.36	10.05	20.02	39.69
31	3.37	10.08	20.08	39.81
32	3.38	10.11	20.14	39.93
33	3.39	10.14	20.19	40.05
34	3.40	10.17	20.25	40.16
35	3.41	10.20	20.31	40.28
36	3.43	10.26	20.43	40.52
37	3.44	10.29	20.49	40.64
38	3.46	10.35	20.61	40.87
39	3.48	10.41	30.73	41.11
40	3.51	10.50	20.91	41.46
41	3.53	10.56	21.03	41.70
42	3.56	10.65	21.21	42.05
43	3.60	10.77	21.45	42.53
44	3.63	10.86	21.62	42.88
45	3.68	11.01	21.92	43.47
46	3.72	11.13	22.16	43.94
47	3.78	11.31	22.52	44.65
48	3.84	11.49	22.88	45.36
49	3.90	11.67	23.23	46.07
50	3.98	11.91	23.71	47.02
51	4.06	12.16	24.19	47.96
52	4.15	12.41	24.72	49.02
53	4.25	12.71	25.32	50.20
54	4.37	13.07	26.03	51.62

231. Problems

1. Sam Hawley, an ex-service man, had $10,000 of War Term Insurance, and he wishes to convert $4,000 of it into United States Government Insurance. He is 24 yr. old. Which of the three forms shown in the tables will cost least in premiums? Which most?

2. If he chooses the Ordinary Life form, what will his premium be per year if he pays it annually? semi-annually? quarterly? monthly?

3. If he chooses the 20-Year Payment form, answer as in Problem 2.

4. If he chooses the 20-Year Endowment form, answer as in Problem 2.

5. How much will he pay the government in annual premiums in 20 years under the Ordinary Life plan? under the 20-Year Payment plan? under the 20-Year Endowment plan?

6. The annual premium, ordinary life insurance, for a man 25 years old is what per cent of the annual premium for a 20-year endowment policy?

7. Find the simple interest on each annual premium of the 20-Year Endowment policy for $4,000 from the time of its payment to the end of the twenty year period, if money is worth 6%.

Suggestion:

Annual premium, $4\times\$39.22=\$156.88$.

(1) Interest on $156.88 at 6% for 20 years.
(2) Interest on 156.88 at 6% for 19 years.
(3) Interest on 156.88 at 6% for 18 years, etc., to
(20) Interest on 156.88 at 6% for 1 year

Adding, Interest on $156.88 at 6% for $(1+20)\times 10$ years $=\$\_\_\_\_$

Note the method suggested for finding the sum of the 20 interest items. Since the product, $\$156.88\times 6\%$, appears in each of the 20 terms we need only find the sum of $(20+19+18+\ \ldots\ +1)$ years. The rule for finding the

sum of consecutive numbers, or numbers that increase by a fixed addend, is add the first and last numbers, and multiply their sum by half the number of terms; hence, the sum of the first 20 consecutive numbers $= (1+20) \times \frac{20}{2} = 21 \times 10 = 210$.

8. What is the sum of the 20 annual premiums?

9. Find the amount of the premiums, including interest as shown in Problem 7, for a term of 20 years.

232. GRAPHS

I. In order to compare at sight two or more numbers or quantities without making the actual computations, lines drawn to a convenient scale are frequently used. Such lines are called **graphs.** Thus, if train A runs 30 miles an hour and train B runs 60 miles an hour, a comparison of the rates may be shown by two straight lines drawn to scale, thus:

A_____

B_____

FIG. 1

1. How long is line A? line B?

2. What part of line B is line A?

3. Line B is how many times line A?

4. How does the rate of train A compare with that of train B?

5. How does the rate of train B compare with that of train A?

6. Draw lines to represent comparative rates of 30 mi. and 75 mi.

II. Sometimes representations of quantities for purposes of comparison are made by means of pictures. Thus, if a certain high school has 360 boys and girls, the comparative number of each may be represented by the picture on the next page.

FIG. 2

1. How many boys and girls in the picture?

2. What part is boys? What per cent? How many of the 360 students are boys?

3. What part is girls? What per cent? How many of the 360 students are girls?

The figure below represents the comparative production of cotton in America and in all other countries for a recent year:

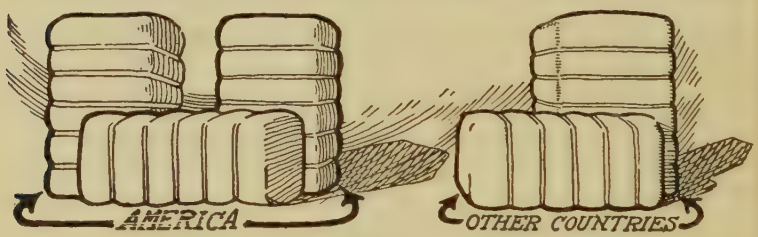


FIG. 3

1. How do the number of bales of cotton representing America's production compare with all the bales? What part? What per cent?

2. How do the number representing the remainder of the world compare with the whole? What part? What per cent?

3. How do the productions of the remainder of the world and of America compare? What part? What per cent?

III. The circle is used to compare (1) quantities among themselves, and (2) each quantity with the sum of all, or the whole. Thus, in the figure below, the entire circle represents the income of a family, and the various divisions of the circle

represent amounts spent for the purposes indicated. The circle is thought of as being divided into 360 equal parts, called degrees, and the scale is based on the degree.

Fig. 4 represents a family budget suggested for an income equal to the total of the various items.

1. What per cent of the income is the amount set apart for (a) food? (b) rent? (c) clothes? (d) operating expenses? (e) advancement? (f) insurance and savings?

2. What per cent of the food allowance is the rent allowance?

FIG. 4

3. The clothes allowance is what per cent of the insurance and savings allowance?

4. Use the circle for comparing items with each other and for comparing items with total income for the following budget:

Income	Food	Rent	Clothes	Operating Expenses	Advance-ment	Insurance and Savings
$4000	$875	$600	$600	$725	$400	$800

(a) What part of the circle will be used to represent the allowance for food?

Suggestion: Fractional part of circle for food $= \frac{875}{4000} = \frac{7}{32}$, or a little less than $\frac{1}{4}$.

(b) What part will be used for rent? clothes? operating expenses? advancement? insurance and savings?

(c) Will the size of the circle have any effect upon the fractional parts?

(d) Construct the circle showing the above relations and present to your teacher for inspection.

IV. One of the most frequently used forms of comparison is the bar diagram. For example, Fig. 5 shows the comparative quantities of cotton produced by the four principal cotton-producing grand divisions of the world in a recent year expressed approximately in metric tons:

GRAND DIVISIONS	TONS
America	3,240,000
Asia	1,770,000
Africa	340,000
Europe	239,000

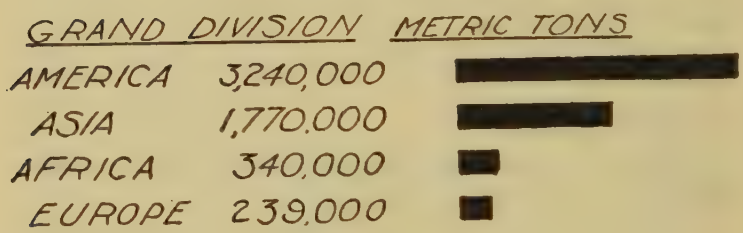


FIG. 5

Answer approximately by reference to the bars:

1. America's bar is about how many times as long as Asia's?

2. Asia's bar is about how many times as long as Africa's?

3. Africa's bar is about how many times as long as Europe's?

4. America's bar is about how many times as long as Africa's? as Europe's?

5. Asia's bar is about how many times as long as Europe's?

6. Asia's bar is about what part of America's?

7. Africa's bar is about what part of Asia's?

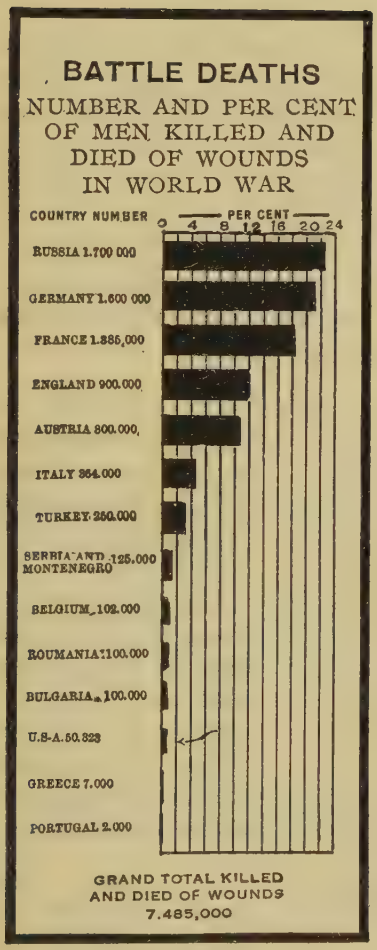


FIG. 6

Here is found another bar graph showing the number and per cent of men killed and died of wounds in the World War for the various countries.

1. By reference to the bars, answer the following questions:

(a) About what per cent of the total deaths did each country suffer?

(b) Verify your estimates by computations with the actual losses as shown.

(c) About what per cent of England's loss was the loss of Turkey?

(d) Verify your estimate by actual computation.

(e) The loss of France was about what per cent of Germany's loss?

(f) Test your estimate by actual computation.

V. If two quantities are related to each other in such way that a change in one causes a change in the other, the relation

of the two changing quantities may be represented by a continuous line.

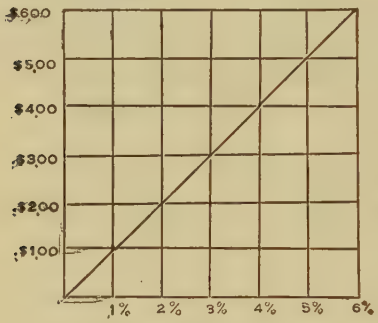


FIG. 7

For example, if the principal and time remain constant, the interest will change as the rate changes. The relation of rate and interest is shown in Fig. 7 for a principal of $100 and a period of 1 year:

1. How much is the interest when the rate is 6%? 5%? 2%? 4%?

2. Construct a graph to show the relation of rate and interest when the principal is $300 and the time 5 years.

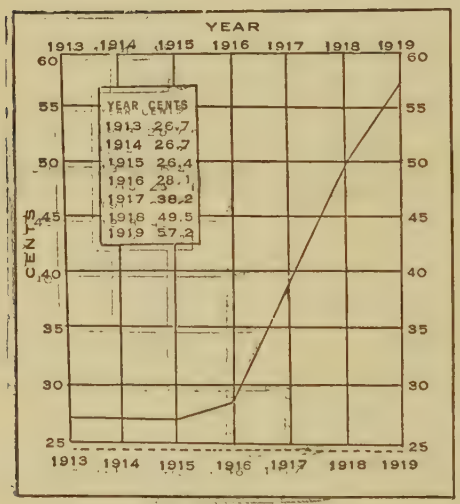


FIG. 8

The graph in Fig. 8 shows the average price of bacon from 1913 to 1919, inclusive, in the United States, as shown from the U. S. Bureau of Labor Statistics.

3. At what time was the price of bacon at its lowest? How is this shown on the graph? During which years was the increase most rapid? How is this shown on the graph? Can you account for the increase in price, 1915 to 1919?

The table below shows the relation between depth and pressure upon bodies submerged in fresh water:

Depth in feet	Pressure in pounds per square foot
2	125
4	250
6	375
8	500
10	625
12	750
15	937.5
20	1250
25	1562.5
30	1875

Draw two axes intersecting at right angles. Call the horizontal axis, *depth in feet*, and the perpendicular axis, *pressure in pounds*.. Use a scale of 1 in. = 4 ft. for the horizontal axis, and 1 in. = 250 lb. for the perpendicular axis. Locate points for the corresponding depth and pressure measurements and connect them with a smooth line.

4. About what is the pressure at a depth of 18 ft.? 24 ft.? At about what depth would the pressure be 1000 lb.?

5. What is the external pressure on a submarine 30 ft. under the surface if its outside surface is 4000 sq. ft.? Express the result in tons.

6. The rank of Tennessee in population as compared with the other states of the Union for the different census years from 1790 to 1920 is as follows:

Year	Rank	Year	Rank
1790	17	1860	10
1800	15	1870	9
1810	10	1880	12
1820	9	1890	13
1830	7	1900	14
1840	5	1910	17
1850	5	1920	19

6. Represent these relations graphically.

7. The national indebtedness and wealth of the principal countries involved in the Great War as reported in the World Almanac for 1920 are shown below, correct to the nearest billions:

	Indebtedness	National Wealth
United States	$ 26,000,000,000	$ 188,000,000,000
France	35,000,000,000	50,000,000,000
Great Britain	38,000,000,000	130,000,000,000
Italy	10,000,000,000	20,000,000,000
Germany	40,000,000,000	83,000,000,000

Represent these statistics graphically.

Suggestion: Use horizontal axis for **country**, and perpendicular axis for both **indebtedness** and **wealth.** You will have two curves.

(a) What do the perpendicular distances between wealth and indebtedness indicate?

(b) Which country has the greatest margin of wealth over indebtedness?

(c) What part of the wealth is the indebtedness in each country?

(d) How does the graph show this result for each country?

8. In a certain grammar school, the pupils were distributed by grades as follows: 1st, 156; 2nd, 141; 3d, 130; 4th, 120; 5th, 110; 6th, 94; 7th, 80; 8th, 65.

Make a graph showing the comparative number in each grade.

9. (a) In Problem 8, the number of pupils in the 8th grade is about what per cent of the number in the 1st grade?

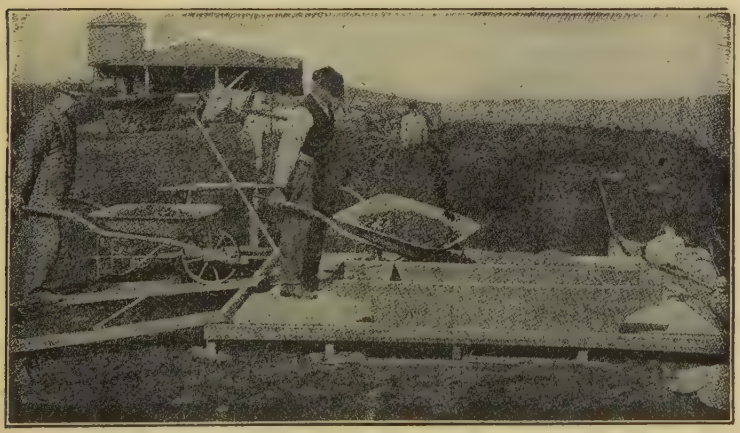
(b) Test your estimate by actual computation.

233. CONCRETE CONSTRUCTION

One of the most important materials used in construction work at the present is **concrete.**

Concrete may be defined as an artificial stone prepared by mixing cement, sand, and pebbles or broken stone in certain proportions, depending upon the nature of the work for which the concrete is to be used.

Concrete is used for a variety of purposes, such as building streets, roads, bridges, sidewalks, houses, foundations for buildings, silos, etc.



Cement is usually put up in standard packages (cloth sacks and paper bags) weighing 94 pounds net and considered as 1 cubic foot when proportioning mixtures by volume, which is the common method.

Four sacks of cement make one barrel. In the process of making the mixture for concrete, a sack of cement and the proportion of sand desired are thoroughly mixed and then the proper proportion of gravel or crushed stone is added and these three elements are thoroughly mixed together. This mixture is called a **batch**. The proper amount of water is then added to the mixture and thoroughly stirred. The concrete mixture is put into a frame or form where it hardens and takes the permanent shape of the containing form.

It is highly important that the sand be free from dirt and other foreign matter and that the gravel or stone be of the proper size. The amount of cement used in a 1:2:4 mixture, for instance, is 1 cu. ft. (one sack) of cement, 2 cu. ft. of sand, 4 cu. ft. of pebbles or crushed stone. In this book, the mixtures are arranged in the order: cement, sand, stone.

Read and explain the following mixtures: 1:2:3; 1:1:1; 1:1:1$\frac{1}{2}$; 1:3:6; 1:2$\frac{1}{4}$:4; 1:2; 1:2$\frac{1}{2}$.

QUANTITIES OF MATERIALS REQUIRED FOR VARIOUS MIXTURES OF MORTAR AND CONCRETE

Mixed Materials for One Bag Batch				Resulting Volume in Cubic Feet		Quantities of Cement, Sand and Pebbles or Stone Required for One Cubic Yard of Compact Mortar or Concrete				
	Cement in Sacks	Sand cu. ft.	Pebbles or Stone cu. ft.	Mortar	Concrete	Cement in Sacks	Sand		Stone or Pebble	
							Cu. Ft.	Cu. Yd.	Cu. Ft.	Cu. Yd.
1:1½	1	1.5		1.75		15.5	23.2	.86		
1:2	1	2.0		2.1		12.8	25.6	.95		
1:2½	1	2.5		2.5		11.0	27.5	1.02		
1:3	1	3.0		2.8		9.6	28.8	1.07		
1:2:3	1	2.0	3.0		3.9	7.0	14.0	.52	21.0	.78
1:2:4	1	2.0	4.0		4.5	6.0	12.0	.44	24.0	.89
1:2½:4	1	2.5	4.0		4.0	5.6	14.0	.52	22.4	.83
1:2½:5	1	2.5	5.0		5.4	5.0	12.5	.46	25.0	.92
1:3:6	1	3.0	6.0		6.4	4.2	12.6	.47	25.2	.94

Why would not 1 cu. ft. of cement, 1 cu. ft. of sand, and 1 cu. ft. of stone make a volume of 3 cu. ft. of mixture? Suppose you take a quart measure full of pebbles and see if you cannot put some coarse sand in it without taking out any pebbles. After putting in all the sand you can without taking out any pebbles, see if you cannot put in some cement which is much finer than sand; then see if the can will not hold some water. This experiment will show you why a 1:2:3 mixture makes only 3.9 cu. ft. of concrete instead of 6 cu. ft.

Suppose you measure a sidewalk made of 1:2:3 mixture and find it to be 24 ft. long, 5 ft. wide, and 4 in. thick. How many cubic feet of concrete in the walk? How much cement? sand? stone? Length $\times$ breadth $\times$ thickness equals number of units of volume.

$24' \times 5' \times \frac{1}{3}' = 40$ number of cubic feet

Since it takes a 1:2:3 mixture (6 cu. ft.) to make 3.9 cu. ft. concrete, it can readily be seen that to get the number of batches you would divide 40 cu. ft. by 3.9, the number of cubic feet made from one mixture; then multiply the quotient by the proportionate part representing each element of the mixture.

Thus:

No. batches of concrete	$=40 \div 3.9$
No. sacks of cement	$=1 \times (40 \div 3.9)$
No. cubic feet of sand	$=2 \times (40 \div 3.9)$
No. cubic feet of stone	$=3 \times (40 \div 3.9)$

Problems

1. A 1:2:3 mixture is recommended for one-course concrete street, road, walks, or concrete floors. What part of the mixture is cement? sand? crushed stone?

2. What will be the total volume in cubic yards of concrete mixed in 28 one-sack batches of 1:2:3 mixture?

3. How many sacks of cement will be required for a job using 9.36 cubic yards of 1:2:3 concrete? How many cubic yards

of sand will be required? How many cubic yards of pebbles will be required?

4. How many cubic feet in a concrete floor 15 ft. by 16 ft. and 6 in. thick?

5. How many sacks of cement will be required for the floor in Problem 4? How many cubic feet of sand? stone? (1:2:3 mixture.)

6. What would be the cost of the concrete floor at 22¢ a square foot?

7. How many cubic feet of concrete in a side-walk 468 ft. long, 5 ft. wide, 4 in. thick? (1·2:3 mixture.)

8. How many sacks of cement will be required for the side-walk in Problem 7? How many cubic feet of sand? stone?

NOTE: Sand and gravel or stone are usually sold by the cubic yard.

9. What will be the cost of the side-walk at 22¢ a square foot?

10. How many cubic feet of concrete in a mile (5280 ft.) of road 18 ft. wide, 8 in. thick?

11. In problem 10, how many barrels of cement will be required for the construction of the road? How many cubic yards of sand? How many cubic yards of gravel? (1:2:3 mixture.)

12. (a) What will be the cost of the cement in Problem 11 at $4.25 per barrel? (b) What will be the cost of the sand at $2.25 a cubic yard? (c) the crushed stone at $2.50 a cubic yard? (d) If the cost of the labor was $4250, what was the total cost of the mile of road?

13. What would be the cost of the road if built of crushed stone at $3.00 a cubic yard?

14. How many sacks of cement, cubic yards of sand, and cubic yards of stone will be required in a 1:2:3 mixture to build 2750 cu. yd. of concrete?

15. How many sacks of cement, cubic yards of sand, and cubic yards of stone in a $1:2\frac{1}{2}:4$ mixture will be required to build 1875 cu. yd. of concrete?

16. In a 1:3:6 mixture, how many cubic feet of concrete will 48 sacks of cement, 144 cu. ft. of sand, and 288 cu. ft. of stone make?

17. In a 1:2:4 mixture, how many sacks of cement, cubic yards of sand, and cubic yards of stone will be required to make 1275 cu. yd. of concrete for the foundation of a building?

18. Find the cement, sand, and pebbles required for the foundation mentioned in Problem 17, using a 1:2½:4 mixture.

19. How many square feet of side-walk can be made per sack of cement, making the slabs 5 inches thick, of single course construction, of 1:2:3 mixture?

20. How many loads of cement will be required for the foundation mentioned in Problem 17, if the wagons in which hauling is done can carry two tons over the roads which must be traversed in making this delivery?

21. A tank wagon holds 20 barrels of water. Each barrel holds 31½ gallons. How many loads will be required to furnish mixing water for 108 cubic yards of concrete of 1:2½:4 concrete, if 5½ gallons of water are required per sack of cement?

22. Mr. White wishes to build a concrete side-walk (using a 1:2½:4 mixture) 4 in in thickness and 4 ft. wide in front of his 54-foot lot. (a) How many sacks of cement will be required? (b) What will the side-walk cost at 25¢ a sq. ft.?

23. How many cubic yards of concrete will 2 cu. yd. of cement make in a 1:2:3 mixture?

24. How many cubic yards of concrete will a 1:2½:4 mixture make, using 6 cu. yd. of pebbles?

25. How many cubic feet of concrete will a dozen sacks of cement make in a 1:2:3 mixture?

235 SQUARES AND SQUARE ROOT

1. Find the number of square rods in a square field 18 rd. on the side.

Solution: $18 \times 18 = 324$

2. Note that 324 is the product of two equal factors. Such number is called a square. Instead of 18×18 we may express the operation with the exponent, as 18^2.

Exponent form	Equal factors	Squares
1^2	1×1	1
2^2	2×2	4
3^2	3×3	9
4^2	4×4	16

Extend the table until you reach 20^2. How many perfect squares are there between 1 and 100? 100 and 200? 200 and 300? 300 and 400?

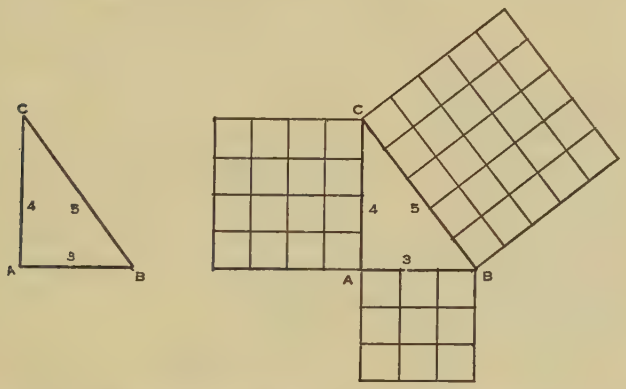
3. One of the two equal factors of a square is called its **square root.** Refer to Exercise 2 and name the square root of 9; of 36; of 144; of 225; of 324; of 400.

4. If a square field has an area of 324 sq. rd., what is the length of its side? (See Exercise 1.)

5. By reference to the work you did in Exercise 2, find the side of a square field whose area is 225 sq. rd.; 144 sq. rd.; 361 sq. rd.; 169 sq. rd.; 289 sq. rd.

6. What *relation* does the side of a square have to its area?

7. Consider this right triangle. Draw a square on each of the three sides, thus:



What is the area of the square on the longest side (hypotenuse)? on the shortest side? on the other side? How does the area of the square on the hypotenuse compare with the sum of the areas on the two sides?

In any right triangle, the square on the hypotenuse equals the sum of the squares on the two sides.

That is, $h^2 = a^2 + b^2$, when *h* stands for the hypotenuse, *a* for one of the sides, and *b* for the other side.

Find the hypotenuse of a right triangle whose sides are:

8. a = 6, b = 8.

Solution: $h^2 = 36 + 64 = 100$

If, $h \times h = 100$, that is, if two equal factors when multiplied = 100,
then, $h = 10$

9. a = 4, b = 3.

10. a = 9, b = 12.

Find the unknown side in each of the following right triangles:

11. $b=6$, $h=10$.

$a^2+b^2=h^2$

Solution: $a^2+36=100$

$a^2=64$ Why?

$a=8$ Why?

12. $a=15$, $h=25$.

13. $a=3$, $h=5$.

The symbol, ($\sqrt{}$), is called a **radical sign** and indicates that the square root is to be found. It resembles somewhat the letter *r* with a bar extending above and to the right. It is thought to have come from the letter *r*, the initial letter of the Latin word *radix*, which means root.

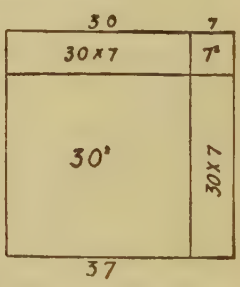
Perform the operations indicated:

(a) $\sqrt{25}$; (b) $\sqrt{100}$; (c) $\sqrt{81}$; (d) $\sqrt{400}$; (e) $\sqrt{225}$.

Squaring a Number of Two Figures

1. Square 37. Note the two ways of squaring.

	(a)		Process (b)		
	37	=	30 + 7		
	37	=	30 + 7		
(1)	49	=		7^2	(1)
(2)	210	=	7×30		(2)
(3)	210	=	30×7		(3)
(4)	900	=	30^2		(4)
	1369	=	$30^2+2\times30\times7+7^2$		



2. How many partial products are there in squaring 37?

3. Into how many parts is the square divided?

4. How many of the parts are squares?

5. What are the other parts called?

6. How do these other parts compare in size?

7. What are the dimensions of each of the sub-divisions of the square?

8. What is the area of each of the sub-divisions?

9. How would you get the area of the whole square from the areas of the sub-divisions?

10. Is that what was done above?

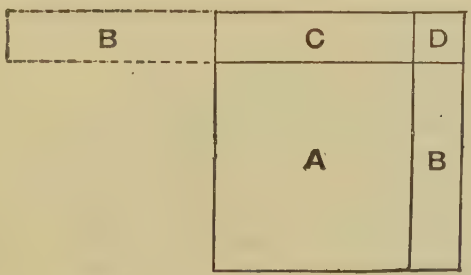
Squares of units	Squares of tens	Squares of hundreds
$1^2 = 1$	$10^2 = 100$	$100^2 = 10000$
$2^2 = 4$	$20^2 = 400$	$200^2 = 40000$
$3^2 = 9$	$30^2 = 900$	$300^2 = 90000$
$4^2 = 16$	$40^2 = 1600$	$400^2 = 160000$
$5^2 = 25$	$50^2 = 2500$	$500^2 = 250000$
$6^2 = 36$	$60^2 = 3600$	$600^2 = 360000$
$7^2 = 49$	$70^2 = 4900$	$700^2 = 490000$
$8^2 = 64$	$80^2 = 6400$	$800^2 = 640000$
$9^2 = 81$	$90^2 = 8100$	$900^2 = 810000$

1. Name the square roots of: 36; 1600; 81; 40000; 3600; 810000.

2. How many figures are required to express the squares of 3? 9? 20? 70? 90? 100? 500? 900?

3. By reference to the table above, you will note that the *number of figures in a square is either twice the number in the square root or one less than twice the number.*

4. How many figures will the square roots of these numbers have: 81; 324; 2025; 6561; 14641.



5. Find the square root of 1369; that is, find the side of a square whose area is 1369. The operation is the reverse

of squaring. By reference to the figure, answer these questions: (1) What is the area of the larger square (A)?

Hint: Its area is the square of tens next below 1369 in the table on page 377.

6. How can you find its side when you know its area?

7. If the area of the larger square be taken away from that of the entire square, what sub-divisions remain?

8. What are the dimensions of rectangle B? of rectangle C? of square D?

9. Since most of what remains is composed of the two rectangles, B and C, and since they each have one dimension (30) equal to the side of the square A, why would 2×30 divided into what remains give approximately the other dimension of B and C?

10. Note that the other dimension of B and C is also the side of the small square (D). Why would the sum of the lengths of B, C, and D, multiplied by the side of the square D, give the area of B, C, and D?

Suggestion: Think of B, C, and D put together end to end. The figure would be a rectangle $30+7$ long and 7 wide.

Process

		Square	Root
		13 69	
Square of tens	=	9 00	30
Trial divisor, 2×30	$=60$	4 69	7
Next figure in root, $469\div60$	$=7$		37 (30+7)
Complete divisor, $60+7$	$=67$		
Complete divisor$\times$next figure, 67×7		$=4\ 69$	

(1) Find the largest square next below 1360 (900).

(2) Write the square root (30) of this square to the right.

(3) Subtract the square found (900) from 1369.

(4) The remainder (469) contains the sum of the areas of two rectangles and a square (B+C+D).

(5) Double the part of the root found (2×30), and using this as a trial divisor into 469 find the next figure in the root (7).

(6) Multiply the sum of the trial divisor and next figure in the root (60+7) by the next term in the root (7).

11. Find the square root of 2137444. How many figures will there be in the root? Why?

Abbreviated Process

	Square	Root
	2′ 13′ 74′ 44′	1462
	1 (omit naughts)	
[2×10=]20	1 13	
[113÷20 =] 4 (about)		
24 [×4 =]	96	
[2×140=]280	17 74	
[1774÷280 =] 6 (about)		
286 [×6 =]	17 16	
[2×1460=]2920	58 44	
[5844÷2920=] 2 (about)		
2922 [×2 =]	58 44	

Note that the part of the root found at any stage of the process is considered as tens, that the tens are doubled for a trial divisor, and that the divisor is completed by adding the next figure of the root. Parts of the process in brackets [ ] may be omitted in ordinary work.

Find the square root of:

12. 12321.

13. 8046.09

Suggestion: Decimals are set off in periods of two figures each, naught being annexed if necessary to make a full period.

14. 63001 **17.** 110.25 **20.** 824.9

15. 1176.49 **18.** 299209 **21.** .7921

16. 0.2809 **19.** 404496

Perform the operation indicated to three decimal places if necessary.

22. $\sqrt{\frac{4}{9}}$. **23.** $\sqrt{\frac{9}{16}}$ **24.** $\sqrt{\frac{25}{49}}$.

Suggestion: Find the square root of numerator and denominator.

25. $\sqrt{\frac{11}{16}}$ **26.** $\sqrt{\frac{6}{25}}$ **27.** $\sqrt{\frac{2}{5}}$

Suggestion: Change to a decimal first.

236. Problems

Find answers to three decimal places.

1. A square field contains 10 acres. Find its dimensions.

2. How many rods of fence will be necessary to enclose a square lot containing 2 acres?

3. If your class-room is 30 ft. by 28 ft., how long is the diagonal on the floor?

4. Use the two dimensions in Problem 3 and a height of 14 ft. for your classroom, and find the distance from one lower corner to the opposite upper corner.

5. The area of a circle is expressed in the formula, $A=\pi\times r^2$, in which A represents the area and r the radius. Find the area of a circle whose radius is 5 ft. $\pi=3.1416$.

Interchange A and $\pi\times r^2$; thus, $\pi\times r^2=A$.

6. If you divide the formula in Problem 5 through by π, note the result:

(a) $\pi\times r^2=A$

(b) $\dfrac{\pi\times r^2}{\pi}=\dfrac{A}{\pi}$

(c) $r^2=\dfrac{A}{\pi}$ (How?)

If you know the area of a circle, how may you find r^2? Then how would you find r?

7. The area of a circle is 28.2744 sq. ft. Find its diameter.

Solution: $r^2=\dfrac{A}{\pi}$

$r^2=\dfrac{28.2744}{3.1416}$

$r^2=9$

$r=\sqrt{9}=3$

$d=2r=6$

8. If two silos are the same height, but the first has four times as large area in its base as the second, how do the capacities compare?

Hint: Volumes of solids having the same height are to each other as the areas of their bases.

9. If the base of the larger is 314.16 sq. ft., find the diameter of the smaller.

10. If the area of the base of a silo be multiplied by the height, expressed in the same units of measure as the diameter, the result is called the volume or capacity, as shown in the formula, $V = \pi r^2 \times h$, where V represents the volume or capacity and h the height.

11. Find the capacity of a silo with a diameter of 8 ft. and a height of 40 ft.

12. A square box containing 30 bu. of grain was filled to a uniform depth of 12 in. Find the length of one side of the box.

237. Applications of Square Root

1. Find the side of a square field containing 13 A. 52 sq. rd. 23 sq. yd.

2. The side of a square is 14 yd. Find its diagonal to two decimal places.

3. Find the side of a square equal in area to a rectangle whose sides are 8 in. and 2 in.

4. Find the side of a square equal in area to a rectangle whose sides are 24.7 ft. and 7138.3 ft.

5. A square field contains 21 A. 121 sq. rds.

(a) Find its perimeter.

(b) How many miles would a man travel in walking around the field 6 times?

(c) How many posts would it take to fence the field putting the posts 15 ft. apart?

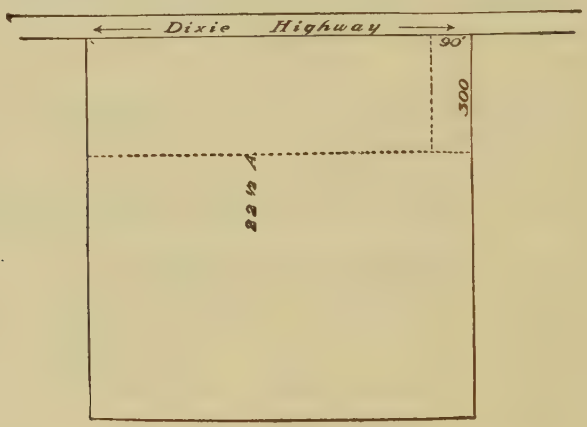
6. Find the length of a side of a square field containing 5 A. 46 sq. rd. $8\frac{1}{2}$ sq. yd.

7. A square plot of land contains 824464 sq. yd.

(a) What is the length of each of its sides?

(b) How many square yards in a driveway 8 yd. wide surrounding the square?

(c) What will it cost to re-surface the driveway at $1.25 a square yard?



8. One side of a square field of $22\frac{1}{2}$ A. abuts on the Dixie Highway.

The side of the field bordering on the road is divided into lots 300 ft. deep with a frontage of 90 ft. each.

(a) How many lots are there?

(b) What is the value of all the lots at $375 each?

(c) What is the value of the remaining part of the field at $275 an acre?

9. How long will it take a surveyor to run the boundary of a square field containing 13 A. 81 sq. yd. if he travels at the rate of 32 yd. a minute?

SUPPLEMENT

238. MENSURATION

NOTE: The supplement includes a more advanced treatment of mensuration and the equation, and also the subjects of cube root, the metric system, and a list of review problems. The teacher may use such of these subjects as the needs of the class require.

In measurements we have already treated plane surfaces and rectangular solids.

A plane surface bounded by straight lines is called a **polygon.**

Polygons are named from the number of sides.

A **triangle** is a polygon of 3 sides.

A **quadrilateral** is a polygon of 4 sides.

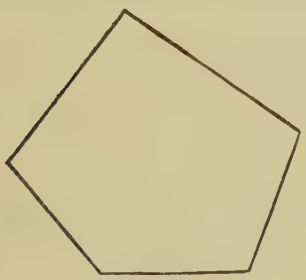


Fig. 1

Fig. 2

A polygon of five sides is called a **pentagon.**

A polygon of six sides is called a **hexagon.**

If the plane figure has equal sides and equal angles, **it is called a regular polygon.**

What kind of a triangle is a regular polygon?

What quadrilateral is a regular polygon?

Draw a circle. Use the radius as a chord and see how many equal arcs on the circumference of the figure it will cut off. Connect the arcs. What kind of a figure have you?

Draw a radius to each vertex and you will have six equal equilateral triangles. Cut out of paper an isosceles or an equi-

lateral triangle. Fold it at the center of the base so that the crease forms a right angle with the base. The two triangles thus formed are what kind of triangles? How do you find the area of each triangle?

The area of any regular polygon is equal to the sum of the areas of the triangles composing it.

1. Find area of a hexagon whose sides are each 10 ft. and the perpendicular distance from the center to one side is 8.66 ft.

2. How far is it from the corner of a square room that is 16 ft. on a side to the center of the room? Draw the figure.

3. Find the area of a triangular field each side of which is 40 rd. Draw the figure.

4. The base of an isosceles triangle is 6 ft. and its equal sides 5 ft. Find its area.

5. Each side of an equilateral triangle is 8 ft. (a) What is its altitude? (b) What is its area? Draw the figure.

6. Find the area of a regular hexagon whose side is 8 ft. and whose distance from its center to side is 6.9 ft.

7. What is the area of a regular hexagon inscribed in a circle whose radius is 6 ft.? (See Figure 2.)

8. Find the area of a regular pentagon whose side is 10 ft. and whose perpendicular distance from the center to each side is 7.75 ft.

9. Find the area of an isosceles triangle whose equal sides are 6 ft. and whose base is 8 ft.

Revolve the rectangle around A B as an axis and you will describe a cylinder. The altitude of a cylinder is the perpendicular distance between the bases.

The curved surface extending from base to base is called the **convex** or **lateral surface.**

The volume of a cylinder is determined by the number of cubic units it contains.

You learned in measurements that you found the volume of a body by multiplying the length by the breadth (area of bottom) by the height. You may find the volume of a cylinder by *multiplying the area of the base by the altitude; that is,* $V = b \times a$, where b = area of base, and a = altitude.

1. How many cubic feet in a cistern 8 ft. in diameter and 12 ft. deep?

Suggestion: No. cu. ft. $= 4^2 \times \pi \times 12$.

2. How many gallons in a cylindrical tank 30 inches in diameter and 6 ft. deep?

3. How many gallons of gasoline in a cylindrical tank 3 ft. in diameter and 5 ft. in height?

4. How many gallons in a cylindrical can 20 in. in diameter and 30 in. deep?

If you will cut a piece of tissue paper in the form of a rectangle, with one side equal in length to the circumference of the cylinder and the other side equal to the altitude, you will find that it will exactly cover the cylinder.

The convex surface of a cylinder is obtained by multiplying the circumference of the base by altitude; that is, con. sur. $= c \times a$.

5. How many square feet of asbestos will be required to cover a pipe 18 in. in circumference and 20 ft. long?

6. What will it cost to paint the lateral surface of a circular silo whose diameter is 8 ft. and whose altitude is 32 ft. at 75¢ a square (100 sq. ft.)?

7. How many cubic feet in a marble column whose diameter is 5 ft. and whose altitude is 30 ft.?

A **cone** is a solid whose base is a circle and whose lateral surface tapers to a point called the **vertex.**

The volume of a cone is $\frac{1}{3}$ the volume of a cylinder having the same base and altitude; that is, $V=\frac{1}{3}\times b\times a$.

To verify this, take the conical vessel, fill it with sand and see if the cylinder will not hold exactly three measures of the conical vessel.

1. Find the volume of a cone whose altitude is 12 in. and the area of whose base is 9 sq. in.

$V=\frac{1}{3}(9\times 12)=36$. No. of cubic inches.

Find the volume of a cone whose altitude is:

2. 12 in. and area of base 20 sq. in.

3. 17.6 in. and area of base 39.75 sq. in.

4. 8 ft. and the radius of whose base is 4 ft.

Hint: How do you find the area of a circle when the radius is given?

5. Find the area of the lateral surface of a cone whose circumference at the base is 20 ft., and whose slant height is 30 ft.

One-half the product of the circumference of base by the slant height equals the lateral surface; that is, $Area=\frac{1}{2}\times c\times s$.

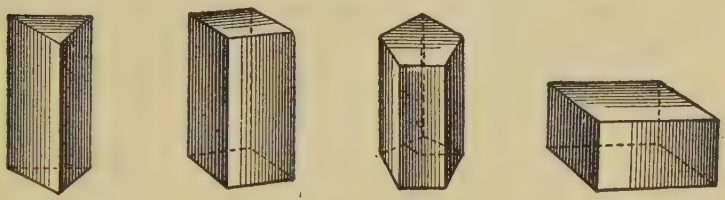
6. How many square inches of tin foil will be required to cover the lateral surface of a cone whose base is 37.5 in. in circumference and whose slant height is 12 in.?

7. How many square yards of cloth will be required to make a conical tent whose circumference is 60 ft. and whose slant height is 18 ft.?

8. How many cubic inches in a conical cup whose base is 3 in. in diameter and whose altitude is 4 in.?

9. Find the cost of painting a cone-shaped tower whose slant height is 24 ft. and whose diameter is 20 ft., at 65¢ a square yard.

Solids



A body that has length, breadth, and thickness is called a **solid.**

The surfaces bounding a solid are called its **faces.**

The area of the faces of a solid is called its **lateral surface.**

The volume of a solid is the number of times it contains its unit of measure.

The unit of measure for solids is the cubic inch, cubic foot, or cubic yard.

A solid whose faces are rectangles and whose bases are equal and parallel polygons is called a **prism.**

Such prisms are named from the number of equal sides found in their bases, as triangular, quadrangular, pentagonal, hexagonal, etc.

Cut a piece of paper in the shape of a rectangle, one of whose sides is the length of the altitude of the prism and the other the length of the perimeter. This rectangular piece of paper will exactly cover the lateral surface of the prism.

The lateral surface of a prism then may be found by multiplying its perimeter by its altitude and that product by its unit of measure; that is, Lat. Sur. $= p \times a$.

1. Find the lateral surface of a regular triangular prism each side of which is 8 ft., height 12 ft.

2. Find the lateral surface of a pentagonal prism with side 8 ft., altitude 12 ft.

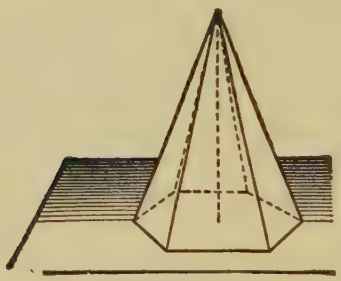
3. Find the volume of a quadrangular prism whose base is 5 ft. square, and whose height is 11 ft.

4. Find the volume of a pentagonal prism, the area of whose base is 30 sq. ft., altitude 15 ft.

5. Find the lateral area of a hexagonal prism. each side of whose base is 8 ft., and whose altitude is 20 ft.

6. Find the lateral surface of a triangular prism each of whose sides is 6 ft., height 8 ft.

Pyramid



A pyramid is a solid whose base is a regular polygon and whose faces are composed of triangles which meet at a point called the vertex.

Pyramids are named from the number of sides found in the base, as triangular, quadrangular, pentagonal, hexagonal, etc.

The altitude of a pyramid is the perpendicular distance from the vertex to the base.

The slant height of the pyramid is the altitude of the triangles that bound it.

The lateral area of a pyramid is equal to $\frac{1}{2}$ *the product of the perimeter by the slant height multiplied by its unit of measure; that is, Lat. Area* $= \frac{1}{2} \times p \times s$.

The volume of a pyramid is one-third the volume of a prism that has the same base and altitude. To verify this statement, take a hollow pyramid, fill it with sand, and see if you cannot empty three measures into a hollow prism having the same base and altitude as the pyramid.

The volume of a pyramid then is found by taking $\frac{1}{3}$ of the product of its altitude and area of its base; that is, $V = \frac{1}{3} \times a \times b$.

1. Find the volume of a triangular pyramid the area of whose base is 25 sq. ft. and whose altitude is 10 ft.

2. Find the lateral surface of a pentagonal pyramid with each side 6 ft., and slant height 12 ft.

3. Find the volume of a pyramid whose base is 10 ft. square and whose altitude is 12 ft.

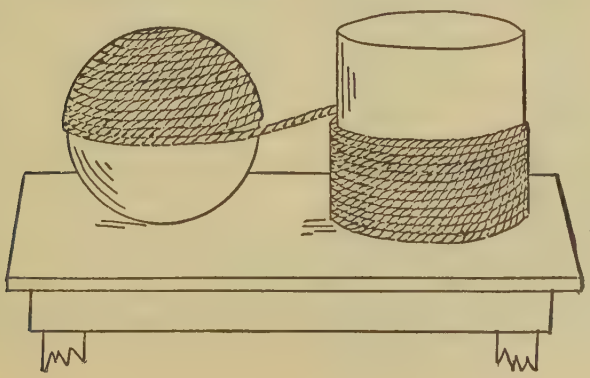
4. How many cubic yards in a rectangular prism whose base is 6 ft. by 4 ft. and whose altitude is 10 ft.?

5. What is the lateral surface of a pentagonal pyramid each side of whose base is 5 ft., slant height 10 ft.?

239. Spheres

A **sphere** is a solid bounded by a curved surface, every point of which is equally distant from a point within called the center.

Comparative Surfaces of Cylinder and Sphere



The surface of a sphere is equal to the convex surface of a cylinder whose height and diameter are the same as the diameter of the sphere.

The truth of this statement is demonstrated in geometry.

This can be shown by taking a cylinder with a diameter of 6 in. and altitude of 6 in. and a sphere whose diameter is 6 in. and comparing their surfaces thus: Wind a cord around the cylinder till it is covered. Unwind the cord from the cylinder on to the sphere as shown in the figure on page 389.

When the cord is unwound the distance of 3 inches or $\frac{1}{2}$ the altitude of the cylinder, it covers $\frac{1}{2}$ the surface of the sphere.

1. Find the convex surface of a cylinder the diameter of whose base is 6 in. and whose height is 6 in., or of a sphere whose diameter is 6 in.

Convex surface of cylinder $= c \times h$.

Circumference $= 6 \times \pi$.

Height $= 6$. Surface $= 6 \times 6 \times \pi = 6^2\pi$.

$6^2\pi = 6^2 \times 3.1416 = 113.0976$: use d for its equal, 6, and we have the formula $d^2\pi =$ convex surface of a sphere.

Hence, the square of the diameter multiplied by 3.1416 = the convex surface of a sphere; that is, Surface of sphere $= d^2 \times \pi$.

2. Find the surface of a globe 18 in. in diameter.

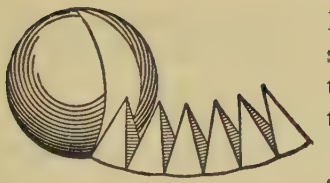
3. At 45¢ a square yard, find the cost of painting a dome in the shape of a hemisphere whose diameter is 20 ft.

4. How much leather is required to cover a ball 4 in. in diameter?

5. Find the area of the surface of the earth if its diameter is 8000 miles.

6. A large globe representing the earth is 8 ft. in diameter. What is the area of its convex surface?

7. Find the convex surface of a hemisphere 3 ft. in diameter.



From the figure, it is shown that a sphere may be cut up into pyramids, the sum of whose bases is equal to the convex surface of the sphere.

The volume of each pyramid is equal to $\frac{1}{3}$ of the area of the base multiplied by the altitude. The altitude of these pyramids is the radius of the sphere.

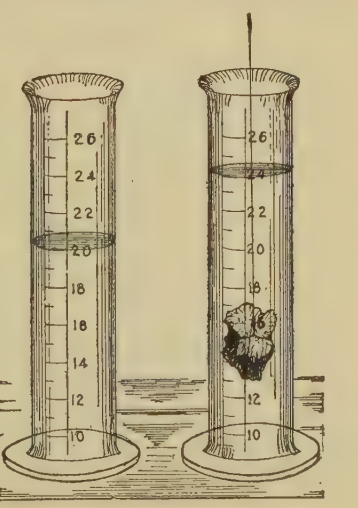
Hence, the volume of a sphere is equal to $\frac{1}{3}$ of the surface area multiplied by the radius; that is, $V = \frac{1}{3} \times s \times r$.

8. Find the number of cubic inches in a ball 5 inches in diameter.

9. Find the number of cubic feet in a globe 2 feet in diameter.

240. To Find the Volume of An Irregular Body

Since any body submerged in water displaces a volume of water equal to its own volume, find the number of cubic units of water displaced and you have the number of cubic units in the irregular body. Thus, if a cylindrical jar 6 inches in diameter and 26 inches deep contains 20 in. of water, and you place in the jar a stone which causes the water to rise in the jar to 24 in., the 4 in. increase in the depth of the water is due to the volume of the stone.



π r^2 × increase of depth = volume.

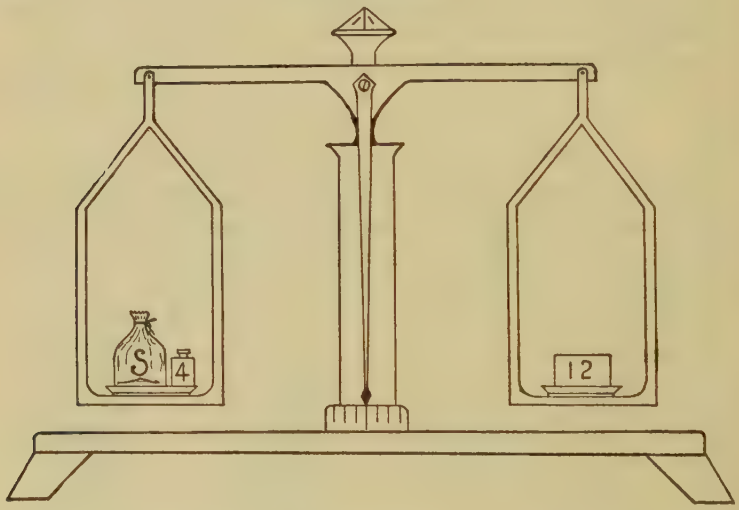
3.1416 × 9 × 4 = 113.0976 No. cu. in.

1. A stone was submerged in a jar 8 inches in diameter and caused the water to rise 4 inches; what was the volume of the stone?

2. A piece of iron put in a cylindrical jar 6 inches in diameter caused the water to rise 2 inches. Find the volume of the iron.

3. A lump of coal was placed in a cylindrical jar 2 ft. in diameter and caused the water to rise 6 in. What was the volume of the lump of coal?

241. The Equation



1. Before anything was placed on the pans, the scales shown in the figure above were perfectly balanced. A 12-lb. weight was placed in one pan and a bag of sugar, S, placed in the other.

The sugar, S, was found to be lighter than 12 lb., and a 4-lb. weight placed in the pan with the sugar brought the scales back in balance.

2. If 4 lb. could be taken from each pan, would the scales still be in balance?

3. By what weight would the sugar then be balanced?

4. What is then the weight of the sugar, S?

5. The relation of the objects in the pans may be expressed as follows: $S+4=12$. Why?

6. The expression, $S+4=12$, is called an **equation.** If 4 were taken away from each side of the equation what would the remainders on each side be?

Hint: $S+4-4=12-4$.

7. Would the remainders be equal? Read and give the meaning of $S=8$.

8. An equation always states an equality between two expressions. The two expressions, one on either side of the sign of equality, are called **members** of the equation, the one on the left the **first member** and the one on the right the **second member.**

I. If the same number be subtracted from both members of an equation the remainders are equal.

Find the value of x in the following equations:

1. $x+3=10$.

Solution: Subtract 3 from each member of the equation.

Thus: $x+3=10$

$x=7$

2. $x+2=8$

3. $x+6=15$

4. $x+1\frac{1}{2}=4\frac{1}{2}$

5. $x+16=20$

6. $x+1=10$

7. $x+19=20$

8. $x+3\frac{1}{2}=5\frac{1}{2}$

9. $x+4=4$

10. $x+2\frac{1}{8}=4\frac{2}{8}$

11. $x+1.5=4.5$

1. Three balls of the same weight each were placed in one pan of the scales and exactly balanced a 12-lb. weight. What equation represents the relation if we let x stand for the weight of 1 ball?

2. If $3x=12$, what is the value of x?

3. Observe that $3x$ means $3 \times x$. If we wish to divide each member of the equation by 3, we might show the operation as follows:

$\frac{3x}{3} = \frac{12}{3}$;

Or, $x = 4$. Why?

II. If both members of an equation be divided by the same number, the quotients are equal.

What are the quotients in Ex. 3?

Solve the following equations: (To solve an equation means to find the value of the unknown letter.)

1. $2x = 10$

Solution: $2x = 10$. Dividing each member by 2, $x = 5$.

2. $4x = 8$

3. $3x = 18$

4. $5x = 15$

5. $7x = 21$

6. $2x = 7$

7. $3x = 8$

8. $1.2x = 12$

9. $.03x = .03$

10. $.03x = 3$

11. $2x + 4 = 14$

Solution: $2x = 10$ Why?

$x = 5$ Why?

12. $3x + 1 = 10$

13. $5x + 2 = 17$

14. $7x + 2 = 23$

15. $6x + 4 = 28$

16. $4x + 3 = 15$

17. $6x + 2 = 20$

18. $9x + 1 = 19$

19. $7x + 10 = 45$

1. If 4 castings of equal weight are placed in one pan of the scales and a 30-lb. weight in the other, and if it requires an upward pull of 2 lb. on the pan containing the castings to balance the scales, which is heavier, the 4 castings or the 30-lb. weight? How much heavier?

2. The relation may be expressed in the equation: $4x - 2 = 30$, in which x stands for the weight of 1 casting. Why?

3. If 2 were added to both members of the equation, thus, $4x - 2 + 2 = 30 + 2$, what is the resulting equation?

4. What is the value of x (the weight of one of the castings)?

III. If equal numbers be added to both members of an equation, the sums are equal.

What are the sums in Ex. 3?

Solve for x in the following:

1. $3x-2=10$

Solution: $3x-2+2$ Why?
$3x=12$ Why?
$x=4$ Why?

2.	$7x-1=20$	**7.**	$4x-11=37$
3.	$6x-3=27$	**8.**	$5x-4=16$
4.	$4x-7=13$	**9.**	$3x-9=27$
5.	$5x-2=18$	**10.**	$8x-1=15$
6.	$9x-\frac{1}{2}=17\frac{1}{2}$	**11.**	$10x-2=28$

1. If $3x=12$, $x=4$. Why?
2. If $x=4$, $3x=12$. Why?

IV. If both members of an equation are multiplied by the same number, the products are equal.

What are the products in Ex. 2?

Solve for x in the following:

1. $\frac{1}{2}x=3$.

Solution: If each member of the equation is multiplied by 2, we have $x=6$.

2.	$\frac{1}{8}x=6$	**7.**	$\frac{1}{4}x=6$
3.	$\frac{1}{5}x=5$	**8.**	$\frac{1}{6}x=7$
4.	$\frac{1}{8}x=\frac{2}{8}$	**9.**	$\frac{1}{5}x=\frac{2}{5}$
5.	$\frac{2}{8}x=4$	**10.**	$\frac{1}{5}x=6$
6.	$\frac{3}{5}x=9$	**11.**	$\frac{1}{10}x=.5$

1. The four principles used so far may be summarized as follows:

I. *If equals are added to equals, the sums are equal.*

II. *If equals be subtracted from equals, the remainders are equal.*

III. *If equals be multiplied by equals, the products are equal.*

IV. *If equals be divided by equals, the quotients are equal.*

2. By the application of these principles to the equation, it becomes a very useful instrument in solving problems.

Solve these problems by use of the equation:

1. Find the principal that will yield \$36 interest in 1 yr. at 6%.

Solution: Let x = the principal

then, $\frac{6}{100}x = \$36$. (1) Why?

Applying principle III, $6x = \$3600$ (2)

Applying principle IV, $x = \$600$ (3)

2. Find the principal that will yield \$?6 interest in 1 yr. at 5%.

3. 50 is $\frac{2}{7}$ of what number?

Hint: $\frac{2}{7}x = 50$. Why?

4. 50 is 6% of what number?

5. If the length of a rectangular field is 60 rd. and its area is 840 sq. rd., what is its width?

Hint: $60x = 840$. Why?

6. (a) Interpret the formula, $P \times R \times T = I$, as used in Interest.

(b) Find the interest on \$800 for $2\frac{1}{2}$ yr. at 5%.

Suggestion: $\$800 \times \frac{5}{100} \times 2\frac{1}{2} = I$.

(c) If the interest on \$800 for $2\frac{1}{2}$ yr. is \$100, what is the rate?

Suggestion: Let R = the rate

then, $800 \times 2\frac{1}{2} \times R = 100$ Why?

and $8 \times 2\frac{1}{2} \times R = 1$ Why?

and $20 \times R = 1$ Why?

$R = \frac{1}{20} = ?\ \%$

7. Find the time required for \$600 to yield \$96 interest at 4%.

8. 25 is $\frac{5}{9}$ of what number?

Hint: $\frac{5}{9}x=25$. Why?

9. 30 is $\frac{5}{8}$ of what number?

10. What principal will yield \$72 interest in 3 yr. at 6%?

11. A horse and buggy are together worth \$360. If the horse is worth 3 times as much as the buggy, find the value of each.

Solution: Let x = value of buggy.
then $3x$ = value of horse. Why?
and $4x$ = \$360. Why?
x = \$90. Why?
$3x$ = \$270. Why?

12. If a house and lot together cost \$7500 and if the house costs 5 times as much as the lot, find the cost of each.

13. The circumference of a tree is 15.708 ft. Find its diameter.

Hint: $3.1416 \times d = 15.708$. Why?

14. The smaller of two numbers is to the larger as 5 to 8. The larger number is 40; what is the smaller number?

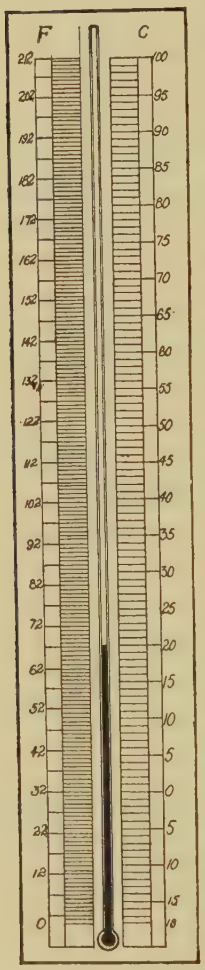
Suggestion: $x : 40 = 5 : 8$. Why?
$8x = 200$. Why?

15. The larger of two numbers is 3 times the smaller, and their sum is 20. What are the two numbers?

16. A man's hat and shoes cost \$15. If the shoes cost twice as much as the hat, what was the cost of each?

17. What is the number of which 60 is $\frac{4}{5}$?

242. THERMOMETERS



The figure shows the two kinds of thermometers most used by the leading countries. The Fahrenheit thermometer is in common use in all English speaking countries. The Centigrade thermometer is in general use in science.

The Fahrenheit scale for measuring temperature was introduced by Fahrenheit about 1714. He believed the lowest possible degree of cold was obtained by mixing salt and snow. He called this 0° on the scale, and the boiling point 212°. The scale was divided into 212 equal parts called degrees. The freezing point was then found to be 32°, thus making 180° between the freezing and boiling points.

The Centigrade scale was designed by Celsius in 1742. He fixed the freezing point at 0° and the boiling point at 100°, thus making 100° between the freezing and boiling points.

Since the same difference in temperature (boiling point—freezing point) is divided into a greater number of equal parts on the Fahrenheit scale, it follows that a given number of degrees Centigrade is equivalent to a greater number Fahrenheit. In changing from degrees Centigrade to degrees Fahrenheit, we find a greater number of degrees, and in changing from degrees Fahrenheit to degrees Centigrade we find a fewer number of degrees.

The following equations or formulas enable us to change a given number of degrees from one scale to the other:

$100°$ C. $= 180°$ F. $\qquad$ $180°$ F. $= 100°$ C.

(I) $1° C. = \frac{9}{5}° F.$ Why? $\qquad$ (II) $1° F. = \frac{5}{9}° C.$ Why?

1. If the readings above 0° on either scale are written thus, +4° C., +25° F., etc., how should we write the readings if they were the same distance below 0°?

2. Besides being signs of operation for addition and subtraction, (+) and (−) are used to denote quantities above and below 0 in value. Point out on the scales in the figure: +50° F.; +45° C.; −3° F.; −4°C.

3. Change to Fahrenheit equivalents:
(a) 80° C.; (b) 45° C.; (c) 55° C.; (d) 13° C.

Suggestion: Since $1° C. = \frac{9}{5}°$F., $80° C. = 80 \times \frac{9}{5}°$ F.

4. Change to Centigrade equivalents:
(a) 45° F.; (b) 36° F.: (c) 27° F.; (d) 81° F.

Suggestion: Since $1° F. = \frac{5}{9}°$ C., $45° F. = 45 \times \frac{5}{9}°$ C.

5. In the exercises above, we have been dealing with degrees and not with readings. To change thermometer readings from one scale to another, account must be taken of the different positions of 0° on the two scales.

Suppose the reading on the F. scale is 41°. Since 32° F. = 0° C., if we take 32° from 41° the remaining 9° will correspond to so many degrees above 0° on the C. scale. By formula (I), $1° F. = \frac{5}{9}°$ C., the reading will be, $9° F. = 9 \times \frac{5}{9}° = 5° C.$ Why?

6. The process in Problem 5 is expressed in the formula: (III) $\frac{5}{9}(F. - 32°) = C.$ reading. Explain.

7. Find readings on the C. scale for the following.
(a) 50° F.; (b) 38° F.; (c) 95° F.; (d) 86° F.

Suggestion: $\frac{5}{9}(50°-32°)=C.$
$\frac{5}{9}\times 18°=C.$ Why?

8. If the C. reading is known, we may find the number of degrees on the F. scale that corresponds to the C. reading by multiplying the C. reading by $\frac{9}{5}$. But since 0° on the C. scale is 32° on the F. scale, we will need to add 32° to the result to find the F. reading. The process is expressed in the formula:
(IV) $\frac{9}{5}\ C.+32°=F.$ reading.

9. Find the F. readings for the following:
(a) 15° C.; (b) 30° C.; (c) 45.5° C.; (d) 81.5° C.

Suggestion: $\frac{9}{5}\times 15°+32°=C.$ reading.

243. MEASUREMENTS

NOTE: Dimensions are considered as abstract numbers. Linear units must correspond to surface units.

Lines

To find—

The side of a square when the area is known: $s=\sqrt{A}$.

The diagonal of a square when the side is known: $d=s\sqrt{2}$.

The diagonal of a rectangular prism when the length, width, and height are known: $d=\sqrt{l^2+w^2+h^2}$.

The circumference of a circle or sphere when the diameter is known: $c=\pi\times d$.

The diameter of a circle or sphere when the circumference is known: $d=\frac{c}{\pi}$.

The hypotenuse of a right triangle when the sides are known: $h=\sqrt{a^2+b^2}$. *a* represents altitude; *b*, the base.

One of the sides of a right triangle when the hypotenuse and other side are known: $a=\sqrt{h^2-b^2}$; $b=\sqrt{h^2-a^2}$.

Surfaces

NOTE: Dimensions must be like numbers.

To find the area of—

A parallelogram when the base and altitude are known: $A = b \times a$.

A triangle when the base and altitude are known: $A = \frac{1}{2} \times b \times a$.

A trapezoid when the bases and altitude are known: $A = \frac{a \times (b_1 + b_2)}{2}$. b_1 represents upper base; b_2, lower base.

A circle when the radius is known: $A = \pi \times r^2$.

The lateral surface of a prism when the perimeter of the base and the altitude are known: L. S. $= p \times a$.

The lateral surface of a cylinder when the circumference of the base and the altitude are known: L. S. $= c \times a$.

The lateral surface of a pyramid when the perimeter of the base and the slant height are known: L. S. $= \frac{1}{2} \times p \times s$.

The lateral surface of a cone when circumference of the base and the slant height are known: L. S. $= \frac{1}{2} \times c \times s$.

The surface of a sphere when the circumference and the diameter are known: $S = c \times d$, or $S = \pi \times d^2$.

Solids

To find the volume of a—

Rectangular solid when the length, width, and height are known: $V = l \times w \times h$.

Prism or cylinder when the area of the base and the altitude are known: $V = b \times a$.

Pyramid or cone when the area of the base and the altitude are known: $V = \frac{1}{3} \times b \times a$.

Sphere when the diameter is known: $V = \frac{1}{6} \times \pi \times d^3$.

To find the number of board feet in a piece of lumber when the length, width, and thickness are known: Bd. ft. $=$ l (in ft.) $\times$ w (in ft.) $\times$ t (in inches).

244. GENERAL NUMBER

Add:

1.	2.	3.
\$3	3¢	3x
\$2	2¢	2x
\$5	5¢	5x
\$6	6¢	6x

In Exercise 3, if x stands for *four*, we would have:

4.		5.		6.
3 fours		3, 4's		3×4
2 fours		2, 4's		2×4
5 fours	or	5, 4's	or	5×4
6 fours		6, 4's		6×4
16 fours		16, 4's		16×4

7. Do you know another way to solve Exercise 6? Which is shorter?

8. If James has *a* marbles, John three times as many, and Henry twice as many, how many have all of them together?

Hint: a+3a+2a=?a.

9. If James has 5 marbles, John three times as many, and Henry twice as many, how many have they all?

Hint: 1×5+3×5+2×5=?×5.

10.	11.	12.
.5x	2.4x	$1\frac{1}{2}$x
.06x	.1x	$2\frac{1}{4}$x
.25x	35.19x	$7\frac{3}{4}$x
.14x	21.01x	$3\frac{1}{2}$x

Add: (Use the shortest and easiest way.)

13.
2×5
4×5
$6\frac{1}{2} \times 5$
$3\frac{1}{2} \times 5$

14.
$.1 \times 6$
$.15 \times 6$
2.05×6
7.2×6

15.
$7 \times 1\frac{1}{8}$
$8 \times 1\frac{1}{8}$
$11 \times 1\frac{1}{8}$
$4 \times 1\frac{1}{8}$

16. It is the custom of some banks to allow interest on daily balances in the case of large deposits. A certain bank allowed the Plymouth Co. 2% interest on daily balances as follows:

June 25, balance \$1890.33 at 2% for 1 da.
June 26, balance 2071.17 at 2% for 1 da.
June 27, balance 3178.19 at 2% for 1 da.
June 28, balance 963.18 at 2% for 1 da.
June 29, balance 1028.11 at 2% for 1 da.
June 30, balance 906.12 at 2% for 1 da.

Find the total interest on the daily balances the easiest way you can. Would it be necessary to find the interest for each day separately?

17. If the base of a rectangle is 9 ft. and its height 7 ft., how would you express its area? If its base is 9 ft. and its height x ft., how would you express its area?

18. If the area of a rectangle is 40 sq. ft., its base 8 ft., and its height x ft., what does x stand for? Draw the figure.

19. If a boy bought a knife for 25¢ and sold it at a profit of 5¢, how much did he get for the knife? If he buys it for 25¢ and sells it at a profit of x¢, how much does he get for it?

20. The base of a triangle is 6 ft. and its altitude is x ft. How would you express its area? If its area is 30 sq. ft., what is its altitude? Draw the figure.

21. James sold 8 papers at x¢ each. How much did each paper sell for if he got 56¢ for all of them?

245. THE METRIC SYSTEM

1. (a) Add:

	6 dollars	9 dimes	7 cents
	3 "	8 "	5 "
	2 "	6 "	7 "
	13 "	4 "	9 "

(b) Add:

	6 rds.	9 ft.	7 in.
	3 "	8 "	5 "
	2 "	6 "	7 "
	12 "	$8\frac{1}{2}$ "	7 "

2. Which is more difficult, (a) or (b)? Why?

3. If 10 in. were equal to 1 ft., and 10 ft. equal to 1 rd., which would be more difficult? Why?

Our money units of measure have a decimal relation to each other. That is, 10 units of one order make one of the next higher.

4. Note the short way of adding (a):

$$\begin{array}{r} \$6.97 \\ 3.85 \\ 2.67 \\ \hline \$13.49 \end{array}$$

Could we use the short way for (b) in Problem 1? Why?

The principle of addition, as well as of all other operations, is the same in our monetary and linear systems. In each, we find the sum of the numbers under the lowest order, find how many this sum makes of the next higher order, write what remains under the lowest order, and carry the number of the next higher order up to its place, etc., until the sum is expressed in integral numbers of the highest orders possible. But because the scale in linear measure is not decimal or uniform, the actual process is more complicated.

5. Do you know of any other units of measure that have the decimal scale? Do our units for dry measure? for avoirdupois weight?

6. What is the scale used in our Hindu-Arabic system of numbers? Answer from your observation of the following process in addition: (**u** stands for **units**, **t** for **tens**, **h** for **hundreds.**)

h	t	u
6	9	7
3	8	8
1	6	7
12	5	2

To the teacher: Most pupils are unconscious of any scale in operations with the Hindu-Arabic system for the reason that carrying has become automatic, which fact is one of the strongest arguments for the metric system.

246. History

About 1783 a great English scientist, named James Watt, who invented the steam engine, designed a system of units of weights and measures decimally related. The French government adopted the system in 1799.

The system is based upon the **meter,** which is a unit of length equal to $\frac{1}{10000000}$ of the distance from the equator to the north pole along the meridian passing through Paris. While a slight error was made in securing this standard unit, the science and usefulness of the system are in no sense affected.

This system is now used by nearly all civilized nations except the United States and Great Britain. Its use is required in some of the government departments of the United States, and officially permitted in others. By direction of law, all our common units of weights and measures are standardized by comparison with the international standards kept at the International Bureau of Weights and Measures near Paris.

Some of the advantages of this system of weights and measures are:

(1) Its simplicity. There are only five tables, and only one scale, except for surface and volume measures, where the squares and cubes of the one scale are quite logically used.

(2) The definiteness of the units. It is said we have 130 kinds of bushels in the United States. A pound of one commodity may not be a pound of another.

An English gallon is larger than an American gallon by about 20% of the latter. In the place of much confusion such as this, the metric system offers absolute definiteness of units and of terms.

(3) Its close similarity to our decimal system. After some practice in the metric system, much of the work will become automatic, and much of it will become simply a matter of moving decimal points, all of which correlates with our decimal system.

The Metric System was named from its fundamental unit, the Meter. All units of weights and measures are derived from the meter.

The standard unit of length, surface, or volume is the **meter** in one, two, or three dimensions.

The standard unit of capacity is the **liter** (lee-ter), a cubical vessel $\frac{1}{10}$ of a meter on the edge.

The standard unit of weight is the **gram,** the weight of a cube of water $\frac{1}{100}$ of a meter on the edge.

Latin and Greek prefixes are used to indicate sub-divisions and multiples of the standard units.

The Latin prefixes denote sub-divisions. Thus, deci-meter, deci-liter, or deci-gram means $\frac{1}{10}$ of a meter, liter, or gram; centi-meter, centi-liter, or centi-gram means $\frac{1}{100}$ of a meter, liter, or gram; milli-meter, milli-liter, or milli-gram means $\frac{1}{1000}$ of a meter, liter, or gram.

The Greek prefixes denote multiples. Thus, Deka-meter, Deka-liter, or Deka-gram means 10 meters, liters, or grams;

Hekto-meter, Hekto-liter, or Hekto-gram means 100 meters, liters, or grams;

Kilo-meter, Kilo-liter, or Kilo-gram means 1000 meters, liters, or grams.

In the tables which follow, the units of measure in common use are in *italics*.

247. Measures of Length

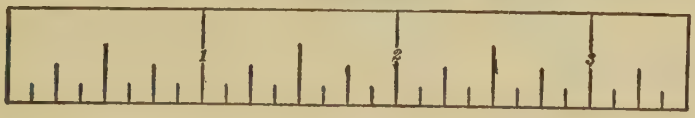
The standard unit is the *meter*.

10 *millimeters* (mm.)	= 1 *centimeter* (cm.)
10 centimeters	= 1 decimeter (dm.)
10 decimeters	= 1 *meter* (m.)
10 meters	= 1 dekameter (Dm.)
10 dekameters	= 1 hektometer (Hm.)
10 hektometers	= 1 *kilometer* (Km.)

Table of Equivalents

1 inch = 2.54 centimeters	1 centimeter = .39 inches
1 yard = .91 meters	1 meter = 39.37 inches
1 mile = 1.61 kilometers	1 kilometer = .62 miles

1 DECIMETER.



INCHES

Oral Exercises

1. Read: (a) 2.7 m.; (b) 75 mm.; (c) 67 dm.; (d) 2.5 dm.

2. How many decimeters in a meter? in a centimeter?

3. Reduce to millimeters: (a) 2.7 m.; (b) 67 dm. (c) 25 dm.

4. How many millimeters in a centimeter? in a decimeter?

5. How many meters in a kilometer? in a hektometer?

6. Reduce to meters: (a) 12 Km.; (b) 1.5 Km.; (c) .7 Hm.

7. Reduce 1575 mm. to meters; to kilometers.

8. It is 295,320 meters from Paris to Calais; how far is it in kilometers?

9. Reduce 1.15 Dm. to meters; to millimeters.

10. 1 m.=39.37 in.; 1 cm.= ? in.

Written Exercises

1. Find the cost of 8 m. of cloth at $2.50 a meter.

2. Find the cost of 6.5 m. of lace at $0.85 a meter.

3. Add: 12 Km., 9 Hm., 6 Dm., 5 m., expressing the sum in kilometers.

4. Add: 9 m., 6 dm., 3 cm., 4 mm., expressing the sum in meters.

5. If the scale of a French map is 1 to 50,000 and if the distance between two cities is 9.3 cm. on the map, find the actual distance in kilometers.

6. A French train runs 65.3 Km. per hour. What is its rate in miles?

7. Divide 144 m. by 1.44 cm.

8. An American regiment was 1000 meters back of the front line. Express the distance in miles.

9. A certain state is 110 miles wide. If we used the metric system, how would we express the width?

10. The bore of a French gun is 20 cm. Find its bore in inches.

11. Find your height in decimeters.

Measures of Area

The principal unit is the *square meter.*

The square dekameter and the square hektometer when used to measure land are called, respectively, the *are* and the *hektare* (ha.).

100 square millimeters (sq. mm.) = 1 *square centimeter* (sq. cm.)
100 square centimeters = 1 square decimeter (sq. dm.)
100 square decimeters = 1 *square meter* (sq. m.)
100 square meters = 1 square dekameter (sq. Dm.)
100 square dekameters = 1 square hektometer (sq. Hm.)
100 square hektometers = 1 *square kilometer* (sq. Km.)

Table of Equivalents

1 square inch	= 6.45 square centimeters
1 square yard	= .84 square meters
1 acre	= .405 hektares
1 square centimeter	= .155 square inches
1 square meter	= 1.2 square yards
1 hektare	= 2.47 acres

1 sq. in.

1 sq. cm.

Oral Exercises

1. Reduce 1.2 sq. m. to square decimeters; to square millimeters.

2. Reduce 1 sq. Km. to square meters; to square centimeters.

3. Reduce 1850 sq. mm. to square meters.

4. Reduce 1963 sq. m. to square kilometers.

5. Reduce 5360 sq. m. to acres; to hektares.

Written Exercise

1. Find the width and length of your desk top in centimeters.

2. Find the area of your desk top in square centimeters; in square meters.

3. Find the value of 5 ha. of land at $120 per acre.

4. Find the side of a square field that contains 1.44 ha.

5. How many square meters are there in a rectangle 30 ft. by 15 ft.?

6. Find the area in square meters of a side-walk 1.5 m. wide and 1 km. long.

7. A French town in the shape of a rectangle is 2000 m. long and 1250 m. wide. Find its area in square kilometers.

8. How many hektares are there in a rectangular field 1260 m. by 1600 m.?

9. Find the side of a square in inches if its area is 81 sq. dm.

10. Find the area of your schoolroom floor in square meters.

11. Which is cheaper, land at $100 per acre or at $250 per hektare? How much?

Measures of Volume

The principal unit is the *cubic meter*.

1000 cubic millimeters (cu. mm.) = 1 cubic centimeter (cu. cm.)

1000 cubic centimeters = 1 *cubic decimeter* (cu. dm.)

1000 cubic decimeters = 1 *cubic meter* (cu. m.)

The cubic meter is called a *stere* when used in measuring wood.

Table of Equivalents

1 cubic inch	= 16.4 cubic centimeters
1 cubic foot	= 28.3 cubic decimeters
1 cubic yard	= .76 cubic meters
1 cord	= 3.622 steres
1 cubic centimeter	= .061 cubic inches
1 cubic decimeter	= .035 cubic feet
1 cubic meter	= 1.31 cubic yards
1 stere	= .276 cords

Oral Exercises

1. Reduce 1 cu. m. to cubic decimeters; to cubic dekameters.

2. Reduce 89 cu. cm. to cubic decimeters.

3. How many cubic centimeters in a cubic foot? How many cords in 100 steres?

4. How many cubic centimeters in a cubic foot?

5. Reduce 678 cu. m. 95 cu. dm. to cubic decimeters.

Written Exercises

1. How many steres in a pile of wood 6 m. long, 2 m. high, and 1 m. wide?

2. Find the cost of excavating a cellar 15 m. long, 12 m. wide, and 1.8 m. deep, at $0.50 a cubic meter.

3. Find the value of a pile of wood 20 ft. long, 6 ft. high, and 4 ft. wide, at $2 a stere.

4. How many cubic meters in a rectangular box 5 m. long, 320 cm. wide, and 200 cm. deep?

5. What is the value of a stere of wood, if a cord costs $3.50.

Measures of Capacity

The principal unit is the *liter*, which is in volume a cube 1 decimeter on the edge, or 1 *cubic decimeter*.

10 *centiliters* (cl.)	= 1 deciliter (dl.)
10 deciliters	= 1 *liter* (l.)
10 liters	= 1 dekaliter (Dl.)
10 dekaliters	= 1 hektoliter (Hl.)
10 hektoliters	= 1 kiloliter (Kl.)

Table of Equivalents

1 dry quart	= 1.1 liters	1 liter	= .91 dry quart
1 liquid quart	= .95 liter	1 liter	= 1.06 liquid qts.
1 bushel	= .35 hektoliter	1 hektoliter	= 2.84 bushels

The *liter* is used to measure berries and other small fruits; the *hektoliter* is used to measure grain, vegetables, and liquids in large amounts.

Oral Exercises

1. How many liters will 100 dry quarts make? 100 liquid quarts?

2. Reduce 9 l. 6 dl. 4 cl. to centiliters; to deciliters; to liters.

3. How many liters will a cubical box one meter on the edge contain?

4. Which is cheaper, milk at 9 cents a quart or 10 cents a liter? How much?

5. Read as liters: 9 l. 6 dl. 4 cl.

Written Exercises

1. A dairyman in France sells 50 l. of milk a day. Find the value of the milk at the American price of 12¢ a quart.

2. It requires 1 gal. of gasoline to run an American automobile 16 miles. On the same basis, how many liters of gasoline will be needed for a trip of 96 kilometers?

3. What is the value of 200 Hl. of wheat at $1.20 a bushel?

4. A rectangular box is 6 m. long, 30 dm. deep, and 25 dm. wide. How many liters will it hold? How many dry quarts?

5. If I buy wheat at $1.40 a bushel and sell it at $3.00 a hektoliter, do I gain or lose? How much?

6. A cylindrical tank is 120 dm. high and has a diameter of 120 cm. How many liters will it hold? How many gallons?

Measures of Weight

10 *centigrams* (cg.)	= 1 decigram (dg.)
10 decigrams	= 1 *gram* (g.)
10 grams	= 1 dekagram (Dg.)
10 dekagrams	= 1 hektogram (Hg.)
10 hektograms	= 1 *kilogram* (Kg.)
1000 kilograms	= 1 *metric ton* (T.)

The principal unit is the *gram*, which is the weight of 1 cubic centimeter of distilled water at its greatest density. The kilogram is the weight of a cubic decimeter of distilled water at its greatest density, and the metric ton is the weight of a cubic meter of such water.

Table of Equivalents

1 pound avoirdupois	= .45 kilogram
1 kilogram	=2.2 pounds avoirdupois
1 ton	= .91 metric ton
1 metric ton	=1.1 ton

The *gram* is used to weigh medicines, precious metals, letters, etc.

The *kilogram*, usually called *kilo*, is used to weigh the usual commodities of trade, and the *metric ton* is used to weigh heavy articles.

Oral Exercises

1. Reduce 1 g. to centigrams; to hektograms.
2. How many grams in 1 lb. avoirdupois?
3. Reduce 1 metric ton to grams.
4. Find the number of centigrams in 1 Kg. 5 g. 6 dm. 8 cm.
5. How many metric tons of coal in 100 tons of coal?

Written Exercises

1. A merchant in New Orleans imported 630 Kg. of cheese; how many pounds did he import?
2. Which is cheaper, coal at $10.00 a metric ton or $9 a ton?
3. Find the value of 1 Kg. of butter at 60¢ per lb.
4. For how much should sugar retail per kilogram in France, if an importer there pays 15¢ a pound for it in New Orleans and adds 50% of the net cost to cover all charges and profits?

248. CUBES AND CUBE ROOTS

NOTE: This topic is included principally for reference and may be omitted if deemed advisable.

If a number is used three times as a factor, the product is called a cube. Thus, in $3\times3\times3=27$, 27 is called a cube. A cube always has three equal factors. One of the three equal factors of a number is called the **cube root** of that number.

Instead of writing $3\times3\times3$, we may write 3^3; it is read, *the cube of 3, or the third power of 3.*

When we wish to ask for one of the three equal factors, or cube root, of 27, we write it thus, $\sqrt[3]{27}$; it is read, *the cube root of 27.*

Find the roots indicated, and check by cubing.

1. $\sqrt[3]{8}=2$;

check: $2^3=2\times2\times2=8$

2. $\sqrt[3]{64}$

3. $\sqrt[3]{27}$

4. $\sqrt[3]{125}$

5. $\sqrt[3]{1000}$

6. $\sqrt[3]{216}$

7. $\sqrt[3]{\frac{8}{27}}=\frac{2}{3}$;

check: $(\frac{2}{3})^3=\frac{2}{3}\times\frac{2}{3}\times\frac{2}{3}=\frac{8}{27}$

8. $\sqrt[3]{.125}$

9. $\sqrt[3]{\frac{64}{27}}$

10. $\sqrt[3]{\frac{1}{8}}$

11. $\sqrt[3]{\frac{8}{1000}}$

12. $\sqrt[3]{\frac{27}{64}}$

This table shows some cube roots that are easily remembered.

$\sqrt[3]{1}=1$	$\sqrt[3]{1000}=10$
$\sqrt[3]{8}=2$	$\sqrt[3]{8000}=20$
$\sqrt[3]{27}=3$	$\sqrt[3]{27000}=30$
$\sqrt[3]{64}=4$	$\sqrt[3]{64000}=40$
$\sqrt[3]{125}=5$	$\sqrt[3]{125000}=50$
$\sqrt[3]{216}=6$	$\sqrt[3]{216000}=60$
$\sqrt[3]{343}=7$	$\sqrt[3]{343000}=70$
$\sqrt[3]{512}=8$	$\sqrt[3]{512000}=80$
$\sqrt[3]{729}=9$	$\sqrt[3]{729000}=90$

The cubes are the numbers under the radical signs; the cube roots are the numbers outside the radical signs.

1. If it requires one figure to write a cube, how many are required to write a cube root?

2. If it requires two figures to write a cube, how many are required to write its cube root?

3. Answer for a cube with three figures; four figures; five figures; six figures.

4. A cube will always have three times as many figures as its cube root, or three times as many less one, or three times as many less two. Thus, 12^3 will have either six, or five, or four figures; 80^3 will have either six, or five, or four figures. Find the number of figures in each of these cubes.

5. How many figures will be necessary to express the cube root of a cube with one figure, as $\sqrt[3]{8}$? two figures, as $\sqrt[3]{64}$? three figures, as $\sqrt[3]{512}$? four figures, as $\sqrt[3]{1000}$? five figures, as $\sqrt[3]{64000}$? six figures, as $\sqrt[3]{512000}$?

6. Write three equal factors of each of the following: (a) 27; (b) 8; (c) 216; (d) 729; (e) 343.

7. By reference to the table of cube roots on page 415, write answers to the following:
(a) $\sqrt[3]{\frac{1}{216}}$; (b) $\sqrt[3]{\frac{27}{1000}}$; (c) $\sqrt[3]{\frac{1000}{27}}$; (d) $\sqrt[3]{\frac{125}{8}}$; (e) $\sqrt[3]{\frac{343}{729}}$.

8. Find and test answers to the following: (a) $\sqrt[3]{.008}$; (b) $\sqrt[3]{.216}$; (c) $\sqrt[3]{.027}$; (d) $\sqrt[3]{.001}$; (e) $\sqrt[3]{.343}$.

9. Between what two numbers is each of the following: (a) $\sqrt[3]{16}$? (b) $\sqrt[3]{140}$? (c) $\sqrt[3]{6500}$? (d) $\sqrt[3]{100000}$? (e) $\sqrt[3]{800}$?

10. Since $\sqrt[3]{216}=\sqrt[3]{2\times2\times2\times3\times3\times3}$, therefore $\sqrt[3]{216}=2\times3$. Why? Factor first and show that (a) $\sqrt[3]{64000}=4\times10$; (b) $\sqrt[3]{343000}=7\times10$.

11. Find the cube root of 57512456. How many figures will the cube root have? There will be one figure in the root for every group of three figures, and for the additional group of one or two figures that may be left in the cube.

Note the grouping. The figures in parentheses ( ) refer to the explanation below.

Process			Cube	Root
			57 512 456	386
		3^3 = 27	(1)	
3×30^2	2700	(4)	30 512	(3)
[30512÷2700=8 (about)]		(5)		
$3\times30\times8$	= 720	(6)		
8^2	= 64	(7)		
[2700+720+64=]	3484	(9)		
[3484×8=]			27 872	(10)
3×380^2	=433200		2 640 456	
[2640456÷433200=6 (about)]				
$3\times380\times6$	= 6840			
6^2	= 36			
[433200×6840×6=]	440076			
[440076×6=]			2 640 456	

(1) Find the largest cube in left hand group.

(2) Write its cube root as left hand figure in the root.

(3) Subtract cube and annex next group.

(4) Consider part of root found as tens, square, and multiply by 3.

(5) Use product as trial divisor, remembering that it must be considerably increased for complete divisor.

(6) Multiply part of root found, considered as tens, by 3 and by quotient found by trial divison.

(7) Square quotient of trial division.

(8) Write this quotient as next figure in root.

(9) Add results of (4), (6), (7).

(10) Multiply the result of (9) by the last figure in the root.

(11) Begin with (3) and follow same directions until all groups have been used. [Parts in brackets may be omitted in pratice.]

12. Find cube roots of: (a) 15625; (b) 39304; (c) 4913; (d) 110592; (e) 357911.

249. TEST PROBLEMS

1. A trader bought 20 dairy cows for \$3000, and sold them for \$175 each; find his per cent profit.

2. A bankrupt's debts are \$6500; his assets are valued at \$2600. How much can he pay on the dollar?

3. A bankrupt's debts amounted to \$7500; what were his assets if he could pay $12\frac{1}{2}¢$ on the dollar?

4. A bankrupt's assets were \$6250; he was able to pay 40¢ on the dollar. What were his liabilities?

5. A man's salary is \$2500 a year. How much will he save if his expenses are \$6 a day?

6. How much will it cost to build a street 600 feet long and 24 feet wide, at \$0.35 a square foot?

7. What per cent of the weight of 1 lb. of lead is 1 lb. of gold?

8. When gold is pure, it is said to be 24 carats (K). What per cent pure is 10 carat gold? 14 carat? 18 carat?

9. How many square yards of linoleum will be required to cover a room 17 ft. by 18 ft.?

10. What is the product of $\frac{.6}{\frac{1}{4}} \times \frac{\frac{1}{8}}{\frac{3}{5}}$?

11. If the divisor is .67 and the quotient is .92, what is the dividend?

12. Smith loaned Brown \$600 for 6 months. How long should Brown loan Smith \$150 to compensate him?

13. A owned $\frac{4}{11}$ of the stock of a bank. He sold $\frac{1}{2}$ of $\frac{1}{8}$ of his stock for \$39000. At that rate, what was the value of the whole bank stock?

14. What fraction of a yard is 2 ft. and 6 in.?

15. If a train runs 45 mi. in 1 hr. and 10 min., how long will it take the train to run 125 mi.?

16. How many degrees Centigrade will correspond to 95° Fahrenheit?

17. Divide \$2800 among A, B and C, so that B shall have twice as much as C, and A twice as much as B.

18. Divide \$65 between 3 boys, so that each shall share in the ratio of $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$.

19. Divide \$39 among 5 girls, so that 2 of them may each receive \$2 more than each of the other 3.

20. At a certain election for congress, there were 21375 votes cast for two candidates. The successful candidate received a majority of 2365 votes. How many votes did each candidate receive?

21. Divide 43¢ between A, B and C, so that A will have 7¢ more than B, and B will have 3¢ more than C.

22. In a certain election for Governor, there were 351,000 votes cast for 3 candidates. The successful candidate received 17,500 more votes than his nearest competitor, who received 25,000 more votes than the hindmost man in the race. How many votes did each candidate receive?

23. A can do a piece of work in 4 days, B in 5 days and C in 6 days. In what time can they all do the work together?

24. The smaller of two numbers is 5 and their difference is 7. What is their product?

25. What is the least number that can be divided by 6, 7, 8, or 9?

26. A sugar dealer had 20 T. and 200 lb. of sugar which he placed into packages each weighing 2 lb. 10 oz.; how many packages were there?

27. If we **add** the difference of two numbers to their sum, we obtain twice the greater; but if we **subtract** the difference from the sum, we obtain twice the less. Explain why this is so.

28. What part of an acre is embraced in a rectangular garden 96 ft. by 80 ft.?

29. Express .275 of a mile in whole units of a lower denomination.

30. Out of a certain sum of money, a boy spent $\frac{3}{5}$ and then $\frac{1}{4}$ of the remainder. He had \$30 left. How much had he at first?

31. If a train runs 54 mi. an hour, how long will it take it to run $278\frac{1}{4}$ mi.?

32. Simplify: $\frac{\frac{1}{2}+\frac{1}{5}}{\frac{7}{8}-\frac{1}{8}}$.

33. A cistern that will hold 60 gallons is $\frac{1}{2}$ full; if a supply pipe runs into the cistern $3\frac{4}{5}$ gal. a minute and a discharge pipe runs out $2\frac{1}{5}$ gal. a minute, how long will it take to fill the cistern if both pipes are left open?

34. A certain county has an area of 450 sq. mi. and a population of 263250. What is the population to the square mile?

35. Multiply $\frac{7}{8}$ by the reciprocal of $\frac{3}{4}$.

36. A loaded truck weighs 16 T. 800 lb., and the weight of the load is 7 times the weight of the truck. What is the weight of the truck?

37. Divide \$289 among 3 people in the ratio $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{9}$.

38. How many bricks 8 in. by 4 in. by 2 in. will be required for a wall 30 ft. long, 20 ft. high and $1\frac{1}{2}$ ft. thick, deducting 10 per cent for the space filled with mortar.

39. A merchant bought pocket knives at 75¢ each, which he sold at a profit of 40%. How much was his profit on a dozen knives?

40. If knives are bought at \$9.60 a dozen and are sold for \$1 each, what is the rate of profit?

41. If knives are bought at \$10 a dozen and are sold at the rate of 10 knives for \$12, what is the rate of gain?

42. Find the simple interest on \$450 for 2 yr. 9 mo. 27 da. at 6%.

43. On what day will \$400, loaned April 1, 1920 at 6%, amount to \$408?

44. A man loaned \$600, and 3 months from the date of lending he collected \$612. At what rate did he lend his money?

45. If we allow 200 cu. ft. of air space to each pupil, how many pupils could occupy a room 25 ft. by 30 ft. and 12 ft. in height?

46. A man wanted to invest \$1500 for 5 years; which is the better, to invest at 6% per annum simple interest or $5\frac{3}{4}\%$ per annum compound interest? How much would the difference be?

47. A man built a house for \$4500. For how much must he rent it per month to make 6% annually on his investment after allowing 10% for insurance, taxes and repairs?

48. Find the diagonal of a 4 inch square.

49. If a leaf in your book is 6 in. by 8 in., find the diagonal.

50. Find the sum of $\frac{2}{3}$ and its reciprocal.

51. Which is the better investment and by how much, Northern Pacific stock at $77\frac{7}{8}$ and yielding $3\frac{1}{2}$ per cent dividend, or New York Central stock at 71 and yielding $3\frac{1}{4}$ per cent dividend?

52. What is the bank discount of \$400 due in 6 months at 6%?

53. A one-year note of \$750, dated January 1, 1919 and bearing interest from date at 6%, was discounted at a bank June 15 following at 7%. What were the proceeds of the note?

54. When the thermometer registers 80° Fahrenheit, what is its equivalent in Centigrade?

55. What would it cost at \$1.75 a yard to carpet a floor $12\frac{1}{2}$ ft. by 15 ft. with carpet $\frac{3}{4}$ yd. in width, the strips running lengthwise?

56. What is the area of a triangle whose base is $6\frac{1}{2}$ ft. and whose altitude is $5\frac{1}{4}$ ft.?

57. A flag pole 42 ft. in height casts a shadow 32 ft. long. What is the height of a tree whose shadow is 60 feet?

58. In a 3-inch rainfall, how many gallons of water would fall on a square yard?

59. What is the difference in area between a circle 6 inches in diameter, and a square each side of which is 6 inches?

60. How long will it take a pipe that discharges 5 gal. a second to empty the water from a tank 8 ft. by $3\frac{1}{2}$ ft. by 2 ft.?

61. If bonds bearing 5% interest are bought at 10% premium, what rate per cent does the buyer make on his investment?

62. What is the simple interest on \$1250 for 2 yr. 6 mo. 24 da. at 6%?

63. What is the compound interest in Problem 62?

64. What is the difference in volume between a 6-inch cubical box and a sphere 6 inches in diameter?

65. How long must \$360 be at interest to earn \$103.92 at 6%?

66. A merchant sold a suit for \$35 and made 25% on the cost. At what price should he have sold the suit to make $33\frac{1}{3}$%?

67. A merchant sold a suit for \$35, which was $14\frac{2}{7}$% less than his marked price. What was his marked price?

68. What is the cost of an article listed at \$75 with discounts 20% and 15%?

69. How many acres in a rectangular field whose perimeter is 240 rd. and whose length is 3 times its width?

70. What is the cost of an automobile listed at \$1800 with 20% and 5% discounts?

71. A boy bought newspapers at the rate of 3 for 10¢ and sold them at 5¢ each. What per cent on the cost did he make?

72. John said 35¢ was $\frac{5}{7}$ of his money. How much money had he?

73. The sides of a rectangle are 40 ft. and 90 ft. Find the side of a square equal in area to the rectangle.

74. Find the diagonal of a square with an area of 1 sq. in.

75. How many rods of fence will be required to enclose a square field containing 40 A ?

76. Find the net cost of a bed room suit listed at \$350 with cash discounts of 20% and 15%, if the freight and drayage cost \$4.95.

77. Find the cost in our money of a French Government bond of 2000 francs at 32% of its face value.

78. Find the cost of 20 U. S. $4\frac{1}{4}$ Liberty Bonds at $93\frac{1}{2}$.

79. Find the rate of income on cost in Problem 78.

80. An agent received \$26.50 commission for selling 500 bushels of wheat at \$1.06 a bushel. What was his rate of commission?

81. A broker received $2\frac{4}{5}\%$ commission for selling cotton. His commission was \$226.08. What was the amount of the sale?

82. A rectangular field is 60 rd. by 40 rd. If the length and width are each increased 10%, what is the per cent of increase in the area?

83. A merchant asked 25% more for a suit of clothes than it cost him. He later sold the suit for \$45 which was 10% less than he asked for it. What was the cost of the suit?

84. Find the compound interest on \$7650 for 2 yr. 9 mo. 15 da. at 6%.

85. How many feet of picture molding will be required for a square room whose ceiling contains 625 sq. ft.?

86. What is the diagonal of the ceiling in Problem 85?

87. Find the area of a trapezoid whose lower and upper bases are 60 ft. and 40 ft. respectively and whose altitude is 18 ft.

88. A who owns $\frac{3}{7}$ of an estate, sells $\frac{4}{9}$ of his share to B, who sells $\frac{3}{5}$ of his purchase for \$20,000; what is the whole estate worth at the same rate?

89. If I contract to do a piece of work in 35 days, what part of the work can I do in $12\frac{1}{2}$ days?

90. A certain fraction multiplied by $\frac{5}{9}$ gives a product of $\frac{7}{11}$. What is the fraction?

91. The number of boys in a school is such that they can be arranged in equal classes of either 24, 30 or 35 boys each; what is the least possible number of boys in the school?

92. There are 500 boys and 640 girls in a high school. The teacher wanted to classify the boys and girls separately, placing the largest number possible of each in classes of equal size. How many pupils in a class?

93. A carpet 15 ft. by 14 ft. is put down in a room 18 ft. square; the remainder of the floor is varnished. How many square feet of the floor are varnished?

94. What per cent of the whole floor in Problem 93 is varnished?

95. Find the cost of painting the walls of a room 16 ft. by 16 ft. and 12 ft. high at 15¢ a square yard.

96. A herdsman, who had 3500 sheep, sold 20% to A, 30% to B, and 40% of the remainder to C. How many had he left?

97. What principal at 6% interest will produce $350 in 2 yr. 8 mo. 12 da.?

98. A stock trader bought 12 horses at an average cost of $175. He sold 4 of them at an average price of $180. At what price must he sell the others so as to gain 10% on the whole cost?

99. A dairyman sold two cows for $125 each. On one he gained 25% and on the other he lost 25% of the cost. Did he gain or lose in the transaction and how much?

100. How many shares of 8 per cent bank stock at 106 can I buy for $848, and what is the rate per cent on the cost?

101. What length of molding is required to frame a picture which is 2 ft. 10 in. long and 1 ft. $7\frac{1}{2}$ in. wide?

102. How many lengths of wire each 1 ft. 6 in. can be cut from 50 yards?

103. How many pieces of cheese each weighing 3 lb. 6 oz. can be cut from a larger piece weighing 44 lb. 4 oz.?

250. EXAMINATION QUESTIONS

A selected list of questions that have been used on teachers' examinations is given in promiscuous order.

They cover practically every phase of the important topics of arithmetic. Teachers may use them for purposes of drill, review, and application, selecting such questions as will serve to secure the particular end desired.

1. Why does multiplying both terms of a fraction not change the value of the fraction? Illustrate this.

2. Express in words: .60032, 12.0507, 1.07118, 2.00104.
Express in figures: MCLXIV, nine hundred sixteen thousandths.

3. What will 36 pieces of timber, each 2 in. by 4 in. by 12 ft., cost at $50 per M. board feet?

4. Express in two other ways each of the following:
.25, $\frac{3}{8}$, 125%, .005, 12.5 per cent.

5. If a man's tax rate for schools is 5.8 mills on the dollar, how much will he pay on $8,500?

6. A note is made for $150 on May 24, 1913, at 6% interest. How much will be required to settle the debt in full on February 12, 1915?

7. A man bought a house for $2,000 and sold it for $2,750. What was the per cent of gain?

8. Write in figures and then find the sum of: 38 hundred-thousandths, 3 and 6 tenths, 31 thousandths, 21 hundred-thousandths, and 16 millionths.

9. I sent my agent $1,364.76 to be invested in pork at $6.00 a barrel. After deducting his commission of 2 per cent, how many barrels of pork did he buy?

10. A man invests $6,000.00 in sheep at $6.00 a head. If 20 per cent of the sheep die, at what price per head must he sell the remainder to make $33\frac{1}{3}$ per cent on the transaction?

11. A laborer received $3.00 a day for his work and paid $5.00 a week for expenses. He saved $100.00 in ten weeks.

How many days was he idle? (Note.—Six work days make a week).

12. In what time will \$450.00 at 6 per cent yield \$90.00 interest?

13. Phillip's pig weighed 40 lb. and cost \$8.00. It gained 1.6 lb. a day for 95 da., at a cost of 7¢ for each pound gained. The live pig sold at 16 cents per pound. Find the profit.

14. How many cubic yards of earth are removed in digging a ditch 54 ft. long, $3\frac{1}{2}$ ft. wide, and 2 ft. deep?

15. How many cakes of soap 4 in. by 3 in. by 2 in. can be packed in a box whose inside dimensions are 4 ft. by 3 ft. by 2 ft.?

16. If the carpet costs 45 cents a running yard, find the cost of carpeting a room 24 ft. long by 18 ft. wide with carpet 27 in. wide, the strips running lengthwise.

17. A merchant sold a watch for \$40, and lost 20 per cent of the cost. With the \$40 he bought another watch, which he sold at a gain of 20 per cent of the cost. What was the merchant's gain or loss per cent?

18. Two men can pick a field of cotton in eighteen days. If after they have worked ten days, they hired another and finished the work in five days, how many days would it have taken the third man alone to pick the field of cotton?

19. If 1800 men have provision to last $4\frac{1}{2}$ mo., at the rate of 1 lb. 4 oz. a day to each, how long will five times as much last 3,500 men, at the rate of 12 oz. a day to each man?

20. How many shares of B. & O. R. R. stock can be purchased for \$718.75 at $15\frac{1}{2}$ per cent, brokerage $\frac{1}{8}$ per cent?

21. Divide 3.1416 by 5.236 and multiply the quotient by .166.

22. (a) How many rods of fence are required to enclose a square lot whose area is 5,184 sq. ft.?

(b) What will it cost to fence the field at \$3.30 a rod?

23. What will 56 pieces of timber, each 2 in. by 4 in. by 12 ft., cost at $32.50 per thousand board feet?

24. If a man's tax rate for roads is 3.6 mills on the dollar, how much will he pay on $7,200?

25. A man bought a house for $2,000 and sold it for $2,750. What was the per cent of gain?

26. How many yards of carpet $2\frac{1}{4}$ ft. wide will cover a floor 17 ft. by 15 ft. if strips run across the room?

27. For what price per pair must shoes be sold to gain 25 per cent if 15 per cent of the cost is lost when they are sold for $2.55 per pair?

28. If $21 worth of bread will supply 6 persons for 4 mo. when flour is $7.00 a barrel, how many persons can be supplied for 8 mo. for $50.00 when flour is $5.00 per barrel?

29. Compute the interest on $625.00 for five months at 6 per cent per annum.

30. Find the simple interest on $600 for 2 mo. and 15 da., 6 per cent.

31. Divide .7854 by .02 and multiply the quotient by 1.3.

32. A triangular field has a base 280 rd. and an altitude of 24 rd. How many acres does it contain?

33. If a cubic foot of marble weighs 173 lb., what is the weight of a cylindrical marble column that is 10 ft. high and 16 in. in diameter?

34. The area of a trapezoid is 55 sq. ft. and the length of one of the bases is 7 ft., and the altitude is 5 ft. Find the length of the other base.

35. A wholesale jeweler advertised his stock to be sold at 50 per cent and 25 per cent off. I bought a watch for $15. What was the list price?

36. How many yards of carpet 1 yd. wide will be required for a room 21 ft. long and 18 ft. wide?

37. If a man's property is assessed at $5,400 and his tax is $135, what is the rate of taxation?

38. A man and two boys earn $160 and agree to divide it as follows: 5 parts to the man, 2 parts to the first boy, and 1 part to the second boy. How much should each receive?

39. A boy buys apples at the rate of 5 for 2 cents and sells them at the rate of 8 cents a dozen. How many must he buy and sell to make $2?

40. A man has $1\frac{3}{4}$ mi. to go; after he has gone $\frac{\frac{1}{2}\times\frac{2}{5}\times 1\frac{1}{4}}{\frac{3}{4}\times 1\frac{7}{8}\div\frac{5}{8}}$ of a mile, how far has he yet to go?

41. Simplify: $(.08\frac{1}{4}+1.2\frac{1}{2})\div(.006\frac{1}{4}\times.016)$.

42. Reduce 2 pk. 3 qt. 1.2 pt. to the decimal of a bushel.

43. A man sold two horses for $200 each. On one he gained 50% of the cost, and on the other he lost 50%. Did he gain or lose by these sales, and how much?

44. A merchant sends his agent $10,246.50 with which to buy flour. After his commission of $3\frac{1}{2}$% has been deducted, how many barrels of flour at $5.50 a barrel can be purchased?

45. A note of $850 with interest payable annually at 5% was paid 3 yr. 3 mo. 18 da. after date, and no interest had previously been paid. What was the amount due?

46. What is the exact interest on $600 at 5% for 90 days?

47. A man receives $630 as his annual dividend from 7% stock. How many shares of $100 each does he hold?

48. A sewing machine cost a retailer $18, and the freight charge was $2. If the retailer knows that his selling expense is $12\frac{1}{2}$ per cent of the net cost, at what price shall he sell the machine to make 15 per cent on the net cost?

49. If we allow 1.5 cu. ft. of silage per cow per day, how many days' feed for 30 cows can be stored in a silo 12 ft. in diameter and 25 ft. high?

50. A given fertilizer contains 1.6 per cent of nitrogen, 7 per cent of available phosphoric acid, and 2 per cent of available potash. What is the value of the fertilizer per ton if

nitrogen is worth 15 cents a pound, phosphoric acid 5 cents a pound, and the potash 5 cents a pound?

51. An investor borrowed $6,000 at 6 per cent to place an article upon the market. At the end of 3 yr. 2 mo. and 20 da., he sold his invention for $18,000. Find the amount he made after paying back the borrowed money with interest.

52. How much must be invested in 6 per cent stock, at 120, brokerage one-eighth per cent, to yield an annual income of $720.00?

53. A boy agreed to work for a merchant for 20 weeks on the condition that he should receive $40.00 and a suit of clothes. At the end of 12 weeks, when the boy stopped work, it was found that he was entitled to $16.00 and the suit. Find the value of the suit.

54. A holder of a note bearing 6% interest payable semi-annually collected $24 interest every six months. What was the face of the note?

55. A man paid $30 interest on a $600 note bearing interest at 6%. How long did the note run?

56. A note of $600 drew $24 interest in 6 mo. What was the rate?

57. What is the cost of a sewing machine listed at $75 with discount 20% and 10%?

58. What is the diagonal of a square field containing 36 A.?

59. A man paid $3750 for a farm which he sold for $5000. What was his per cent of gain?

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ANSWERS TO
NEW SCHOOL ARITHMETIC
BOOK TWO

Page 7.—2. $1648. 3. 135 gal. 4. 660 mi. 5. $369. 6. 64 yr. 7. 191 gal. 8. $380. 9. 288 mi. 10. 887 A. 11. 619,637 sq. mi. 12. 1,019,766.

Page 8—13. 72,813. 14. 458,631. 1. 386,902,137. 2. 381,099,699. 3. 38,888,885. 4. 279,938,977. 5. 38,740,547. 6. 336,011,324. 7. 342,428,522. 8. 339,878,153. 9. 197,855,602. 10. 275,675,534. 11. 353,349,865. 12. 69,963,378.

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Page 11.—1. 17,359. 2. 119,098. 3. 34,832. 4. 195,531. 5. 139,546. 6. 104,276. 7. 157,768. 8. 144,687.

Page 13.—2. 337,747. 3. 99,098. 4. 221,402. 5. 609,049. 6. 361,906. 7. 11,433. 8. 410,908. 9. 109,099. 10. 139,088. 11. 24,919. 12. 105,091. 13. 573,191. 14. 389,832. 15. 119,513. 16. 273,568. 17. $5,233.65. 18. $844.92. 19. $1,106.75. 20. $11,918.95. 21. $8,587.79. 22. $4,499.89.

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Page 15.—5. $345. 6. $6,500. 7. $6,500. 8. 297 yr. 9. $15. 10. $1.50. 11. $80. 12. $5. 13. $12.02. 14. $7.75. 15. 2,874,977. 16. 125 yr.

Page 17.—19. $1.50. 20. $.50. 21. $4.75. 22. $.75. 23. $8.50. 24. $3.78.

Page 20.—5. 6,190,450. 6. 2,254,500. 7. 10,353,174. 8. 7,849,178. 9. 8,056,125. 10. 43,797,512. 11. 3,559,048. 12. 998,998. 13. 43,139,096. 14. 3,704,634. 15. 3,782,403. 16. 21,074,141. 17. 3,314,853. 18. 6,048,000. 19. 32,087,124. 20. 2,458,350. 21. 2,533,689. 22. 48,748,656. 23. 8,016,230. 24. 796,536. 25. 69,121,976. 26. 3,303,300. 27. 5,549,445. 28. 54,998,594.

Page 21.—29. 26,690,664. 30. 591,949,325. 31. 7,911,431,064.

Page 22.—2. 1.225. 3. 8.481. 4. 7.569. 5. 9.604. 6. 625. 7. 106.929. 8. 229.441. 9. 390,625. 10. 974,169. 11. 7,219,969. 12. 31,668,129. 13. 4,754,929,936. 14. 5,859,596,304. 15. 65,940,390,521. 16. 42,875. 17. 636,056. 18. 970,299. 19. 1,000,000. 1. 9,600 lb.; 79,200 lb. 2. 4,800 lb. 3. 6,600 lb.

Page 23.—1. 364. 2. $8,000. 3. $13.75. 4. $1,800. 5. $1,125. 6. $11.25. 7. *a*. $44; *b*. $6.00. 8. $65.00. 9. $81.60. 10. $390,468.75. 11. 600 mi. 12. 27,300. 13. $228,000. 14. $621.

Page 26.—1. *a*. 339; *b*. 333; *c*. 104; *d*. 207; *e*. 115; *f*. 323. 2. *a*. 316; *b*. 146; *c*. 109; *d*. 161; *e*. 147; *f*. 103.

Page 27.—3. *a*. 102; *b*. 91; *c*. 198; *d*. 106; *e*. 109; *f*. 106. 4. *a*. 112; *b*. 114; *c*. 196; *d*. 140; *e*. 97; *f*. 77. 5. *a*. 204; *b*. 108; *c*. 117; *d*. 81; *e*. 162; *f*. 98. 7. $74\frac{4}{5}$. 8. 11.578. 9. $1.987\frac{1}{2}$. 10. $1.402\frac{1}{2}$. 11. $8.698\frac{4}{5}$. 12. $5.489\frac{1}{2}$. 13. $7980\frac{3}{5}$. 14. $7.112\frac{1}{2}$. 15. 7.906. 16. $10.87\frac{1}{2}$. 17. $12.640\frac{3}{5}$. 18. $9.304\frac{1}{2}$. 19. 8.054. 20. $8.551\frac{1}{2}$. 21. $21.405\frac{1}{2}$. 22. $33.272\frac{1}{2}$. 23. $7.88\frac{1}{2}$. 24. $2.241\frac{1}{2}$.

Page 28.—2. *a*. $235.664\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *b*. $848.249\frac{4}{5}\frac{4}{5}$; *c*. $96.001\frac{1}{2}\frac{1}{2}$; *d*. $97.304\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *e*. $108.062\frac{1}{2}\frac{1}{2}\frac{1}{2}$. 3. *a*. $4.136\frac{1}{2}\frac{1}{2}$; *b*. $1.2654\frac{1}{2}\frac{1}{2}$; *c*. $1.528\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *d*. $1.345\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *e*. $2.598\frac{1}{2}\frac{1}{2}\frac{1}{2}$. 4. *a*. $927.641\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *b*. $788.950\frac{4}{5}\frac{4}{5}$; *c*. $633.645\frac{1}{2}\frac{1}{2}$; *d*. $455.937\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *e*. $423.541\frac{1}{2}\frac{1}{2}\frac{1}{2}$. 5. *a*. $12.797\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *b*. $16.045\frac{4}{5}\frac{4}{5}$; *c*. $20.183\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *d*. $12.512\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *e*. $10.923\frac{1}{2}\frac{1}{2}\frac{1}{2}$. 6. *a*. $1.284\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *b*. $673\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *c*. $454\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *d*. $894\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *e*. $448\frac{1}{2}\frac{1}{2}\frac{1}{2}$. 7. *a*. $1.088,579\frac{4}{5}\frac{4}{5}\frac{4}{5}$; *b*. $822.349\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *c*. $711.440\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *d*. $1.002.600$; *e*. $599,420\frac{1}{2}\frac{1}{2}\frac{1}{2}$. 8. *a*. $35.406\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *b*. $27.263\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *c*. $55.711\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *d*. $49.533\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *e*. $33.344\frac{1}{2}\frac{1}{2}\frac{1}{2}$. 9. *a*. $109.935\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *b*. $178.783\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *c*. $93.906\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *d*. $235.861\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *e*. $488.208\frac{4}{5}\frac{4}{5}$. 10. *a*. $159.981\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *b*. $149.389\frac{4}{5}\frac{4}{5}\frac{4}{5}$; *c*. $119.455\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *d*. $133.702\frac{1}{2}\frac{1}{2}\frac{1}{2}$; *e*. $99.324\frac{1}{2}\frac{1}{2}\frac{1}{2}$.

Page 30.—2. $14\frac{15}{16}$. 3. $26\frac{5}{16}$. 4. $92\frac{5}{16}$. 5. $13\frac{7}{16}$. 6. $20\frac{1}{16}$. 7. $40\frac{15}{16}$. 8. $242\frac{7}{16}$. 9. $145\frac{1}{16}$. 10. $2.918\frac{7}{16}$. 12. $2.85. 13. $4.95. 14. $.80. 15. $.20. 16. $2.50. 17. $20. 18. $40.03. 19. $.20. 20. $1.13. 21. $2.04. 22. $2.92. 23. $.20. 24. $.25. 25. $3.75. 26. $1.83. 27. $3.14\frac{4}{16}$. 28. $82. 29. $.45. 30. $6.25. 31. $9.05. 2. 15. 4. 83. 6. $900. 8. 340.

Page 31.—10. $126. 12. 4. 14. 25. 16. 21 mi. 18. 923. 20. 2,646.

Page 32.—22. $356.25. 24. $80. 26. 236 A. 28. 22c. 30. $8.875. 32. 50. 34. $14.40. 36. $21.000.

Page 33.—38. $173. 40. $48; $384.

Page 36.—1. 2^3, 3, 7. 2. 2^4, 67. 3. 5^3. 4. 5, 59. 5. 2^3, 37. 6. 3^4. 7. 2, 5^3. 8. 2^4, 3, 5^3. 9. 5^3, 11. 10. 3, 11^3. 11. 2^3, 3^3. 12. 2^8. 13. 5^3, 7. 14. 5^4. 15. 3^3, 7. 16. 2^4, 3^3. 17. 5^3, 3. 18. 2, 3^3, 5, 7. 19. 2^3, 5. 20. 3^3, 2^3, 13.

Page 38.—2. 25. 3. 8. 4. 16. 5. 56. 6. 2. 7. 4. 8. 18. 9. 6. 10. 8. 11. 2. 12. 9. 13. No. G. C. D. 14. 4. 15. 13. 16. No G. C. D. 17. 18. 18. 8. 19. 25. 20. No G. C. D.

Page 40.—3. 252. 4. 880. 5. 240. 6. 360. 7. 60. 8. 180. 9. 360. 10. 504. 11. 540. 12. 1,008. 13. 2,352. 14. 105. 15. 476. 16. 720. 17. 120. 1. 72. 2. James, 2 hr.; Thomas, 3 hr. 3. 60. 4. 30.

Page 41.—5. 205 mi. 6. 360 ft. 7. 8 ft. 8. 792. 9. $20. 10. 13. 11. 288 in. 12. 3 A. 13. 34. 15. 24″x40″.

Page 42.—1. 24. 2. 80. 3. 24. 4. 189. 5. 12. 6. 20. 7. 82. 8. 6. 9. 4. 10. 24. 11. 12. 12. *a*. 8; *b*. 8; *c*. $4\frac{4}{15}$.

Page 43.—13. $30\frac{3}{8}$ lb. **14.** $25\frac{3}{8}$ da. **15.** 8 doz. **16.** $82\frac{1}{8}$ A. **17.** 3 bbl. **18.** $38\frac{2}{3}$ yd. **19.** $4\frac{1}{2}$.

Page 47.—2. $\frac{4}{8}, \frac{2}{8}, \frac{6}{8}, \frac{8}{8}$. **3.** $\frac{9}{18}, \frac{6}{18}, \frac{15}{18}, \frac{16}{18}, \frac{12}{18}, \frac{3}{18}, \frac{14}{18}$. **4.** $\frac{12}{24}, \frac{8}{24}, \frac{18}{24}, \frac{20}{24}, \frac{21}{24}, \frac{15}{24}, \frac{22}{24}$. **5.** $\frac{15}{30}, \frac{10}{30}, \frac{25}{30}, \frac{27}{30}, \frac{28}{30}, \frac{18}{30}, \frac{20}{30}$. **6.** $\frac{12}{36}, \frac{27}{36}, \frac{30}{36}, \frac{28}{36}, \frac{33}{36}, \frac{34}{36}, \frac{32}{36}$. **7.** $\frac{10}{20}, \frac{15}{20}, \frac{16}{20}, \frac{18}{20}, \frac{12}{20}, \frac{14}{20}, \frac{8}{20}$.

Page 48.—2. $\frac{1}{8}, \frac{1}{4}, \frac{1}{3}, \frac{2}{3}, \frac{2}{5}, \frac{2}{3}, \frac{13}{15}, \frac{2}{3}, \frac{7}{9}, \frac{3}{4}$. **3.** $\frac{4}{5}, \frac{5}{6}, \frac{9}{10}, \frac{6}{7}, \frac{5}{6}, \frac{7}{8}, \frac{6}{7}, \frac{5}{6}, \frac{4}{5}, \frac{15}{16}$. **4.** $\frac{2}{3}, \frac{1}{4}, \frac{5}{11}, \frac{3}{4}, \frac{8}{9}, \frac{2}{3}, \frac{8}{9}, \frac{5}{6}, \frac{7}{8}, \frac{5}{8}$. **5.** $\frac{4}{7}, \frac{10}{13}, \frac{56}{83}, \frac{1}{2}, \frac{6}{7}, \frac{8}{9}, \frac{4}{5}, \frac{7}{8}, \frac{5}{6}, \frac{1}{10}, \frac{4}{5}$. **6.** $\frac{28}{25}$. **7.** $\frac{9}{22}$. **8.** $\frac{33}{38}$. **9.** $\frac{1}{8}$. **10.** $\frac{1}{12}$. **11.** $\frac{2}{7}$. **12.** $\frac{2}{3}$. **13.** $\frac{2}{3}$. **14.** $\frac{3}{7}$. **15.** $\frac{5}{11}$. **16.** $\frac{267}{899}$. **17.** $\frac{1}{4}$. **18.** $\frac{5}{6}$. **19.** $\frac{6}{7}$. **20.** $\frac{12}{13}$. **21.** $\frac{7}{8}$. **22.** $\frac{5}{6}$. **23.** $\frac{1}{3}$. **24.** $\frac{5}{7}$. **25.** $\frac{4}{7}$. **26.** $\frac{3}{8}$. **27.** $\frac{5}{7}$. **28.** $\frac{7}{9}$. **29.** $\frac{8}{9}$. **30.** $\frac{5}{8}$. **31.** $\frac{9}{10}$. **32.** $\frac{1}{4}$. **33.** $\frac{4}{5}$. **34.** $\frac{17}{19}$. **35.** $\frac{29}{36}$. **36.** $\frac{21}{32}$. **37.** $\frac{17}{21}$. **38.** $\frac{2}{5}$. **39.** $\frac{2}{3}$. **40.** $\frac{7}{8}$. **41.** $\frac{1}{3}$.

Page 49.—1. 56. **2.** 1,202. **3.** 4. **4.** $27\frac{8}{15}$. **5.** $20\frac{7}{13}$. **6.** $25\frac{1}{40}$. **7.** $21\frac{8}{8}$. **8.** $41\frac{10}{18}$. **9.** $80\frac{11}{108}$. **10.** 27. **11.** $32\frac{5}{18}$. **12.** $21\frac{3}{4}$. **13.** $4\frac{1}{7}$. **14.** $5\frac{1}{2}$. **15.** $5\frac{5}{8}$. **16.** $13\frac{3}{8}$. **17.** 14. **18.** $13\frac{1}{4}$. **19.** $4\frac{1}{6}$ lb. **20.** $22\frac{5}{6}$. **2.** $\frac{30}{6}$. **3.** $\frac{3}{3}, \frac{12}{3}, \frac{15}{3}, \frac{18}{3}, \frac{21}{3}, \frac{24}{3}$.

Page 50.—4. $\frac{6}{6}, \frac{30}{6}, \frac{42}{6}, \frac{48}{6}, \frac{54}{6}, \frac{60}{6}$. **5.** $\frac{15}{15}, \frac{30}{15}, \frac{75}{15}, \frac{105}{15}, \frac{120}{15}, \frac{150}{15}$. **7.** $\frac{17}{7}$. **8.** $\frac{54}{7}$. **9.** $\frac{33}{5}$. **10.** $\frac{27}{6}$. **11.** $5\frac{1}{2}$ yd. **12.** $1\frac{13}{18}$. **13.** $2\frac{29}{4}$. **14.** $3\frac{38}{5}$. **15.** $\frac{50}{8}$. **16.** $1\frac{14}{14}$. **17.** $\frac{75}{2}$. **18.** $\frac{100}{3}$. **19.** $\frac{200}{3}$. **20.** $\frac{279}{4}$. **21.** $\frac{493}{2}$. **22.** $1\frac{75}{2}$. **23.** $4\frac{31}{5}$. **24.** $17\frac{8}{6}$. **25.** $1\frac{607}{12}$. **26.** $2\frac{93}{3}$. **27.** $5\frac{42}{7}$. **28.** $3\frac{1}{3}$. **29.** $1\frac{879}{10}$. **30.** $\frac{800}{9}$. **2.** $\frac{2}{8}, \frac{4}{8}, \frac{1}{8}$. **3.** $\frac{10}{20}, \frac{5}{20}, \frac{12}{20}$. **4.** $\frac{30}{50}, \frac{18}{50}, \frac{25}{50}$.

Page 51.—5. $\frac{8}{24}, \frac{21}{24}, \frac{22}{24}$. **6.** $\frac{12}{18}, \frac{15}{18}, \frac{14}{18}$. **7.** $\frac{8}{16}, \frac{6}{16}, \frac{7}{16}$. **8.** $\frac{6}{36}, \frac{16}{36}, \frac{21}{36}$. **9.** $\frac{9}{30}, \frac{2}{30}, \frac{28}{30}$. **10.** $\frac{15}{21}, \frac{14}{21}, \frac{18}{21}$. **11.** $\frac{16}{18}, \frac{17}{18}, \frac{15}{18}$. **12.** $\frac{11}{44}, \frac{12}{44}, \frac{10}{44}$. **13.** $\frac{18}{30}, \frac{27}{30}, \frac{28}{30}$. **14.** $\frac{18}{18}, \frac{12}{18}, \frac{15}{18}, \frac{16}{18}$. **15.** $\frac{28}{42}, \frac{18}{42}, \frac{39}{42}, \frac{6}{42}, \frac{42}{42}$. **16.** $\frac{24}{96}, \frac{12}{96}, \frac{32}{96}, \frac{18}{96}, \frac{15}{96}, \frac{66}{96}$. **17.** $\frac{25}{30}, \frac{21}{30}, \frac{24}{30}, \frac{14}{30}, \frac{30}{30}$. **18.** $\frac{18}{42}, \frac{14}{42}, \frac{6}{42}, \frac{20}{42}, \frac{42}{42}$. **19.** $\frac{12}{36}, \frac{6}{36}, \frac{8}{36}, \frac{10}{36}, \frac{21}{36}$. **20.** $\frac{11}{44}, \frac{12}{44}, \frac{10}{44}, \frac{8}{44}, \frac{44}{44}$. **21.** $\frac{24}{84}, \frac{35}{84}, \frac{28}{84}, \frac{21}{84}$. **22.** $\frac{10}{32}, \frac{24}{32}, \frac{28}{32}, \frac{30}{32}, \frac{29}{32}$. **23.** $2\frac{6}{12}, 3\frac{4}{12}, 5\frac{8}{12}$. **24.** $\frac{21}{24}, \frac{24}{24}, \frac{20}{24}, 1\frac{12}{24}$. **25.** $\frac{18}{84}, \frac{17}{18}, 5\frac{3}{18}, 2\frac{12}{18}$. **26.** $\frac{10}{20}, \frac{18}{20}, \frac{12}{20}, \frac{16}{20}$. **27.** $37\frac{4}{8}, 3\frac{3}{8}, 3\frac{1}{8}$. **28.** $\frac{27}{80}, 4\frac{2}{30}, \frac{29}{30}, \frac{25}{30}$. **29.** $9\frac{6}{12}, 10\frac{8}{12}, 12\frac{3}{12}, 15\frac{9}{12}$. **10.** $1\frac{5}{12}$. **11.** $1\frac{8}{16}$. **12.** $1\frac{1}{14}$.

Page 52.—13. $1\frac{18}{18}$. **14.** $\frac{16}{21}$. **15.** $1\frac{5}{6}$. **16.** $1\frac{115}{134}$. **17.** $1\frac{18}{30}$. **18.** $2\frac{1}{3}$. **19.** $2\frac{9}{16}$. **20.** $2\frac{1}{4}$. **21.** $1\frac{23}{30}$. **22.** $1\frac{18}{30}$. **23.** $\frac{6}{9}$. **24.** $1\frac{1}{210}$. **25.** 2. **26.** $2\frac{3}{4}$. **27.** $\frac{53}{8}$. **28.** $1\frac{5}{63}$. **29.** $2\frac{33}{72}$. **31.** $110\frac{19}{24}$. **32.** $758\frac{3}{7}$. **33.** $56\frac{11}{24}$. **34.** $188\frac{5}{48}$. **35.** $1090\frac{19}{30}$. **36.** $288\frac{5}{21}$. **37.** $166\frac{1}{7}$. **38.** $332\frac{3}{8}$. **39.** $216\frac{21}{56}$. **40.** $205\frac{13}{24}$. **41.** $149\frac{23}{60}$. **42.** $350\frac{5}{84}$.

Page 53.—1. $4\frac{5}{8}$ bu. **2.** $118\frac{19}{24}$ lb. **3.** $963\frac{1}{8}$ bu. **4.** $13\frac{3}{4}$. **5.** $50\frac{1}{2}$ gal. **6.** $407\frac{7}{8}$ A. **7.** $30\frac{1}{4}$. **8.** $28\frac{1}{2}$ yr. **9.** 37 da. **10.** Works $\frac{5}{12}$ da.; sleeps $\frac{1}{3}$ da. **11.** $147\frac{4}{24}$ bu. **12.** $74\frac{23}{24}$ mi. **13.** $1\frac{1}{15}$.

Page 54.—14. $20\frac{1}{10}$ yd. **15.** $\frac{1}{4}$ dif. **16.** $17\frac{1}{20}$. **17.** $1\frac{1}{25}$. **18.** $8\frac{1}{4}$ ft. **19.** $23\frac{7}{24}$. **20.** $165\frac{1}{2}$ rd. **21.** $22\frac{5}{12}$ in. **22.** $109\frac{3}{8}$ ft. **23.** $63\frac{3}{8}$ ft. **24.** $23\frac{1}{24}$ mi. **25.** $6\frac{1}{8}$. **26.** $49.

Page 56.—1. $\frac{5}{24}$. **2.** $\frac{1}{12}$. **3.** $\frac{11}{18}$. **4.** $\frac{7}{24}$. **5.** $\frac{5}{18}$. **6.** $\frac{3}{5}$. **7.** $\frac{599}{1000}$. **8.** $\frac{9}{50}$. **9.** $\frac{10}{36}$. **10.** $\frac{13}{68}$. **11.** 0. **12.** $\frac{5}{90}$. **13.** $\frac{13}{18}$. **14.** $\frac{17}{24}$. **15.** $\frac{13}{18}$. **16.** $\frac{5}{42}$. **17.** $\frac{1}{15}$. **18.** $\frac{1}{50}$. **19.** $\frac{85}{85}$. **20.** $\frac{85}{1008}$. **21.** $\frac{11}{105}$. **22.** $\frac{1}{8}$. **23.** $\frac{109}{252}$. **24.** $\frac{43}{210}$. **25.** $\frac{1}{14}$. **26.** $\frac{23}{63}$. **28.** $11\frac{7}{8}$. **29.** $8\frac{5}{8}$. **30.** $\frac{7}{8}$. **31.** $6\frac{17}{18}$. **32.** $4\frac{43}{72}$. **33.** $1\frac{13}{18}$. **34.** $3\frac{3}{16}$. **35.** $5\frac{7}{16}$. **36.** $127\frac{3}{77}$. **37.** $9\frac{19}{20}$. **38.** $4\frac{233}{400}$. **39.** $8\frac{1}{8}$. **40.** $9\frac{1}{12}$. **41.** $8\frac{11}{24}$. **42.** $10\frac{19}{90}$. **43.** $4\frac{7}{9}$. **44.** $6\frac{17}{40}$. **45.** $12\frac{19}{36}$. **46.** $1\frac{11}{12}$. **47.** $13\frac{13}{48}$. **48.** $55\frac{5}{24}$. **49.** $10\frac{5}{12}$. **50.** $6\frac{3}{4}$. **51.** $9\frac{11}{48}$.

Page 57.—2. $249\frac{1}{2}$. **4.** $358\frac{7}{16}$. **6.** $17\frac{23}{25}$. **8.** *a.* Increases value by $\frac{1}{72}$; *b.* increases any proper fr.; *c.* decreases any proper fr. **10.** $57\frac{9}{20}$ mi. **12.** $13\frac{5}{8}$ yr.

Page 58.– 14. $11\frac{1}{2}$. 16. 541 lb. 18. 30c. 20. $\frac{1}{4}$ hr. 22. $\frac{5}{8}$ mi. 24. $92\frac{1}{2}$ A. 26. $\frac{1}{12}$. 28. 6 words more.

Page 59.– 1. $1\frac{1}{4}$. 2. $6\frac{1}{8}$. 3. $3\frac{1}{2}$. 4. $8\frac{1}{12}$. 5. $6\frac{5}{8}$. 6. $1\frac{1}{2}$. 7. $4\frac{1}{2}$. 8. $9\frac{7}{8}$. 9. 24. 10. $6\frac{3}{4}$. 11. $6\frac{1}{12}$. 12. $5\frac{1}{12}$. 13. $11\frac{1}{2}$. 14. $16\frac{1}{2}$. 15. $18\frac{1}{4}$. 16. $18\frac{1}{2}$. 18. $8\frac{1}{2}$. 19. 8. 20. $11\frac{1}{2}$. 21. $14\frac{1}{4}$. 22. $12\frac{1}{2}$. 23. $84\frac{1}{2}$. 24. $15\frac{1}{12}$. 25. 30. 26. $35\frac{1}{2}$. 27. $4\frac{1}{2}$. 28. 25. 29. $15\frac{1}{2}$. 30. $15\frac{1}{4}$. 31. $2\frac{1}{12}$. 32. 36. 33. 21. 34. $37\frac{1}{2}$. 35. 42. 36. 45. 37. $13\frac{1}{12}$. 38. $60\frac{1}{2}$. 39. $54\frac{1}{2}$. 40. 35. 41. $15\frac{1}{12}$.

Page 60. 45. $\frac{1}{12}$. 46. $\frac{1}{2}$. 47. $24\frac{1}{2}$. 48. 33. 49. $\frac{1}{2}$. 50. $\frac{1}{2}$. 51. $46\frac{1}{4}$. 52. $21\frac{1}{12}$. 53. $15. 54. 49 ft. 55. $8\frac{1}{2}$. 56. $30\frac{1}{12}$. 57. 111. 58. $111\frac{1}{4}$. 59. $892\frac{1}{2}$. 60. 20 yd. 61. 24 bu. 62. $50. 63. 10 mo. 2. 650 mi. 3. $98. 4. $8.46.

Page 61.—5. $40.50. 6. 19.58\frac{1}{2}$. 7. $18.56. 8. 33.82\frac{1}{2}$. 9. 11.50\frac{1}{2}$. 10. 5.687\frac{1}{2}$. 2. 50\frac{1}{2}$. 3. $412\frac{1}{2}$ ft. 4. 2.541 sq. yd. 5. 9.161$\frac{1}{2}$ da. 6. $154\frac{1}{2}$ A

Page 62.—7. $598\frac{1}{2}$ gal. 8. $81\frac{1}{2}$ yd. 9. $81\frac{1}{2}$ A 10. 249\frac{1}{2}$. 11. $295\frac{1}{2}$ mi. 1. 846. 2. $106\frac{1}{2}$ 3. 948. 4. 132.029. 5. 47.701$\frac{1}{2}$. 6. 4.081. 7. 6.966. 8. 2.886$\frac{1}{2}$. 9. 4,283,839. 10. 2,103$\frac{1}{2}$. 11. 17.919. 12. 8.900. 13. 74.583$\frac{1}{2}$. 14. 15.570. 15. 28.064$\frac{1}{2}$. 16. 43.200. 17. 77.672$\frac{1}{2}$. 18. 82.417$\frac{1}{2}$. 19. 55.000. 20. 34.745$\frac{1}{2}$. 21. 92.616. 22. 669.093$\frac{1}{2}$. 23. 21.555$\frac{1}{2}$. 24. 32.500$\frac{1}{2}$. 25. 59.065$\frac{1}{2}$ 26. 156.886$\frac{1}{2}$. 27 66.110. 28. 123.744$\frac{1}{2}$. 29. 83.600. 30. 519.-067$\frac{1}{2}$. 31. 66.867. 32. 40.987$\frac{1}{2}$. 33. 315.465$\frac{1}{2}$.

Page 63.– 3. $\frac{1}{16}$. 4. $\frac{1}{12}$. 5. $\frac{1}{12}$. 6. $\frac{1}{2}$. 7. $\frac{1}{12}$. 8. $\frac{1}{12}$. 9. $\frac{1}{16}$. 10. $\frac{1}{2}$ 11. $\frac{1}{12}$. 12. $\frac{1}{12}$. 13. $\frac{1}{12}$. 14. $\frac{1}{2}$. 15. $\frac{1}{2}$.

Page 64.—16. $\frac{1}{2}$. 17. $\frac{1}{12}$. 18. $\frac{1}{2}$. 19. $\frac{1}{2}$. 20. $\frac{1}{2}$. 21. $\frac{1}{2}$. 22. $\frac{1}{2}$. 23. $\frac{1}{2}$ 24. $\frac{1}{2}$. 25. $\frac{1}{2}$. 26. $\frac{1}{2}$. 28. 94$\frac{1}{12}$. 29. 466$\frac{1}{2}$. 30. 120$\frac{1}{12}$. 31. 825. 32. 26$\frac{1}{2}$ 33. 28. 34. 18$\frac{1}{2}$. 35. 4.893$\frac{1}{2}$. 36. 30$\frac{1}{2}$. 37. 18. 38. 44. 39. 138$\frac{1}{2}$. 1. 31$\frac{1}{2}$ in. 2. 199\frac{1}{12}$. 3. 30\frac{1}{2}$. 4. 1.51\frac{1}{2}$. 5. 702$\frac{1}{2}$ sq. 6. $6.45. 7. $\frac{1}{2}$ sold. $\frac{1}{2}$ left.

Page 65.–8. 1.92\frac{1}{2}$. 9. $400. 10. $117. 11. 19$\frac{1}{2}$ mi. 12. 836$\frac{1}{2}$ T. 13. 1.43\frac{1}{2}$. 14. $1.50. 15. 1$\frac{1}{2}$.

Page 66. 1. $\frac{5}{8}$. 2. $\frac{5}{8}$. 3. $\frac{5}{8}$. 4. $\frac{5}{8}$. 5. $\frac{5}{8}$. 6. $\frac{5}{8}$. 7. $\frac{5}{8}$. 8. $\frac{5}{8}$ 9. $\frac{5}{8}$. 10. $\frac{5}{8}$. 11. $\frac{5}{8}$. 12. $\frac{5}{8}$. 13. $\frac{5}{8}$. 14. $\frac{5}{8}$. 15. $\frac{5}{8}$. 16. $\frac{5}{8}$. 17. $\frac{5}{8}$ 18. $\frac{5}{8}$. 19. $\frac{5}{8}$. 20. $\frac{5}{8}$. 22. 18. 23. 12$\frac{1}{2}$. 24. 82. 25. 49. 26. 30. 27. 82. 28. 81. 29. 70. 30. $\frac{5}{8}$. 31. $\frac{5}{8}$. 32. $\frac{5}{8}$. 33. 48. 35. $\frac{5}{8}$. 36. $2\frac{1}{2}$. 37. $1\frac{5}{8}$. 38. $\frac{5}{8}$. 39. $\frac{5}{8}$. 40. $\frac{5}{8}$. 41. $1\frac{5}{8}$. 42. 20. 43. $1\frac{5}{8}$. 44. $\frac{5}{8}$. 45. $\frac{5}{8}$. 46. $\frac{5}{8}$. 47. $\frac{5}{8}$. 48. 44. 49. $\frac{5}{8}$. 50. 180. 51. $\frac{5}{8}$. 52. 30.

Page 67.—54. $3\frac{1}{2}$. 55. $\frac{5}{8}$. 56. $\frac{5}{8}$. 57. 2. 58. $\frac{1}{2}$. 59. $4\frac{1}{2}$. 60. $\frac{5}{8}$. 61. $15\frac{5}{8}$. 62. $10\frac{1}{2}$. 63. $\frac{5}{8}$. 64. 12. 65. $\frac{5}{8}$. 1. 8 da. 2. $12\frac{1}{2}$ yd. 3. $\frac{5}{8}$. 4. $113\frac{1}{2}$ in. 5. 80. 6. $120.000. 7. $5\frac{1}{2}$ T. 8. 8 bu.

Page 68. 9. $8\frac{1}{2}$ ft. 10. $2.800. 11. $.56. 12. $8\frac{1}{2}$. 13. 2. 14. 1. 15. $\frac{1}{2}$. 16. 30 bu. 17. $2\frac{1}{2}$ doz. 18. $2\frac{1}{2}$ doz. 19. $\frac{5}{8}$. 20. $\frac{5}{8}$. 21. $\frac{1}{2}$. 23. 75c.

Page 69.– 24. 7 da. 25. 25 da. 26. 40c. 27. $2.70. 28. $8.25. 29. $1. 30. 152 31. 427. 32. $ .25 33. 360. 34. $800. 35. 110 A. 36. 4.12\frac{1}{2}$. 37. 4 bu. 38. 28 ft. long; 19$\frac{1}{2}$ ft. wide.

Page 70.–8. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. $\frac{1}{2}$. 9. $\frac{1}{2}$. 10. $\frac{1}{2}$. 11. $\frac{1}{2}$. 12. $\frac{1}{2}$. 13. $\frac{1}{2}$.

Page 71.—14. $2; $\frac{2}{3}$; $\frac{2}{5}$. 15. $\frac{1}{6}$. 16. $\frac{7}{20}$. 18. $\frac{3}{4}\frac{2}{9}\frac{2}{3}$. 19. $\frac{1}{4}\frac{5}{0}$. 20. $\frac{2}{10}\frac{1}{4}$. 21. $\frac{1}{10}$. 22. $\frac{1}{4}$. 23. $\frac{1}{3}\frac{2}{1}\frac{3}{9}$. 25. $\frac{4}{5}$. 26. $\frac{6}{6}\frac{5}{9}$. 27. $\frac{2}{4}\frac{0}{5}$. 28. $\frac{1}{2}\frac{7}{1}$. 29. $\frac{2}{2}\frac{2}{4}\frac{4}{8}$. 30. $\frac{1}{2}$. 31. $\frac{1}{1}\frac{6}{6}$. 32. $\frac{3}{17}$. 33. $\frac{5}{6}$. 34. $\frac{1}{1}\frac{4}{3}$. 35. $\frac{5}{8}$. 36. $\frac{1}{5}$.

Page 72.—2. $\frac{1}{2}\frac{5}{8}$. 3. $\frac{1}{3}$. 4. $\frac{1}{2}$. 5. $\frac{4}{17}$. 6. $\frac{2}{8}\frac{5}{8}$. 7. $\frac{3}{5}$. 8. $\frac{7}{20}$. 9. $\frac{7}{10}$. 10. $\frac{3}{5}$. 11. $\frac{1}{2}$. 12. $\frac{3}{8}$. 13. $\frac{7}{11}$. 14. $\frac{7}{3}\frac{7}{6}$; $\frac{7}{1}\frac{7}{8}$. 15. $\frac{1}{2}$.

Page 73.—3. 108. 4. $2\frac{1}{2}$. 5. 155. 6. $4\frac{1}{5}$. 7. 1199. 8. 4. 9. $3\frac{3}{10}$. 10. 169. 11. $2\frac{3}{4}$. 12. 1575. 13. $1\frac{11}{17}$. 14. 100. 15. $1\frac{1}{2}$. 16. $\frac{3}{7}$. 17. 6. 18. $8\frac{1}{3}$. 19. 640. 20. $24\frac{1}{2}$. 21. $31\frac{1}{4}$. 22. 176. 23. 784. 24. $27\frac{1}{3}\frac{3}{3}$. 25. $8\frac{1}{3}$. 26. $94\frac{1}{2}$. 27. 105 mi. 28. $\$\frac{7}{10}$. 29. 325.

Page 74.—30. $.80. 31. $4,200. 32. $2.61. 33. 50. 34. $5,500. 35. $1.60. 36. 28 yr. 37. 49 yr. 38. $\$2\frac{1}{10}$. 39. 30. 40. $4.20. 41. 18. 42. $.50. 43. 8. 44. 300 A.

Page 75.—45. 35. 46. 42 yr. 47. 25. 48. $210. 49. 30. 50. 33. 51. $\frac{2}{2}\frac{3}{5}$. 52. $\frac{1}{3}$. 53. $\frac{3}{5}$. 54. $\frac{9}{1}\frac{0}{6}$. 55. 36. 56. $\frac{6}{7}$. 57. 80,000. 58. $\frac{3}{4}$. 59. 36. 60. 16 yr.; 12 yr.

Page 76.—2. $8. 3. 6¢. 4. 7¢. 5. 25¢. 6. $14. 7. 12. 8. 22,785. 9. 28 bu. 10. $12,500. 11. 70,200. 12. 18 yr. 13. $42. 14. $2,250.

Page 77.—3. 2. 4. 12. 5. 5. 7. $\frac{1}{4}\frac{5}{4}$. 8. $1\frac{1}{8}$. 9. 5. 10. 5. 11. 16. 12. 16. 13. 0. 14. 2. 15. $\frac{2}{3}\frac{3}{5}$. 16. $\frac{5}{7}\frac{9}{7}$. 17. $1\frac{8}{1}\frac{5}{5}$. 18. $\frac{3}{2}\frac{1}{1}$. 19. $\frac{4}{15}$. 20. $\frac{7}{3}\frac{7}{0}$. 21. 4. 22. 2. 23. 100. 24. 5. 25. $1\frac{1}{1}\frac{3}{3}$. 26. $\frac{3}{4}\frac{3}{0}$. 27. $\frac{2}{5}$.

Page 78.—1. $\frac{2}{3}\frac{7}{2}$. 2. $\frac{1}{1}\frac{6}{9}$. 3. $1\frac{1}{6}$. 4. $1\frac{3}{8}$. 5. $1\frac{3}{8}$. 6. $\frac{7}{1}\frac{7}{8}$. 7. $2\frac{3}{7}\frac{3}{7}$.

Page 79.—8. $\frac{7}{1}\frac{7}{1}\frac{8}{9}$. 9. $\frac{6}{1}\frac{3}{2}\frac{3}{1}$. 10. $\frac{3}{4}$. 11. 16. 12. $16\frac{4}{1}\frac{1}{7}$. 13. $\frac{1}{2}\frac{1}{8}$. 14. $1\frac{4}{1}\frac{1}{2}\frac{1}{0}$. 15. $1\frac{1}{8}$. 16. $\frac{2}{3}\frac{1}{8}$. 17. $1\frac{1}{2}\frac{1}{0}$. 18. $3\frac{3}{7}$. 19. $\frac{4}{1}\frac{4}{0}\frac{0}{3}$. 1. $\frac{1}{4}$. 2. $\frac{3}{5}$. 3. $\frac{8}{1}\frac{1}{0}$. 4. 70 bu. 5. 9 yr. 6. 9 yr. 7. 5. 8. 41. 9. $97\frac{1}{8}$ mi.

Page 80.—10. $5\frac{3}{4}$ yd. 11. 12 da. 12. $1,024,000. 13. $18,666\frac{2}{3}$. 14. $\$52\frac{5}{1}\frac{1}{2}$. 15. $\frac{1}{8}$. 16. *a*. 27 yr.; *b*. 30 yr. 17. A, 25 yr.; B, 20 yr. 18. 10. 19. $1\frac{2}{1}\frac{3}{5}$. 20. $3\frac{7}{1}\frac{7}{6}$. 21. 15. 22. 16. 23. 24. 24. $4\frac{3}{6}\frac{3}{1}$. 25. $\frac{7}{1}\frac{7}{0}$.

Page 81.—26. Diminished by $\frac{1}{2}\frac{1}{0}$. 27. Increased by $\frac{1}{3}\frac{1}{0}$. 28. Increased by $\frac{1}{2}\frac{1}{0}$. 29. Diminished by $\frac{1}{8}\frac{1}{0}$. 30. $37,500. 31. $4,900. 32. 80 pear; 100 apple; 160 peach; 40 plum; 400 all. 33. $\frac{7}{1}\frac{7}{0}$. 34. $48,600. 35. John, 75¢; James, 50¢. 36. $600; $2,000. 37. $50. 38. 160.

Page 82.—39. $210. 40. 40. 41. 84. 42. $20.25. 43. 75¢. 44. $15.20. 45. 60¢. 46. $5. 47. $360. 48. 20 yr. 49. 24. 50. $\$5\frac{5}{8}$. 51. 77. 52. $1\frac{3}{8}$. 53. $58\frac{1}{3}$. 54. $7\frac{1}{2}\frac{8}{3}\frac{3}{9}$ hr. 55. $1,125. 56. $2.50 gain. 57. $343.

Page 83.—58. 20. 59. $176. 60. $27.80. 61. $5. 62. $1.25. 63. 30¢. 64. $12.50. 65. 60 mi. 66. 6 hr. 67. Sarah, 45¢; Mary, 30¢. 68. $150.

Page 88.—1. 2. 2. 3. 3. 2. 4. 3. 5. 2. 6. 2. 7. 2. 8. 8. 9. 8. 10. 3. 11. 3. 12. $\frac{2}{8}\frac{7}{5}$. 13. $3\frac{1}{5}$. 14. 4. 15. $12\frac{3}{8}$. 16. 6. 17. 8. 18. $7\frac{1}{3}$. 19. $\frac{3}{4}$. 20. $\frac{1}{1}\frac{2}{6}\frac{1}{8}$. 21. 2. 22. $\frac{1}{2}$. 23. 3. 24. 7. 25. 40. 26. 12. 27. 9. 28. 12. 29. 9. 30. 100. 31. 4. 32. 8. 33. $9\frac{1}{8}$. 34. $\frac{5}{1}\frac{5}{6}$. 35. $2\frac{1}{2}$. 36. 4. 37. $\frac{8}{1}\frac{8}{1}$. 38. $1\frac{1}{5}$. 39. $\frac{1}{3}$. 40. $\frac{3}{4}$. 41. 7. 42. $\frac{1}{3}$. 43. 20. 44. 4.

Page 89.—2. $63. 3. $12.60. 4. $49. 5. $81. 6. $93.75. 7. $20. 8. $5. 9. $5.20; $10.40. 10. $10. 11. $255. 12. $250. 13. $.60. 14. 180 mi. 15. $.80. 16. $2,000. 17. $6. 18. $2,000. 19. $.81. 20. $17.50. 21. $5. 22. $13.50.

Page 90.—1. $6.00. **2.** $54. **3.** $4.50. **4.** $450. **5.** $10,800. **6.** $15. **7.** $30. **8.** $.80; $1; $1.60. **9.** $63. **10.** $6; $3.60; $7.20. **11.** $300; $480. **12.** $8.40. **13.** $6; $3; $10; $12. **14.** 210 yd. **15.** 135 da. **16.** 3,840 rd. **17.** $80. **18.** 162. **19.** $25.

Page 96.—1. $12.30. **2.** $40.19. **3.** $396.80.

Page 103.—2. 111.586. **3.** 95.3458. **4.** 92.075 yd. **5.** 42.4475. **6.** 72.77 in. **7.** 110.4825. **8.** $4530.95. **9.** 4112.4046. **10.** 181.84534. **11.** 169.2733. **12.** 460.486.

Page 104.—13. 865.063 mi. **14.** 109.961 rd. **15.** 38.006 A. **16.** .18. **17.** $2.54. **18.** .9195. **19.** 288.699 bu.; 96.233 bu. **20.** 460.69 A. **21.** 122.99 bu. **22.** 16.823. **23.** $9.68. **24.** 603.66 mi.

Page 105.—2. *a.* .18; *b.* 92.75. **3.** *a.* .477; *b.* 389.375. **4.** *a.* .857; *b.* 54.894. **5.** *a.* 4.33; *b.* 46.517. **6.** *a.* 23.06; *b.* 2854.44. **7.** *a.* 2.175; *b.* 121.25. **8.** *a.* .0375; *b.* $21.75. **9.** *a.* .0184; *b.* 3.485. **10.** *a.* 982.9359; *b.* 24.5. **11.** *a.* 5.1037; *b.* 1.65. **12.** *a.* 211.9; *b.* 14.25. **13.** *a.* $.35; *b.* 4.75. **14.** *a.* 4900.735; *b.* 745.103 **15.** *a.* 22.75; *b.* 2.789. **16.** *a.* 4.83; *b.* 525.25. **17.** *a.* .08174; *b.* 746.25. **18** *a.* .64834; *b.* 1.81. **19.** *a.* 7.82342; *b.* 3.185. **20.** *a.* 22.5045; *b.* .0655. **21.** .1 bu. **22.** .58.

Page 106.—23. .335. **24.** .33. **25.** $.25. **26.** 3.25 mi. **27.** 14.18 T. **28.** 38.47 bu. **29.** 1.171.

Page 107.—1. .0396. **2.** 5.04. **3.** .0015. **4.** 7.175. **5.** 7.2. **6.** 5.695. **7.** .02231. **8.** 376.875. **9.** 35.7. **10.** 2606.325. **11.** 345. **12.** 3750. **13.** 57.7044. **14.** 136.3411. **15.** 28.2744. **16.** .25. **17.** .1. **18.** .01. **19.** 72.9975. **20.** 37.8125. **21.** .336. **22.** 785.4. **23.** .0000288. **24.** 2906.25. **25.** 7865.6. **26.** .38315. **27.** 483,528.771. **28.** 311.8104. **29.** 9.8568. **30.** 274.6375. **31.** $2,851.875. **32.** 330. mi. **33.** $66.389. **34.** $289.50. **35.** 5708.65 in. **36.** 62,500 lb. **37.** $49.40. **38.** $4.05.

Page 108.—39. 121.1 mi. **40.** 247.8 knots. **41.** 161 T. **42.** $45. **43.** 6. **44.** $69.4375. **45.** 42.84 in. **46.** $45. **47.** $68. **48.** 1.

Page 109.—3. 12.02. **4.** 5.07. **5.** .009. **6.** .006. **7.** 1609. **8.** .078$\frac{1}{4}$. **9.** .08. **10.** 1900. **11.** .019. **12.** .002. **13.** 150. **14.** 2260. **15.** 4.5 **16.** .5. **17.** .1805+. **18.** .7854. **19.** .04. **20.** .009+. **21.** .0279. **22.** 9.312. **23.** .152168. **24.** .86. **25.** .0704. **26.** $.50. **27.** $.22 **28.** 1453 lb.

Page 110.—29. 9 lb. **30.** 30 sq. rd. **31.** 50 bu. **32.** 22.83+ hr. **33.** 120.98+ mi. **34.** 28.877+ hr. **35.** $1.08+. **36.** 603.42. **37.** 18+. **38.** 20 boxes. **39.** 10 gal. **40.** 325. **41.** $21.39.

Page 111.—1. 4. **2.** 1.13. **3.** *a.* $2,775; *b.* $21,275. **4.** *a.* $10; *b.* $\frac{1}{2}$; *c.* $\frac{1}{3}$. **5.** $312.50. **6.** $88.99. **7.** 805.125 mi. **8.** $292.30. **9.** $5.92. **10.** 27.29+ hr. **11.** *a.* 15¢; *b.* $\frac{1}{4}$; *c.* .25. **12.** 3.003. **13.** Brown sold 14.4 A. more. **14.** .69.

Page 117.—2. $162.50. **3.** $55.02. **4.** 170 A. **5.** 275.2. **6.** 45. **7.** $3.80. **8.** 1,492.64 bu. **9.** $53.85. **10.** 80 men. **11.** $6.94. **12.** 1.5742. **13.** $1.64.

Page 118.—15. $4. **16.** 120 sheep. **17.** $13.59. **18.** 96 bu. **19.** $57.11. **20.** $322. **21.** $18.70. **22.** $297. **23.** 36 yr. **24.** 18. **25.** 240 bu. **26.** 4 doz. **27.** 20. **28.** 126 A. **30.** $506. **31.** 6,600. **32.** 50.25. **33.** 406 bu. **35.** $2. **36.** $4.50.

Page 119.—37. 4.2. **38.** $16.80. **39.** $13.14. **40.** $.06. **41.** $9. **2.** 50%. **3.** 66⅔%. **4.** 66⅔%. **5.** 66⅔%. **6.** 80%. **7.** 87½%. **8.** 90%. **9.** 75%. **10.** 80%. **11.** 50%. **12.** 25%. **13.** 90%. **14.** 40%. **15.** 25%. **16.** 33⅓%. **17.** 30%. **18.** 38⅜%. **19.** 16⅔%. **20.** 83⅓%. **21.** 96%.

Page 120.—22. 76%+. **23.** 75%. **24.** 30%. **25.** 25%. **26.** 51⅔%. **27.** 30%. **28.** 62½%. **29.** 66⅔%. **30.** 83⅓%. **31.** 20%. **32.** 83⅓%. **33.** 30%. **34.** Son, 33⅓%; daughter, 22⅔%.

Page 121.—25. 2,250. **26.** 20,100. **27.** 3,708. **28.** 4,887.

Page 122.—29. $7,867.35. **30.** 1,728 ft. **31.** $2,896. **32.** 3,876. **33.** $8,687. **34.** 3,750 ft. **35.** $120. **36.** 120. **37.** $75.

Page 123.—2. $135. **3.** $125. **4.** $130. **5.** $234.24. **6.** $7,728. **7.** $1,026.60. **8.** $1,184.25. **9.** $328.35. **10.** $1,836. **11.** $3,344.46. **12.** $706.50. **13.** $229.90. **15.** $437.40. **16.** $1,398.13. **17.** $961.40. **18.** $967.44. **19.** $754.13. **20.** $851.72. **21.** $144.50. **22.** $1,056.53. **23.** $410.63.

Page 124.—24. $1,191.03. **25.** $748.13. **26.** $1,042.50.
2. $288.17. **3.** $286.27. **4.** $99.75. **5.** $510.38. **6.** $244.28. **7.** $63.68. **8.** $422.84. **9.** $388.67. **10.** $6.25. **11.** $550.76. **12.** $2,992.26. **13.** $99.23. **14.** $132. **15.** $108.69.

Page 125.—1. $2,940. **2.** $183.12. **3.** $483.05. **4.** $550.47. **5.** $975.07.
Page 128.—2. 174 in. **3.** 4,342 in. **4.** 127⅞ ft. **5.** 160 rd. **6.** 352,566 in. **7.** 534,715 in.

Page 129.—9. 7 mi. 224 rd. 5 ft. 4 in. **10.** 21 mi. 158 rd. 5 yd. **11.** 4 mi. 105 rd. 14 ft. 6 in. **12.** 15 mi. 187 rd. 10 in. **13.** 16 mi. 11 rd. 3 yd. 1 ft. 6 in. **14.** 10 mi. 21 rd. 7 ft. 3 in. **16.** 213 rd. 5 ft. 6 in. **17.** 8 ft. 3 in. **18.** 203 rd. 3 yd. 1 ft. 6 in. **20.** $\frac{14}{28}$. **21.** $\frac{11}{12}$. **22.** $\frac{48}{56}$. **23.** $\frac{4471}{7020}$.

Page 130.—1. 3,648 ft. **2.** 120 ft. **3.** $\frac{8}{88}$. **4.** 4,000 ft. **5.** 192 ft.

Page 132.—2. 250,204⅜ sq. ft. **3.** 304.923 sq. ft. **4.** 5,685.307 sq. in. **5.** 550,500 sq. rd. **6.** 96,800 sq. yd. **7.** 1,089,000 sq. ft. **8.** 4,900,500 sq. in. **9.** 55,794,915 sq. ft. **10.** ⅞ A. **11.** ½ sq. mi. **12.** $\frac{100}{121}$ sq. rd. **13.** 30 sq. mi. 59 A.

Page 133.—15. 11 A. 61 sq. rd. 8 sq. yd. 108 sq. in. **16.** 10 A. 9 sq. rd. 3 sq. yd. 6 sq. ft. 108 sq. in. **17.** 51 A. 107 sq. rd. **18.** 20 sq. rd. 5 sq. ft. 136 sq. in. **19.** 8 sq. mi. 283 A. 52 sq. rd. **20.** 23,040 A.; 640 A.

Page 136.—3. 490 sq. ft. **4.** 830 sq. yd. **5.** 79⅛ sq. ft. **6.** 1200 sq. rd. **7.** 10 sq. ft. **8.** 38$\frac{28}{36}$ sq. rd. **9.** 200 sq. ft. **10.** 81 sq. rd. **11.** 4,830 sq. rd. **12.** 41⅜ sq. ft.

Page 137.—13. 3.000 sq. ft. **14.** 40 A. **15.** 1,900 sq. rd. **16.** 2,000 sq. ft. **17.** 658 sq. ft. **18.** 30 sq. ft. **19.** 16 ft. **20.** 30 rd. **21.** 24 rd. **22.** 30¼ sq. yd. **23.** 12,900 sq. ft.

Page 138.—24. 87,600 sq. ft. **25.** $3,036. **26.** $108,000. **27.** 32 rd. **28.** 80 ft. **29.** $187.50. **30.** $1,236. **31.** 120 sq. ft.

Page 140.—1. 45½ sq. ft. **2.** 615 sq. rd. **3.** 39 sq. mi. **4.** 33¾ sq. in. **5.** 183⅜ sq. ft. **6.** 16,706¼ sq. ft. **7.** 153 sq. ft. **8.** 75 sq. rd. **9.** 130 sq. in. **10.** 264 sq. ft. **11.** 120 sq. ft. **12.** 3,187½ sq. rd. **13.** 4,040 sq. rd. **14.** 16 ft. **15.** 40 rd. **16.** 40 rd. **17.** 30 cards. **18.** 20 cards. **19.** 20 cards.

Page 143.—**4.** 62½ sq. ft. **5.** 648 sq. rd. **6.** 10 sq. yd. **7.** 407½ sq. ft. **8.** 371½ sq. ft. **9.** 817$\frac{1}{12}$ sq. ft. **10.** 68$\frac{19}{36}$ sq. rd. **11.** 371$\frac{23}{32}$ sq. ft. **12.** 32,793¾ sq. ft. **13.** 109⅜ sq. yd. **14.** 625 sq. ft. **15.** 660 sq. ft. **16.** 6¼ sq. ft. **17.** 16¼ sq. ft. **18.** 30 sq. ft.; ratio 4 : 1.

Page 144.—**1.** 300 sq. ft. **2.** 6,600 sq. ft.

Page 145.—**3.** 6,800 sq. rd. **4.** 14,375 sq. ft.; 13,125 sq. ft.; 11,875 sq. ft.; 10,625 sq. ft.; 9,375 sq. ft.; 8,125 sq. ft.; 6,875 sq. ft. **5.** 37½ A. **6.** 234 sq. ft.; ratio 1 : 4. **7.** 10 ft. **8.** 63¼ sq. rd.

Page 147.—**1.** 9.4248 ft. **2.** 5.729+ ft. **3.** 10.2102 in. **4.** 25.132 in. **5.** 2.944 ft. **6.** 10 ft. **7.** 243 ft. 7.98 in. **8.** 33.51 ft. **9.** 560.224 yd. **10.** 37.699 in. **11.** 18.849 in. **12.** 0.

Page 148.—**1.** 113.0976 sq. ft. **2.** 78.54 sq. ft. **3.** 38.4846 sq. ft. **4.** 78.54 sq. ft.

Page 149.—**5.** 38.4846 sq. rd. **6.** 38.4846 sq. ft. **7.** 3848.46 sq. ft. **8.** 20106.24 sq. ft. **9.** 452.3904 sq. ft. **10.** 1963.5 sq. ft. **11.** 31.416 sq. ft. **12.** 113.0976 sq. ft. **13.** 23.75835 sq. in. **14.** 3.4336 sq. ft. **15.** 706.86 sq. rd. **17.** Square 21.46 sq. ft. greater. **18.** Circle 28.54 sq. ft. larger. **19.** Diameter 7.0028 ft.; area 38.5 sq. ft. **20.** 7854 sq. mi.

Page 151.—**2.** (1) 8 strips; (2) 6 yd.; (3) 48 yd.; (4) $64.80. **3.** $522. **4.** 42 yd. lengthwise; 45 yd. crosswise. **5.** 44⅓ yd. **6.** 80 yd. **7.** 120 yd. **8.** 28 yd. **9.** 14 yd. **10.** 72 yd. **11.** 25⅝ yd.; $45.21. **12.** 30 sq. yd. **13.** $113.75. **14.** 30 yd. **15.** $64.80. **16.** $57.60.

Page 152.—**1.** 18 rolls. **2.** 48 rolls. **3.** $10.15. **4.** $6.12. **5.** $18.70.

Page 153.—**6.** $9.24. **7.** $6.30. **2.** $32.39. **3.** $29.25.

Page 154.—**4.** $75.29. **5.** $17.40. **6.** $12.60. **7.** $6.27. **8.** $155.56.

Page 155.—**1.** 18 sq. in. **2.** 31¼ sq. ft. **3.** 125 sq. ft. **4.** 4 bundles. **5.** 41 bundles. **6.** $367.50. **7.** $132. **8.** $600. **9.** $135. **10.** $160. **11.** $210.

Page 157.—**1.** 9 cu. ft. **2.** 60 cu. ft. **3.** 576 cu. ft. **4.** 9000 cu. ft.; 45 children. **5.** 125 cu. in.; 216 cu. in. **6.** 21,600 cu. ft. **7.** 40. **8.** 64 cu. in. **9.** *a*. $\frac{1}{27}$ cu. ft.; *b*. 27 brick. **10.** 55,000. **11.** 105,600. **12.** 144 cu. ft. **13.** 720.

Page 158.—**14.** 3,128⅜ cu. yd. **15.** 360 cu. ft. **16.** 666⅔ cu. ft. **17.** 133⅓ cu. yd. **18.** $1,800. **19.** 20$\frac{4}{27}$ cu. yd. **20.** 9,065 cu. in. **21.** 432. **22.** 128 cu. ft. **23.** 9 cords.

Page 159.—**24.** 10 cords. **25.** $297. **26.** $25.78. **1.** 100 cu. ft. **2.** 835,974 cu in. **3.** 112,615 cu. in. **4.** 46,656 cu. in. **5.** 21 cu. yd. 2 cu. ft. 615 cu. in. **6.** 1,023$\frac{1}{27}$ cu. yd. **7.** 466,560. **8.** 2 : 3. **9.** 1 : 12. **10.** 25 : 243. **12.** 3 cu. ft. 648 cu. in. **13.** 846.72 cu. in. **14.** 20 cu. ft. 432 cu. in. **15.** 9 cu. ft. **16.** 777.6 cu. in. **17.** 62 cu. ft. 172.8 cu. in.

Page 160.—**1.** ¾ pk. **2.** ¾ bu. **3.** 10 pk. **4.** 255 pt. **5.** 61 pt. **6.** 15 pt. **7.** 618 qt. **8.** 449 pt. **9.** 232 qt. **10.** 2 bu.

Page 161.—**11.** 7 bu. 1 pk. **12.** 11 bu. 1 pk. 1 pt. **13.** 1075.21 cu. in.; 1612.815 cu. in.; 1433.61⅓ cu. in. **14.** 12. **15.** 101.24. **16.** 5 ft. **17.** 7 in. **18.** 72.3. **19.** 44.9+ bu. **20.** 4821.4 bu. **21.** 10.24 in. **22.** 18.81 in. **23.** .48+ bu.

Page 162.—**1.** $\frac{1}{16}$; $\frac{1}{4}$; $\frac{2}{4}$. **2.** 191 gi. **3.** 17 gi. **4.** 82 pt. **5.** 161 gi. **6.** 2 gal. 1 gi. **7.** 11 gal. 3 qt. 1 pt. **8.** 4 gal. 2 qt. **9.** $57\frac{3}{4}$ cu. in. **10.** 67.2 cu. in. **11.** 9.45 cu. in. **12.** 268.8 cu. in.

Page 163.—**13.** 40. **14.** 42.216. **15.** 168,750. **16.** 173,437.1. **17.** 19. **18.** 3,600. bbl. **19.** 30. **20.** 179.5 bbl. **21.** 520. qt. **22.** $6.65. **23.** $13.28. **24.** 45 gal. **25.** 302.4 cu. in. **26.** 9.45 cu. in.; .14+; .16+. **27.** Berries 268.8; milk 231; onion sets 268.8; vinegar 231.

Page 165.—**1.** $\frac{1}{4}$ lb.; $\frac{1}{2}$ lb.; $\frac{3}{4}$ lb. **2.** $1.54. **3.** $90. **4.** 75,210 oz. **5.** 1,119 oz. **6.** 579 lb. **7.** 30,411 oz.

Page 166.—**8.** 51,800. **9.** 1 T. 5 cwt. **10.** 3 T. 15 cwt. 80 lb. **11.** 4,000 T. **12.** 2 T. 16 cwt. 25 lb. **14.** 1,360 oz. **15.** 1,500 lb. **16.** $6.09. **17.** $67.50. **18.** $.04. **19.** $5.20. **20.** 750 bu. **21.** Feathers by 1,240 gr.

Page 167.—**1.** $50. **2.** $62\frac{1}{2}$ bu. **3.** $\frac{1}{4}$ bu. **4.** $.90. **5.** $8. **6.** $71\frac{3}{4}$ bu. **7.** $80. **8.** $85.10. **9.** $93.45. **10.** $.50. **11.** $1.50. **12.** $31.67. **13.** $5.50 loss. **14.** 60 bu. **15.** $.24. **16.** $360.

Page 169.—**1.** 831 hr. **2.** 20,926 sec. **3.** 865,200 sec. **4.** 59,774 hr. **5.** 544,800 sec. **6.** 13,440 min. **7.** 17 hr. 36 min. 1 sec. **8.** 8 wk. 2 da. 14 hr. 11 min.

Page 171.—**3.** $16\frac{2}{3}$. **4.** 12. **5.** $10\frac{1}{2}$. **6.** 40. **7.** $12\frac{1}{2}$. **8.** 15. **9.** 9. **10.** $13\frac{3}{4}$. **11.** 18. **12.** $6\frac{1}{4}$. **13.** 18. **14.** 16. **15.** 160. **16.** 128. **17.** 5280. **18.** 432. **19.** 316.8.

Page 172.—**1.** 100 s.; 150 s.; 500 s. **2.** 60 d.; 78 d.; 150 d. **3.** 25 far.; 50 far., 60 far.; 80 far. **4.** 1,145 d. **5.** 619 s. **6.** £10 6 s. 3 d.

Page 173.—**7.** $1.70. **8.** $17,519.40. **9.** $.32. **10.** £200. **1.** $.772. **2.** $7.72. **3.** $38.60. **4.** 5 fr.; $2\frac{1}{2}$ fr. **5.** $19,300,000,000. **6.** $11.58. **7.** 1,554+ fr. **8.** $135.10.

Page 175.—**1.** 180°; 270°; 135°; 210°; 72°; 120°; 300°; 330°. **2.** 324°; 54°.

Page 176.—**3.** $\frac{1}{12}$; $\frac{1}{8}$; $\frac{1}{6}$; $\frac{1}{4}$; $\frac{3}{8}$; $\frac{1}{2}$. **5.** 72,015 sec. **6.** 1° 6′ 1″. **7.** $\frac{1}{720}$; $\frac{1}{4320}$; $\frac{1}{4500}$. **8.** 2,459″. **9.** $69\frac{4}{9}$ mi. **10.** 6,250 mi. **11.** $277\frac{7}{9}$ mi. **12.** $1.041\frac{2}{3}$ mi. **13.** $347\frac{2}{9}$ mi. **14.** $2,013\frac{3}{9}$ mi. **15.** 52′ 12″. **16.** 4. **17.** 30°. **18.** 90°; 120°; 300°. **19.** 144°. **20.** 288°.

Page 177.—**3.** 7 bu. 3 pk. 2 qt. **4.** 4 hr. 59 min. 7 sec.

Page 178.—**5.** 323 mi. 26 rd. 15 ft. 9 in. **6.** 111 T. 1117 lb. 12 oz. **7.** 1 mi. 19 rd. 15 ft. 4 in. **8.** 18 hr. 11 min. 14 sec. **9.** 1,799 lb. 13 oz. **10.** 12 da. 1 hr. 58 min. 59 sec. **11.** 15° 57′ 5″. **12.** 5 gal. 3 qt. **13.** 61 A. 26 sq. rd. 1 sq. yd. 2 sq. ft. 72 sq. in. **14.** 87 gal. 2 qt. 1 pt. 2 gi. **15.** 83 T. 1,347 lb. 9 oz. **16.** 46 sq. yd. 7 sq. ft. 95 sq. in. **17.** 146 cu. yd. 2 cu. ft. 54 cu. in. **18.** 41 wk. 3 da. 18 hr. 35 min. **19.** 32 cu. yd. 23 cu. ft. 68 cu. in.

Page 179.—**2.** 2 yr. 5 mo. 26 da. **3.** 52 yr. 10 mo. 3 da. **4.** 2 yr. 1 mo. 25 da. **5.** 11 yr. 1 mo. 18 da. **6.** 2 yr. 5 mo. 9 da. **7.** 10 mo. 20 da. **8.** 46 yr. 10 mo. 7 da. **9.** Washington 67 yr. 9 mo. 22 da.; Jefferson 83 yr. 2 mo. 21 da.; Hamilton 47 yr. 6 mo. 1 da.; Webster 70 yr. 9 mo. 6 da.; Clay 75 yr. 2 mo. 17 da. **10.** 65 da.

Page 180.—**2.** 52 mi. 164 rd6 ft. **3.** 132 A 118 sq. rd. 13 sq. yd. 2 sq. ft. 36 sq. in. **4.** 204 bu. 2 pk. 4 qt. 1 pt. **5.** 30 T. 1,925 lb. 10 oz. **6.** 247 da. 22 hr. 6 min. 50 sec. **7.** 262 gal. 2 qt. **8.** 193 cu. yd. 15 cu. ft. 1,488 cu. in. **9.** 34 wk. 6 da. 2 hr. 45 min. **10.** 783° 56′. **11.** £45 2 s. **12.** 284 gal. 2 qt. 1 pt. **13.** 92 bu. 3 qt. 1 pt. **14.** 21 mi. 125 rd. 7 ft. 6 in.

Page 181.—**16.** 4 bu. 2 pk. 5 qt. $1\frac{1}{7}$ pt. **17.** 2 T. 1200 lb. $1\frac{3}{5}$ oz. **18.** 2 mi. 91 rd. $11\frac{5}{8}$ ft. **19.** 2 wk. 6 da. 18 hr. 26 min. 40 sec. **20.** 3 lb. $12\frac{7}{11}$ oz. **21.** 8 gal. 2 qt. $1\frac{5}{8}$ pt. **22.** 10° 30′ 52″. **23.** £2 8 s. 10 d. **24.** 2 bu. 1 pk. 7 qt. $\frac{1}{5}$ pt. **25.** $7\frac{3}{77}$; $2\frac{4}{77}$. **2.** 7. **3.** 5. **4.** 8. **5.** 9. **6.** 9 hr.

Page 182.—**7.** 9. **8.** 21. **9.** 5. **10.** 100. **2.** .25. **3.** .8 mi. **4.** .78125 bu. **5.** .25 wk. **6.** .4848+ rd. **7.** .25 A. **8.** .$66\frac{2}{3}$ cu. yd. **9.** .5. **10.** .5 A. **11.** .4 T. **12.** $\frac{1}{8}$ wk. **13.** $\frac{8}{120}$ da. **14.** $\frac{1}{4}$.

Page 185.—**1.** $\frac{1}{12}$ of circumference, or 30°. **2.** 12. **5.** 60; 720. **6.** 156. **7.** 56 min. **8.** 9:56 A.M. **9.** 7 min. 30 sec. **10.** 3:30 P.M. **11.** 3 hr. 12 min. **12.** 9:25 P.M. **13.** 2:05.

Page 186.—**14.** $36\frac{4}{11}$ mi. **15.** 1892; 1896; 1904; 1908; 1912; 1916; 1920; 1924; 1928. **16.** 1 hr. 10 min. **17.** *a.* 18 mi.; *b.* 1:40 P.M. **18.** 11:35 A.M. **19.** 4:45 P.M. **20.** 2 hr. 30 min. **21.** 4.8 A. **22.** 12 yr. 7 mo. **23.** 23 yr. 1 mo. **24.** 12 yr. 4 mo. 29 da. **25.** 10,080 min. **26.** $26\frac{1}{4}$ mi.

Page 187.—**3.** $9.94. **4.** 15 yr. 10 mo. **5.** .$533\frac{5}{8}$ in.

Page 188.—**1.** 5 bd. ft. **2.** 15 bd. ft. **3.** 27 bd. ft. **4.** 147 bd. ft. **5.** 171 bd. ft. **6.** 720 bd. ft. **7.** $60. **9.** $122.40.

Page 189.—**10.** *a.* 346 bd. ft.; *b.* $56.14. **11.** 750 bd. ft. **12.** $62.40. **13.** $46.76. **14.** 225 bd. ft. **15.** 130.

Page 190.—**16.** 639. **17.** 840. **18.** $\frac{1}{6}$. **19.** $\frac{8}{15}$. **20.** *a.* 540 bd. ft.; *b.* $117.50. **21.** 22 bd. ft.

Page 192.—**2.** $86\frac{1}{4}$ bu. **3.** $75\frac{3}{7}$ bu. **4.** $67\frac{1}{3}$ bu. **5.** $1573\frac{5}{7}$ bu. **6.** 736.3125 gal. **7.** 40 T.

Page 193.—**1.** $1.20. **2.** $1.45.

Page 194.—**4.** $6.10. **5.** $5.60. **6.** $5.04. **7.** $2.85. **8.** $20.55. **9.** $9.52. **11.** $2. **12.** $6. **13.** $3.45.

Page 196.—**1.** $.79. **2.** $.92. **3.** $.34.

Page 197.—**4.** 10¢. **5.** 16¢. **6.** 20¢. **7.** 1200 mi. **9.** $1.45. **11.** 70¢. **12.** 12¢. **13.** 2¢. **14.** 22¢. **15.** 11¢. **16.** 29¢. **17.** 16¢. **18.** 20¢. **19.** No; between 300 and 600 mi. **20.** 34¢.

Page 200.—**1.** 40¢. **2.** $25,000. **3.** $3. **4.** $54.60. **5.** 10. **6.** $648. **7.** 24 marbles. **8.** 615 A. **9.** Frank 40¢; Thomas 50¢. **10.** $1500 and $2000.

Page 201.—**11.** $33. **12.** $11,250. **13.** 2. **14.** 28. **15.** 20 ft. **16.** $3,600. **17.** 20¢. **18.** $9,750. **19.** $1,200. **20.** $\frac{1}{2}\frac{3}{7}$. **21.** 19. **22.** $44\frac{3}{5}$ mi. **23.** $54. **24.** $8,700.

Page 202.—**2.** $33. **3.** 12 trucks. **4.** 3,528 lb. **5.** $432. **6.** $226.80. **8.** 20¢. **9.** 22 mi. **10.** 441 lb. **11.** 44.

Page 203.—12. 18 bu. **13.** 51. **14.** $4.20. **15.** 84. **16.** $183.92. **17.** 24. **19.** 16 hr. 40 min. **20.** 21 da. **21.** 6 hr. 24 min. **22.** 36 da. **23.** 4 min. **24.** 1 min. 24 sec. **25.** 28 min. 30 sec.

Page 204.—26. *a.* $1\frac{1}{2}$ mi.; *b.* 132 ft. **27.** 150 men. **28.** 3 da. **29.** 6 da. **30.** 12 lb. **31.** 5 da. **32.** $7,200. **33.** $5,000. **35.** $\frac{3}{5}$. **36.** $\frac{5}{6}$. **37.** *a.* $2,000; *b.* $200. **38.** 10. **39.** 168 ft.

Page 205.—3. $3\frac{3}{4}$ da. **4.** $6\frac{3}{5}$ min. **5.** $2\frac{4}{45}$ da. **6.** $2\frac{6}{17}$ da. **8.** $\frac{1}{17}$.

Page 206.—9. $\frac{1}{4}$. **10.** $\frac{6}{17}$. **11.** $8,000. **12.** 20. **13.** $\frac{1}{25}$; 4%. **14.** $10. **15.** $.75. **16.** $.50. **17.** $16,000. **18.** *a.* $24; *b.* $36. **19.** 9 ft.; 21.6 ft. **20.** 25 in. **21.** 8 da. **22.** $100 cost of second; $175 cost of third.

Page 207.—1. $5. **2.** $5. **3.** Mr. Riley $5 more. **4.** 5. **5.** $\frac{4}{5}$. **6.** $202.50. **7.** $875. **8.** $65. **9.** $338. **10.** $3. **11.** $1.80.

Page 213.—3. $193.20. **4.** $60. **5.** $477. **6.** 71.57. **7.** $142.50. **8.** $20.30. **9.** 8. **10.** 63. **11.** $25. **12.** $1.50. **13.** $12; $24. **14.** $6.16. **15.** $1,485.

Page 214.—16. 800 lb. lump coal; 1200 lb. slack. **17.** $3360. **18.** 13,377. **19.** $.78. **20.** $571.50. **3.** 25%. **4.** $66\frac{2}{3}$%. **5.** 20%. **6.** $33\frac{1}{3}$%. **7.** $12\frac{1}{2}$%. **8.** 50%. **9.** 50%. **10.** $16\frac{2}{3}$%. **11.** 25%. **12.** $33\frac{1}{3}$%. **13.** 75%. **14.** $33\frac{1}{3}$%.

Page 215.—15. 50%. **19.** $20\frac{5}{8}$%. **1.** $12\frac{1}{2}$%. **2.** 30%. **3.** $12\frac{1}{2}$%. **4.** .5%. **5.** 75%.

Page 216.—6. *a.* $42\frac{1}{2}$%; *b.* $57\frac{1}{2}$%. **7.** 60%. **8.** $6\frac{17}{18}$%. **9.** $33\frac{1}{3}$%. **10.** Won $66\frac{2}{3}$%; lost $33\frac{1}{3}$%. **5.** *a.* 64; *b.* 64.

Page 217.—6. 400. **7.** 600. **8.** 160. **9.** 160. **10.** 300. **11.** $25. **12.** $2.50. **13.** 20 words. **14.** 48 enrolled; 42 present. **15.** $93.75; $108.75. **1.** Food $750; clothing $500; rent $400; books $175; fuel $125; insurance $300; church $250. **2.** $66\frac{2}{3}$%. **3.** $178.

Page 218.—4. Silk dress 15.4+%; serge suit 25.28+%. **5.** Shoes 9.4+%; hats 5.6+%. **6.** Bacon, increase 15¢; 75% increase: bread, increase 5¢; 100% increase: butter, increase 40¢; $133\frac{1}{3}$% increase: eggs, increase 35¢; $116\frac{2}{3}$% increase: flour, increase 85¢; $106\frac{1}{4}$% increase: potatoes, increase 35¢; $87\frac{1}{2}$% increase: sugar, increase 15¢; 300% increase. **7.** Sugar; 300%. **8.** 80 bu.

Page 219.—9. $7.70; 592+%. **10.** 10%. **16.** 18¢ per lb. **17.** 20. **18.** Corn $\frac{1}{3}$; corn land $\frac{1}{2}$ of cotton; cotton land twice corn land. **19.** $3\frac{13}{15}$%; $2722\frac{2}{3}$%.

Page 220.—1. Commission $425; farm $8075. **2.** $66.15; $595.35. **3.** $67.26; $773.49. **4.** $2.50; $2252.50.

Page 221.—5. Com. $52.50; pro. $997.50. **6.** 8%; pro. $345. **7.** Com. $1.25; pro. $998.75. **8.** Com. $150; pro. $2250. **9.** 10%; pro. $6516. **10.** Com. $40; $\frac{1}{4}$%. **11.** Com. $40.58; pro. $284.06. **12.** Sales $460; pro. $391. **13.** Sales $10000; .5%. **14.** Com. $2500; pro. $17500. **15.** Com. $84.56; pro. $750.54. **16.** $12,190.75.

Page 222.—17. $54. **18.** Com. $162; pro. $869. **19.** Com. $787.50; profit $462.50. **20.** *a.* $\frac{1}{8}$¢ per bu. is better; *b.* $3. **21.** $1200. **22.** Investment $4785.71; com. $239.29. **23.** Cost $200; com. $5.

Page 223.—1. $25.50. **2.** $32.40. **3.** $55.965. **4.** $2. **5.** $25. **6.** $11.25. **7.** $630. **8.** 20%. **9.** $100. **10.** $42. **11.** 25%. **12.** 50%. **13.** $15.75. **14.** $23. **15.** $18. **16.** $28.

Page 224.—17. 25%. **18.** $33\frac{1}{3}$%. **19.** $9.60. **20.** $12\frac{1}{2}$%. **21.** 20% **22.** $262.50. **23.** $1.218. **24.** Com. $10; $7190. **25.** $22,053.85. **26.** $177.60. **27.** $600. **28.** $525. **29.** $4.

Page 225.—1. Profit $1200; gain per cent of cost, 20%; gain per cent selling price, $16\frac{2}{3}$%. **2.** Profit $900; per cent total cost, $14\frac{2}{7}$%; per cent selling price, $12\frac{1}{2}$%. **3.** Profit on cost, $14\frac{2}{7}$%; profit on selling price, $12\frac{1}{2}$%. **4.** Profit, $1.08; profit on net cost, 60%; profit on total cost, $42\frac{6}{7}$%; profit on selling price, 30%. **5.** Profit 20%; net profit on cost, 10%.

Page 226.—6. $2.25. **7.** 20% discount; 25% saving. **8.** 50% discount; 100% saving. **9.** $1.37. **10.** $27\frac{1}{11}$%. **11.** Neither. **12.** $32 loss. **13.** $12 loss.

Page 228.—2. $62.89. **3.** $1980. **4.** $515.85. **5.** $1600.35.

Page 229.—1. $18; $.60. **2.** Cereals $20\frac{5}{8}$%; milk $13\frac{5}{8}$%; meat $23\frac{11}{18}$%; vegetables $13\frac{3}{8}$%; fruit $6\frac{17}{18}$%; fats $11\frac{1}{3}$%; sugar $3\frac{17}{36}$%; luxuries $6\frac{1}{4}$%. **3.** $42\frac{8}{9}$%. **4.** Cereals $16\frac{2}{3}$%; milk $15\frac{3}{16}$%; meat 25%; vegetables $12\frac{1}{2}$%; fruits $9\frac{1}{16}$%; fats $11\frac{1}{3}$%; sugar $3\frac{5}{72}$%; luxuries $5\frac{5}{9}$%. **5.** *a.* $46\frac{3}{7}$%; *b.* 50%. **6.** $35\frac{5}{7}$%.

Page 230.—1. $35.20. **2.** $5,850. **3.** 1,000 lb. **4.** 2 tons poor road, 6 tons good road. **5.** (a) $2,346\frac{3}{8}$ cu. yd. (b) $5,280. **6.** $13\frac{1}{3}$ years; **7.** 8,125 mi. **8.** 374 mi.

Page 231.—9. (a) 2%; (b) 41,160 mi. **10.** $40. **11.** 3 sets, $225. **12.** $150. **13.** $32\frac{1}{7}$%. **14.** $14,784. **15.** 9 hours. **16.** $4,752. **17.** $66\frac{2}{3}$%.

Page 235.—4. $27. **5.** $112.50. **6.** $540. **7.** $960. **8.** $1000. **9.** $104. **10.** $585. **11.** $135. **12.** $130. **13.** $61.25. **14.** $275.20. **15.** $70. **16.** $336. **17.** $95.55. **18.** $113.90. **19.** $103.50. **20.** $25. **21.** $92.50. **22.** $888. **23.** $1224. **24.** $60.48. **25.** $102. **26.** $100. **27.** $100. **28.** $100. **29.** $100. **30.** $29.18.

Page 236.—2. $112.79. **3.** $172.50. **4.** $80.14. **5.** $1040. **6.** $1449. **7.** $660. **8.** $9744. **9.** $940.70. **10.** $2236. **11.** $1308. **12.** $845.21. **13.** $1,058.88. **14.** $748.75. **15.** $171.25. **16.** $1140.75. **17.** $468. **18.** $200.70. **19.** $806.93. **20.** $603.75. **21.** $783.75. **22.** $415. **23.** $147.50. **24.** $13.32.

Page 237.—3. $642. **4.** $1044.44. **5.** $94.20. **6.** $77.66. **7.** $200.

Page 238.—3. $169.20. **4.** $93.

Page 239.—5. $40.80. **6.** $93. **7.** $70.80. **8.** $21.25. **9.** $66. **10.** $2.81. **11.** $2.25. **12.** $27. **13.** $.80. **14.** $43.68. **15.** $103.59. **16.** $38.03. **17.** $8.25. **18.** $22.50. **19.** $.38. **20.** $14.40. **6.** $5. **7.** $15. **8.** $7.70. **9.** $3.50.

Page 241.—2. $42.19. **3.** $66.87. **4.** $65.42. **5.** $12.25. **6.** $20.42. **7.** $48.99. **8.** $12.92. **9.** $1.19. **10.** $24. **2.** $500. **3.** $600. **4.** $600. **5.** $400. **6.** $450. **7.** $631.17. **8.** $258.57. **9.** $200. **10.** $62.96.

Page 242.—2. 1 yr. 6 mo. **3.** 4 yr. 1 mo. 3 da. **4.** 3 yr. **5.** 5 yr. 4 mo. **6.** 9 mo. **7.** 6 yr. 8 mo. **8.** 1 yr. 4 mo. **9.** 1 yr. 8 mo. 8 da. **10.** 2 yr. 2 mo. 19 da. **2.** 9%. **3.** 6%. **4.** $1\frac{1}{2}$%. **5.** 3.6%. **6.** 4%. **7.** $5\frac{5}{8}$%. **8.** 10%. **9.** $3\frac{1}{3}$%. **10.** $2\frac{1}{2}$%.

Page 244.—7. $1012.50. **8.** $636 end first year; $672 end second year; $708 end third year; $216 total interest. **9.** $312. **10.** $91.20.

Page 246.—1. $.20. **2.** $20.15. **3.** $20.45. **4.** 10 wk. **5.** 15 mo.; $1. interest. **6.** $2.50. **7.** $.50.

Page 247.—8. Net profit 15¢; net profit of gross cost 21⅜%; net profit of selling price 17$\frac{1}{17}$%. **9.** Net profit 60¢; net profit on gross cost 6⅔%; net profit of selling price 6$\frac{1}{4}$%. **10.** $95.33.

Page 249.—1. Protein 14%; fat 16+%; carbohydrates 70—%. **2.** *a.* 15 grams; *b.* 17.25 grams; *c.* 75 grams. **3.** *a.* 40 grams; *b.* 46 grams; *c.* 200 grams. **4.** *a.* 75 grams; *b.* 86.25 grams; *c.* 375 grams. **5.** *a.* 90 grams; *b.* 103.5 grams; *c.* 450 grams.

Page 250.—1-6. *a.* 11.88%; *b.* 8.09%; *c.* 1.74%; *d.* 73.5%. *e.* 4.76%. **7.** 14.09%. **8.** 3.73%. **9.** 10.35%. **10.** 6.48%. **11.** 60%. **12.** 45%. **13.** 45. **14.** 192.

Page 252.—1. $112.50. **2.** $281.25. **3.** 2$\frac{1}{4}$%. **4.** $72. **5.** $90. **6.** $75. **7.** $273. **8.** $99.90. **9.** $317.06. **10.** $209. **11.** 50 mills. **12.** $325. **13.** 5 mills; $32.50.

Page 253.—14. $4630. **15.** $218. **21.** $291,701.44. **22.** $52,784.07. **23.** $65.63; $11.88.

Page 255.—1. $150. **2.** $2329.20. **3.** $52.50. **4.** $69. **5.** $81.38. **6.** $126, $24; 19+%. **7.** $8. **8.** 42$\frac{8}{7}$%. **9.** $2,129,881.95. **10.** $2800. **11.** $3.19.

Page 256.—12. $25.68. **13.** $1837.80. **14.** $3712.50. **15.** $1,596,000. **16.** $13,077.50. **17.** $17,539. **18.** $8.44. **19.** *a.* $12.50; *b.* $240. **20.** $1212.50. **21.** $1420. **22.** $571.50. **23.** $1428.60.

Page 257.—1. $95.04. **2.** $26. **3.** $114. **4.** $228. **5.** $22.

Page 258.—1. $70. **2.** High grade beef by nearly ½¢ per lb. **3.** $95. **4.** Dressed wt. by $19.20.

Page 259.—5. Dressed wt. by 34¢. **6.** $58.32. **7.** 20¢. **8.** 61%. **9.** 19% chuck; 16$\frac{44}{61}$% round steak; 13$\frac{17}{61}$% porterhouse; 3$\frac{7}{61}$% neck; 15$\frac{45}{61}$% plate; 3$\frac{17}{61}$% shank; 4$\frac{28}{61}$% sirloin. **11.** 55%.

Page 260.—12. 224 lb. **13.** 80%. **14.** $23.38 dressed wt. **15.** 12½¢. **1.** $250. **2.** $3375. **3.** 3½¢. **4.** 21⅔¢. **5.** 2½¢.

Page 261.—6. 30¢. **7.** $2000. **8.** $300. **9.** 33⅓¢. **10.** *a.* 13⅜ bales; *b.* $2720; *c.* $344; *d.* 1195 lb.

Page 262.—11. 81,600. **12.** *a.* $2453.85; *b.* $981.54; *c.* $10,473.75. **13.** *a.* $77.84; *b.* 18$\frac{7}{52}$%; *c.* 27+%.

Page 264.—2. $540.23. **3.** First payment $22; 20 payments. **4.** Second payment $21.90; third payment $21.80.

Page 266.—2. $131.26. **3.** $6.26. **4.** $625.44; $17.16.

Page 267.—1. ½%.

Page 268.—2. $10.13. 3. Ship $450; cargo $281.25. 4. $\frac{5}{8}\%$. 5. $18. 6. $540. 7. Mr. Smith $180.63; Mr. Black $120.42; difference $60.21. 8. $\frac{5}{12}\%$. 9. $65.

Page 269.—10. $102. 11. $2.71; $2.36.

Page 271.—1. $143.76. 2. 20-payment life, $197.88; 20-year endowment, $304.44; 20-year term, $87.66. 3. $3,013. 4. 55 years. 5. 47.09%. 6. 140 : 247. 7. 5.074%. 8. $58.53. 9. $276.30.

Page 272.—1. 12.8¢; 58.7%. 2. 18.5¢; 97.36%. 3. 28.5¢; 91.34%. 4. 6¢; 75%. 5. 26.7¢; 86.68%. 6. 23.6¢; 157.33%. 7. 27.1¢; 111.06%. 8. 4.3¢; 148.27%. 9. 2.9¢; 103.57%. 10. 4.9¢; 98%. 11. $1.28; 154.21%. 12. $1.15; 94.65%. 13. $7.35; 67.43%. 14. $15.75; 217.24%. 15. $0.26; 52%.

Page 275.—1. 14.4 A. 2. $296.40; 53.2A. 3. $81.; 8.1 A. 4. $30.60.

Page 276.—5. $144; 16.2 A. 6. 40 hens; $8.50.

Page 277.—1. $36.20.

Page 278.—2. $29.20. 3. The 1-12-2 by $1.40. 4. $25.60. 5. $28. 6. $263.40. 7. $96.60. 8. 24 T.; $120. 9. $15.625. 10. $36. 11. $107.57. 12. $19.22. 13. Lot I, 51¢; Lot II, 55¢; Lot II by 4¢.

Page 279.—14. Lot I, $1.20; Lot II, $1.09. 15. Lot I, 69¢; Lot II, 54¢. 16. Cotton seed meal. 17. 28 lb. nitrogen; 5.6 lb. phos. acid; 7.5 lb. potash. 18. $23.45. 19. $9.70.

Page 280.—1. $1.75. 2. $63. 3. $120. 4. 24%. 5. $324.

Page 281.—6. $5.40. 7. $723.60. 8. $16.88. 9. The same. 10. $200.

Page 282.—2. $313.40. 3. $11.65

Page 283.—6. $6. 7. $6.36. 8. $9.07. 9. $3.68. 10. $25.59. 11. $21.33. 12. $23.47. 13. $52. 14. $8.27. 15. $2.80. 16. $3.25. 17. $10. 18. $95. 19. $3.17. 20. $9.67.

Page 284.—23. $4.20. 24. $10.80. 25. $10.40. 26. $25. 27. $51. 28. $13.84. 29. $5.85. 30. $18.70.

Page 290.—1. 60 da.; 80¢ dis.; $59.20 proceeds. 2. 121 da.; $1.51 dis.; $73.49 pro. 3. 45 da.; 94¢ dis.; $99.06 pro. 4. 134 da.; $3.35 dis.; $146.65 pro. 5. 51 da.; $3.40 dis.; $296.60 pro. 6. 143 da.; $3.81 dis.; $156.19 pro. 7. 93 da.; $2.71 dis.; $172.29 pro. 8. 113 da.; $4.71 dis.; $195.29 pro. 9. 119 da.; $15.87 dis.; $584.13 pro. 10. 151 da.; $7.55 dis.; $292.45 pro. 11. $300; bank gets $6.

Page 291.—15. $623. 16. $1045.03. 17. $1513.53. 18. $196.45; $15.55. 19. $2500. 20. $849.01.

Page 296.—1. Premium $1.25; $501.25.

Page 297.—3. $6.38. 6. $856.63. 7. *a*. 10¢; *b*. 30¢; *c*. 18¢; *d*. 3¢. 8. $600.60. 11. $740.85.

Page 299.—1. $300 each. 2. Souder $450; Cox $300; Sanders $150. 3. Souder 333\frac{1}{3}$; Cox 266\frac{2}{3}$; Sanders $200. 4. A 184\frac{1}{13}$; B 230\frac{10}{13}$; C $200? D 884\frac{5}{13}$. 6. $\frac{1}{13}$.

Page 300.—7. $50. 8. $250. 9. *a.* 8⅓%. *b.* $4.16⅔; $25; $41.66⅔.

Page 304.—1. $6009.38. 2. $2227.50. 3. $4470. 4. $5550. 5. $8100. 7. $47925. 8. $75 loss. 9. $150.

Page 305.—10. $87.50. 11. $2856.25. 12. 10½%. 13. $1300. 14. 11 7/13%. 15. $2575. 16. 4%. 17. $675; $54. 18. Common by .44+%.

Page 306.—4. U. S. 4's reg. by 1.78%. 5. $811.25. 6. $5; 6.16+%.

Page 307.—7. $423.75; 5.9+%. 8. Cent. Leath. by 1 1/50%. 9. U. S. Steel by ⅛+%. 10. 12. 1. $4.25 for 1st, 3rd, and 4th; $4.00 for 2nd; $4.75 for 5th.

Page 308.—2. 1st, 60; 2nd, 50; 3rd, 20; 4th, 40; 5th, 8. 3. 1st, $2.125; 2nd, $2; 3rd, $2.125; 4th, $2.125; 5th, $2.375. 4. $119. 6. $589.11. 7. Fourth, nearly 3/50%. 8. Texas Co. by 1.68+%. 10. 4.3+%. 11. 4.32+%.

Page 314.—8. Land $3000; improvements $2500. 9. $100. 10. $130; $65. 11. $4545; $4484.94. 12. $75. 13. $48.75. 15. $55.; $10.

Page 317.—4. *a.* 7′6″; 3′0″; *b.* 2′6″x5′; 3′0″x5′0″; 1′6″x2′6″; *c.* 8′6″; *d.* 5′0″; 2′6″. 5. 417 sq. ft. 6. $13.67. 7. $14.30. 8. $13.10. 9. $32.89. 10. 10′. 14. 480 bd. ft. 15. $22.80.

Page 318.—3. 1,730 bd. ft. 5. 16 squares. 6. $219.05.

Page 319.—3. *a.* 335½ bd. ft.; *b.* 1.4 squares. 4. $28.74. 5. 22.

Page 320.—1. 64 cu. in.; 27. 2. 7040. 3. 18,304.

Page 321.—4. $302.02. 5. $114.40. 6. $22.88. 7. $57.20. 8. $496.50. 9. $323.56. 10. $172.94. 11. $111.22. 12. $194.44. 13. $3,240.74. 14. 2,625 cu. ft.

Page 322.—1. 2 hr.; 4 hr.; 5 hr.; 7 hr.; 9 hr.; 10 hr.; 3 hr. 20 min. 3. 3 hr. 14 min. 18 sec. 4. 8 hr. 2 min. 29 sec. 5. 6 hr. 29 min. 11 sec.

Page 323.—6. 11 hr. 43 min. 16 sec. 7. 3 min. 15 sec. 8. 10 hr. 40 min. 31 sec. 9. 7 hr. 41 min. 23 sec. 11. 87° 11′ 30″. 12. 174° 20′. 13. 140° 8′ 45″. 14. 126° 26′ 15″. 15. 64° 23′. 16. 11 A.M.; 1 P.M.; 9 A.M.; 4 P.M.; 7 A.M.; 6 P.M.; 6 A.M.; 5 A.M.; 4 A.M.; 11 P.M.; 1 A.M.

Page 324.—1. 1 hr. 2. 2 hr. 3. 4 hr. 4. 4 hr. 5. 8 hr. 6. 4 hr. 7. 12 hr. 8. 2 hr. 9. 10 hr. 10. 7 hr. 11. 2 hr. 40 min. 12. 3 hr. 20 min.

Page 325.—15. 12 hr. 37 min. 58 3/5 sec. P.M. 1. 4 hr. 53 min. 35⅓ sec. 2. 6 hr. 45 min. 36 sec. 3. 3 hr. 52 min. 2⅔ sec. 4. 3 hr. 13 min. 44⅘ sec.

Page 326.—5. 6 hr. 50 min. 49 1/11 sec. 6. 10 hr. 54 min. ⅛ sec. 7. 5 hr. 8 min. 35⅗ sec. 8. 2 hr. 9 min. 31 3/11 sec. 9. 4 o'clock, 45 min. 38 sec. P.M. 10. 6 o'clock, 20 min. 3 sec. A.M. 11. 76° 20′ 15″. 12. 75°. 13. 58° 45″W. 14. 48° 52′ 30″ East.

Page 328.—1. 8:15 A.M. 2. 4 A.M. 4. 30 min. 5. Set watch forward 1 hr. in crossing each time belt. 6. 9 min. 45 sec. 7. 11 A.M. 1. 3 min. 2. 3 hr. 35⅝ min. 3. 36 2/3 mi. 4. 27.6+ mi. 5. 10:26+ P.M.

Page 330.—4. ⅘. 5. 1¼. 6. ⅝. 7. 3. 8. ⅓. 9. ½. 10. ½. 11. 30. 12. 1/7. 13. ⅜. 14. 100. 15. 3⅓. 16. 10/11. 17. 1/6. 18. ⅛. 19. 1/12.

Page 331.—2. $\frac{8}{75}$. 3. $\frac{104}{133}$. 4. $\frac{4}{7}$. 5. $\frac{808}{875}$. 6. $\frac{75}{1938}$. 7. $\frac{244}{611}$. 8. $\frac{61}{100}$. 9. $\frac{29}{72}$. 10. $\frac{28}{67}$. 2. *a.* 1 : 2; *b.* 1 : 4.

Page 335.—2. 32. 3. 4. 4. 1. 5. 25. 7. 5. 8. 10. 10. $2\frac{2}{3}$. 11. 44. 12. .5. 13. 6. 14. 20. 15. $\frac{13}{18}$. 16. $\frac{1}{9}$. 17. $1\frac{59}{81}$. 18. $\frac{9}{16}$.

Page 338.—8. 40 lb. 9. 105 lb. 1. 60 ft. 3. 518.4 bu. 4. $60. 5. $157.50.

Page 339.—2. David 24; Thomas 15. 3. $40; $45. 4. $500; $750. 5. $625; $375; $500. 6. $.67; $.83; $1. 7. 50 A; 70 A.

Page 340.—9. 21; 28; 35. 10. $5000; $7000; $8000. 11. 20¢; 55¢. 12. 2996; 3210; 3638. 13. $630; $420; $315. 14. 189; 224; 231. 15. $90; $120. 1. A $36; B $40; C $32.

Page 341.—2. $33.88; $30.12. 3. A $14,129.03+; B $15,698.92; C $19,623.66; D $23,548.39. 4. A $1400; B $1125; C $1475. 5. A $2333.33; B $1866.67; C $1400; D $1555.56; E $3344.44. 6. $6750; $8250. 7. A $585.94; B $351.56; C $312.50. 8. A $238.10; B $595.24; C $380.95; D $285.71. 9. A $236.84; B $213.16. 10. A $180; B $180; C $240.

Page 342.—1. $1270 American Can, $310.25 Bendix Aviation, $1847.50 Coca Cola, $426.50 Gen. Asphalt, $957.00 N. C. & St. L. R. R., $187.50 U. S. Rubber.

Page 343.—3. Bendix by $3\frac{1}{3}\%$. 4. $2234.38. 5. Southern R. R. $9\frac{7}{17}\%$, N. Y. Central $4\frac{14}{19}\%$.

Page 344.—1. $226170. 3. $4\frac{3}{8}\%$ on bond at 80, 5% on bond at 110.

Page 345.—1. Beef 2300 lb.; pork 1225 lb.; $579. 2. $1.75. 3. $1.75.

Page 346.—4. $250.

Page 347.—1. 46%. 2. 55%. 3. 28%. 4. 60%. 5. 60%. 6. $62\frac{1}{2}\%$. 7. 80%. 8. 68%. 9. 37%. 10. 30%. 11. 52%. 12. 40%. 13. 70%. 14. 44%. 15. 70%. 16. 40%. 17. 55.9%. 18. $52\frac{1}{2}\%$. 19. 70%. 20. 70%. 21. 46%. 22. Single discount 1%. 23. $45 gain.

Page 348. 2. $1007.36. 3. $1011.20. 4. $450.67.

Page 351.—2. $730.61. 3. $12.19. 4. $44.21. 5. $12.89. 6. $68.40.

Page 360.—2. $59.52 annually; $60.08 semi-annually; $60.32 quarterly; $60.48 monthly. 3. $88.84 annually; $89.60 semi-annually; $89.92 quarterly; $90.24 monthly. 4. $156.88 annually; $158.24 semi-annually; $158.88 quarterly; $159.36 monthly. 5. $1190.40 ordinary life; $1776.80 20 yr. payment; $3137.60 20-yr. endowment. 6. 38.7+%. 7. $1976.69.

Page 361.—8. $3137.60. **9.** $5114.29.

Page 362.—2. $\frac{4}{9}$; $44\frac{4}{9}\%$; 160. **3.** $\frac{5}{9}$; $55\frac{5}{9}\%$; 200. **1.** Ratio 18 to 30; $\frac{3}{5}$; 60%. **2.** Ratio 12 to 30; $\frac{2}{5}$; 40%. **3.** Ratio 12 to 18; $\frac{2}{3}$; $66\frac{2}{3}\%$.

Page 363.—1. *a.* 26%; *b.* 16%; *c.* 12%; *d.* 21%; *e.* $10\frac{3}{5}\%$; *f.* $14\frac{2}{5}\%$. **2.** $61\frac{7}{13}\%$. **3.** $83\frac{1}{3}\%$. **4.** *b.* $\frac{3}{20}$; $\frac{3}{20}$; $\frac{29}{160}$; $\frac{1}{10}$; $\frac{1}{8}$.

Page 365.—1. *b.* Russia 22.7+%; Germany 21.3+%; France 18.5+%; England 12.02 %; Austria 10.69+%; Italy 4.87+%; Turkey 3.34+%; Serbia and Montenegro 1.67+%; Belgium 1.36+%; Roumania 1.33+%; Bulgaria 1.33+%; United States .67+%. *d.* $27\frac{7}{9}\%$. *f.* $86\frac{9}{16}\%$.

Page 367.—4. 1125 lb.; 1500 lb; 16 ft. **5.** 3750 T.

Page 368.—7. *c.* U. S. $\frac{13}{34}$; France $\frac{7}{10}$; England $\frac{12}{65}$; Italy $\frac{1}{2}$; Germany $\frac{49}{85}$. **9.** *b.* $41\frac{2}{3}\%$.

Page 371.—1. Cement $\frac{1}{5}$; sand $\frac{1}{5}$; stone $\frac{1}{2}$. **2.** $4.04\frac{4}{9}$ cu. yd. **3.** 65.52 sacks; 4.867 cu. yd. sand; 7.3+ cu. yd. stone.

Page 372.—4. 120 cu. ft. **5.** 31+; 62+; 93+. **6.** $52.80. **7.** 780 cu. ft. **8.** 200; 400; 600. **9.** $514.80. **10.** 63,360 cu. ft. **11.** Cement $4106\frac{2}{3}$ bbl.; sand 1220.26 cu. yd.; stone 1830.4 cu. yd. **12.** *a.* $17,453.33; *b.* $2745.59; *c.* $4576; *d.* $29024.92. **13.** $7040. **14.** Cement 19,250 sacks; sand, 1,430 cu. yd.; stone, 2,145 cu. yd. **15.** Cement 10,500 sacks; sand 975 cu. yd.; stone, 1,556.25 cu. yd.

Page 373.—16. 307.2 cu. ft. **17.** Cement 7650 sacks; sand 561 cu. yd.; stone 1134.75 cu. yd. **18.** Cement 7140 sacks; sand 663 cu. yd.; stone 1058.25 cu. yd. **19.** 9.36 sq. ft. **20.** 180. **21.** 5.28, or 6 loads. **22.** *a.* 18 sacks; *b.* $54. **23.** $7\frac{5}{9}$ cu. yd. **24.** 7.23. **25.** 46.8 cu. ft.

Page 379.—12. 111. **13.** 89.7. **14.** 251. **15.** 34.3. **16.** .53. **17.** 10.5. **18.** 547. **19.** 636. **20.** 28.7+. **21.** .89. **22.** $\frac{3}{5}$. **23.** $\frac{3}{4}$. **24.** $\frac{5}{7}$. **25.** .829+. **26.** .489+. **27.** .632+.

Page 380.—1. 40 rd. **2.** 71.52+ rd. **3.** 41.036 ft. **4.** 43.35+. **5.** 78.54.

Page 381.—8. First, 4 times as large. **9.** 10 ft. **11.** 2010.624 cu. ft. **12.** 6.11+ft. **1.** 254 yd. **2.** 19.79 + yd. **3.** 4 in. **4.** 419.9 ft. **5.** *a.* 236; *b.* $4\frac{1}{40}$ mi.; *c.* 260 posts.

Page 382.—6. 160 yd. **7.** *a.* 908 yd.; *b.* 29,312 sq. yd.; *c.* $36,640. **8.** *a.* 11 lots; *b.* $4125; *c.* $4,312.50. **9.** $31\frac{3}{5}$ min.

Page 384.—1. 259.8 sq. ft. **2.** 11.3 ft. **3.** 4 A. 52 sq. rd. **4.** 12 sq. ft. **5.** *a.* 6.9+ ft.; *b.* 27.6 sq. ft. **6.** 165.6 sq. ft. **7.** 93.4 + sq. ft. **8.** 193.75 sq. ft. **9.** 17.88 sq. ft.

Page 385.—1. 603.1872 cu. ft. **2.** 220.32 gal. **3.** 264.384 gal. **4.** 40.8 gal. **5.** 30 sq. ft.

Page 386.—6. $6.03+. **7.** 589.05 cu. ft. **2.** 80 cu. in. **3.** 233.2 cu in. **4.** 134.0416 cu. ft. **5.** 300 sq. ft. **6.** 225 sq. in. **7.** 60 sq. yd.

Page 387.—8. 9.4248 cu. in. **9.** $54.45. **1.** 288 sq. ft.

Page 388.—2. 480 sq. ft. 3. 275 cu. ft. 4. 450 cu. ft. 5. 960 sq. ft. 6. 144 sq. ft.

Page 389.—1. 83⅓ cu. ft. 2. 180 sq. ft. 3. 400 cu. ft. 4. 8⅔ cu. yd. 5. 125 sq. ft.

Page 390.—2. 1017.8784 sq. in. 3. $31.42. 4. 50.2656 sq. in. 5. 201,062,400 sq. mi. 6. 201.0624 sq. ft. 7. 14.1372 sq. ft.

Page 391.—8. 65.45 cu. in. 9. 4.1888 cu. ft.

Page 392.—1. 201.0624 cu. in. 2. 56.5488 cu. in. 3. 1.5708 cu. ft.

Page 399.—3. *a*. 144° F.; *b*. 81° F.; *c*. 99° F.; *d*. 23⅔° F. 4. *a*. 25° C.; *b*. 20° C.; *c*. 15° C.; *d*. 45° C.

Page 400.—7. *a*. 10° C.; *b*. 3⅓° C.; *c*. 35° C.; *d*. 30° C. 9. *a*. 59° F.; *b*. 86° F.; *c*. 113.9° F.; *d*. 178.7° F.

Page 408.—1. $20. 2. $5.52. 3. 12.965 Km. 4. 9.634 m. 5. 4.65 Km. 6. 40.486 mi. 7. 10.000. 8. .62+ mi. 9. 177⅛ Km. 10. 7.874+ in.

Page 410.—3. $1482. 4. 120 m. 5. 41.66 sq. m. 6. 1500 sq. m. 7. 2.5 sq. Km. 8. 201.6 ha. 9. 35.4+ in.

Page 411.—11. A. by $1.21.

Page 412.—1. 12 steres. 2. $162. 3. $27.17. 4. 32 cu. m. 5. $0.97.

Page 413.—1. $6.36. 2. 14.136 liters. 3. $681.60. 4. 45.000 l.; 40,950 qt. 5. $0.34 loss per bu. 6. 13,571.712 liters; 3,596.5+ gal.

Page 414.—1. 1386 lb. 2. $0.09 cheaper by T. 3. $1.32. 4. 49½¢.

Page 417.—12. *a*. 25; *b*. 34; *c*. 17; *d*. 48; *e*. 71.

Page 418.—1. 16⅔%. 2. 40¢. 3. $937.50. 4. $15.625. 5. $310. 6. $5040. 7. 82⅔%. 8. 41⅔%; 58⅓%; 75%. 9. 34 sq. yd. 10. $\frac{8}{15}$. 11. .6164. 12. 24 mo. 13. $643,500. 14. ⅝. 15. 3 hr. 14⅔ min. 16. 52⅔° C.

Page 419.—17. C $400; B $800; A $1600. 18. $30; $20; $15. 19. $7; $9. 20. 11.870; 9.505. 21. A 20¢; B 13¢; C 10¢. 22. 137,000; 119,500; 94,500. 23. 1⅞. 24. 60. 25. 504. 26. 15.3144. 28. $\frac{64}{385}$.

Page 420.—29. 88 rd. 30. $100. 31. 5 hr. 9 min. 10 sec. 32. 1⅛. 33. 18¾ min. 34. 585. 35. 1⅛. 36. 2 T. 1 cwt. 37. $153; $102; $34. 38. 21,870. 39. $3.60. 40. 25%. 41. 44%. 42. $76.28. 43. Aug. 1st.

Page 421.—44. 8%. 45. 45. 46. $33.77 more at 5⅔% compound interest. 47. $60. 48. 5.65+. 49. 10 in. 50. 2⅛. 51. N. Y. C. by .083+%. 52. $12. 53. $764.08. 54. 26⅔° C. 55. $52.50. 56. 17⅛ sq. ft. 57. 78¾ ft. 58. 16.8+ gal.

Page 422.—59. 7.7256 sq. in. 60. 83⅓ sec. 61. 4$\frac{6}{11}$%. 62. $192.50. 63. $202.25. 64. Box 102.9 cu. in. larger. 65. 4 yr. 9 mo. 22 da. 66. $37.33. 67. $40.83. 68. $51. 69. 16 A. 140 sq. rd. 70. $1368. 71. 50%. 72. 49¢. 73. 60 ft. 74. 1.4+ in. 75. 320 rd.

Page 423.—76. $242.95. **77.** $123.52. **78.** $1870. **79.** 4.54%. **80.** 5%. **81.** $8,074.29. **82.** 21%. **83.** $40. **84.** $1353.83. **85.** 100 ft. **86.** 35.35+ ft. **87.** 900 sq. ft. **88.** $175,000. **89.** $\frac{5}{14}$. **90.** $\frac{8}{8}$.

Page 424.—91. 840. **92.** 20. **93.** 114 sq. ft. **94.** $35\frac{5}{27}\%$. **95.** $12.80. **96.** 1050 sheep. **97.** $2160.49. **98.** $198.75. **99.** $16.67 loss. **100.** 8 shares; 7.54+%. **101.** 8 ft. 11 in. **102.** 100. **103.** $13\frac{1}{3}$.

Page 425.—3. $14.40. **4.** $\frac{1}{4}$ or 25%; .375 or 37.5%; 1.25 or $\frac{125}{100}$; $\frac{5}{1000}$ or .5%; $\frac{125}{1000}$ or .125. **5.** $49.30. **6.** $165.45. **7.** $37\frac{1}{2}\%$. **8.** 3.631606. 9.223 bbl. **10.** $10. **11.** 10 da.

Page 426.—12. 3 yr. 4 mo. **13.** $12.08. **14.** 14 cu. yd. **15.** 1728. **16.** $28.80. **17.** 4% loss. **18.** 30 da. **19.** $578\frac{4}{7}$ da. **20.** 46. **21.** .0996. **22.** *a.* $17\frac{5}{11}$; *b.* $57.60.

Page 427.—23. $14.56. **24.** $25.92. **25.** $37\frac{1}{2}\%$. **26.** 40 yd. **27.** $3.75. **28.** 10. **29.** $15.63. **30.** $7.50. **31.** 51.051. **32.** 21 A. **33.** 2415.54 lb. **34.** 15 ft. **35.** $40. **36.** 42 yd. **37.** 25 mills.

Page 428.—38. $100; $40; $20. **39.** 750 apples. **40.** $1\frac{28}{80}$. mi. **41.** 13,325. **42.** .6125 bu. **43.** $133.33 loss. **44.** 1800 bbl. **45.** $998.54. **46.** $7.40. **47.** 90. **48.** $24.95. **49.** 62.8+ da. **50.** $13.80.

Page 429.—51. $10,840. **52.** $14,415. **53.** $20. **54.** $800. **55.** 10 mo. **56.** 8%. **57.** $54. **58.** 107+ rd. **59.** $33\frac{1}{3}\%$.

Retail Price	.66
Exchange Price,	.64

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